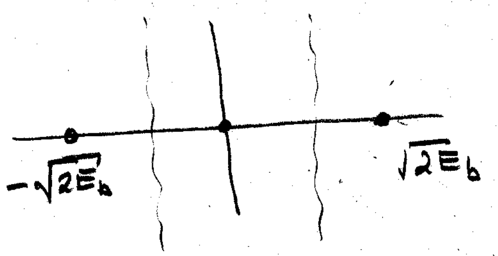
## Performance of the VA

- Is it worth doing? Compare with bit by bit detection
- Bit by bit p 6.4.3

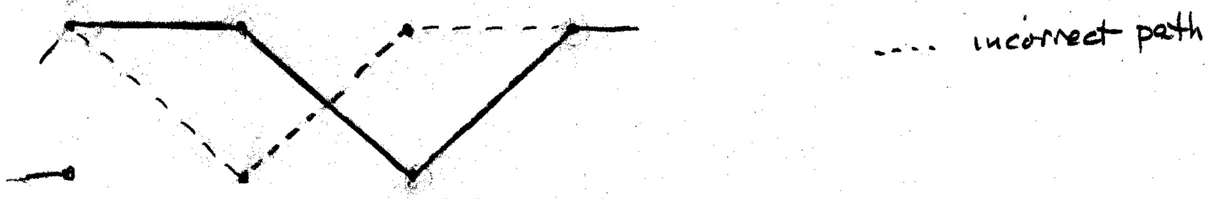



$$P_b = \frac{1}{2}\left(2Q\left(\frac{\sqrt{2E_b}}{2\sqrt{N_0/2}}\right) + Q\left(\frac{\sqrt{2E_b}}{2\sqrt{N_0/2}}\right)\right)$$

$$= \frac{3}{2}Q\left(\sqrt{\gamma_b}\right)$$

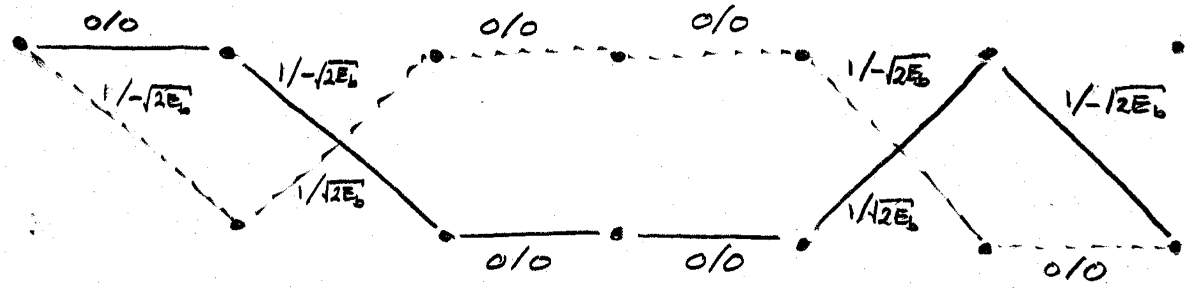
It's 3 dB poorer than binary antipodal.

- Viterbi is more complicated. Suppose that merges have given us the correct state at time k and again at time k+L, but differences in between:



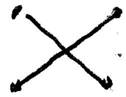

- We call this an "error event". We need:
  - the probability of it happening
  - the number of data bits in error
- Summing over all possible error events gives us a union bound (may be more than one of each length in more general trellises) on prob of event, avg # errors.
- Error events are short compared with inter event intervals, so this approximates the BER.

- Without loss of generality, consider an error event starting in state 0. Here is a typical one.




All events start with a divergence and end with a convergence, each of which produces a bit error.

In between, we have:

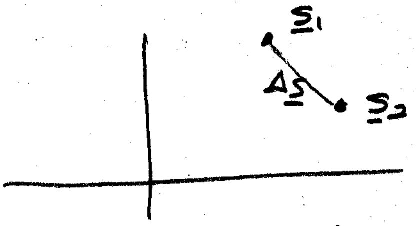
- zero or more parallel branches , none of which causes a bit error
- zero or more crossovers , none of which causes a bit error.

So exactly two bit errors in every possible event.

- What is the probability of that error event? Easier than it looks. Two ways:
  - The two rival paths are points in an N-dimensional signal space, where N is the number of symbols in the event
  - Calculate the metric difference (a random variable because of the noise) and the probability that it is negative.

We'll do both.

- In the signal space approach, we recognize that the paths are signal vectors with components $+\sqrt{2E_b}$, 0, and $-\sqrt{2E_b}$ (following AMI) on the various dimensions. But...




only the separation $|\Delta\underline{s}|$ matters!

The prob of the error event is $Q\left(\frac{|\Delta\underline{s}|}{2}\cdot\frac{1}{\sqrt{N_0/2}}\right)$.
and $|\Delta\underline{s}|$ is the square root of the energy in the difference waveform. For our example,

$$\Delta\underline{s} \rightarrow -\sqrt{2E_b},\ 2\sqrt{2E_b},\ 0,\ 0,\ -2\sqrt{2E_b},\ -\sqrt{2E_b}$$

$$|\Delta\underline{s}|^2 = 2E_b + K\,8E_b + 2E_b \quad \left(\begin{array}{c}K \text{ crossovers} \\ 0 \le K \le N-2\end{array}\right)$$

$$= 4E_b(1+2K)$$

pairwise
$$P_2 = Q\left(\sqrt{\frac{4E_b(1+2K)}{4\cdot N_0/2}}\right) = Q\left(\sqrt{2\gamma_b(1+2K)}\right)$$

Even with $K=0$ crossovers, we are back to the performance of coherent detection.

- Detailed approach based on metric differences. We choose minimum metric, so if

false path metric − true path metric < 0

we have made an error.

The branch metric is $(r-\hat{s})^2$. Denote the true and false signals as $s_i$, $s_i'$. Note $r_i = s_i + n_i$

- True path metric

$$(r_1-s_1)^2+(r_2-s_2)^2+\cdots+(r_N-s_N)^2 = n_1^2+n_2^2+\cdots+n_N^2$$

- False path metric is $(s_i-s_i'+n_i)^2$ on each branch

first branch $(\sqrt{2E_b}+n_1)^2 = 2E_b + 2\sqrt{2E_b}\,n_1 + n_1^2$

crossover br: $(\pm 2\sqrt{2E_b}+n_i)^2 = 8E_b \pm 4\sqrt{2E_b}\,n_i + n_i^2$

parallel br: $(0+n_i)^2 = n_i^2$

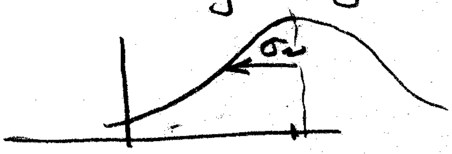
last branch: $(\pm\sqrt{2E_b}+n_N)^2 = 2E_b \pm 2\sqrt{2E_b}\,n_N + n_N^2$

- Difference:

$$4E_b + 8KE_b + \underbrace{2\sqrt{2E_b}\,n_1 \pm 4\sqrt{2E_b}\,n_i \pm \cdots \pm 2\sqrt{2E_b}\,n_N}_{\nu}$$

$$\sigma_\nu^2 = 16E_b(1+2K)\frac{N_0}{2} = 8E_b(1+2K)N_0$$

- Will it go negative?




$4E_b(1+2K$

$$P_2 = Q\left(\frac{4E_b+8KE_b}{\sqrt{8E_b(1+2K)N_0}}\right)$$

$$= Q\left(\sqrt{2\gamma_b(1+2K)}\right)$$

6.5.12

# Relating Pairwise Error Prob to BER

- Normally, we see a long sequence of correct states followed by an error event of some length, starting and ending with a bit error, then back to correct states. Since the average error event length is short compared with average interevent length (normal conditions), we ignore it.

The BER is approximately twice the probability that an error event begins at any correct state.

- Our expression for $P_2$ is only the start, since there are many possible transmitted sequences $\underline{s}$, and error events can be of a variety of lengths.
  - Pairwise events are not necessarily mutually exclusive; more than one false sequence could look better than the true sequence.
  - Union bound.
  - A convenient way to organize the computation is by length

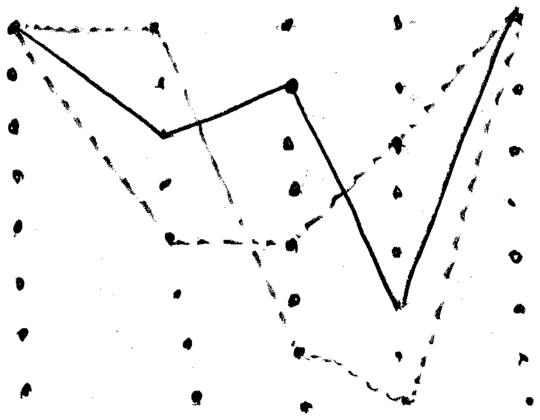
$$\Pr[\text{event}] \leq \sum_{N=2}^{\infty} \Pr[\text{event of length } N]$$

6.5.13

- The Pr[event of length N] depends on the transmitted sequence $\underline{\sigma}_N$, so

$$Pr[E_N] = \sum_{\underline{\sigma}_N} Pr[E_N | \underline{\sigma}_N] \, Pr[\underline{\sigma}_N]$$

- In trellises with many states, there may be several incorrect paths for a $\underline{\sigma}_N$



(suppose all of these are allowable paths)

In this case, we have to union bound $Pr[E_N | \underline{\sigma}_N]$ too:

$$Pr[E_N | \underline{\sigma}_N] \le \sum_{\underline{\sigma}_N'} P_2(\underline{\sigma}_N', \underline{\sigma}_N)$$

We are luckier, with two states

the true sequence determines a unique false sequence

So $Pr[E_N | \underline{\sigma}_N] = Q\left(\sqrt{2\gamma_b(1+2K)}\right)$

- For $Pr[\underline{\sigma}_N]$, all the same, $2^{-N}$. What counts is the number K of crossovers in N-2 symbols. Probability

$$\binom{N-2}{K} 2^{-(N-2)}$$

- Thus $\Pr[E_N] = \sum_{K=0}^{N-2} \binom{N-2}{K} 2^{-(N-2)} Q(\sqrt{2\gamma_b(1+2K)})$

and $P_b \leq 2 \sum_{N=2}^{\infty} \Pr[E_N]$

- For higher SNR, $\Pr[E_N]$ is dominated by the $K=0$ term,

and $P_b \leq 2Q(\sqrt{2\gamma_b}) \underbrace{\sum_{N=2}^{\infty} (N-2)2^{-(N-2)}}_{2}$

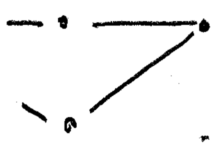
$= 4Q(\sqrt{2\gamma_b})$

So we really have recovered the 3 dB (apart from the factor of 4 - not a significant issue).

How did this happen?

6.5.15

- The characteristics of the VA can be summarized
  - The state transition $\sigma_{k-1}$ to $\sigma_k$ determines what the receiver would measure in the absence of noise
  - The path metric is accumulative, branch by branch, and is non-decreasing
  - At any time, each state has a survivor path leading to it, with the associated metric.
  - A fixed amount of computation in the recursion step means that computation is linear in sequence length
  - Decisions are tentative until a merge, when they can be released, so variable delay before release.
  - Terminate in a known state to bring it to a close



  - Performance is determined by multisymbol "error events", in which an allowable, but incorrect, alternative sequence is selected. Pairwise error prob is easily calculated, then union bound