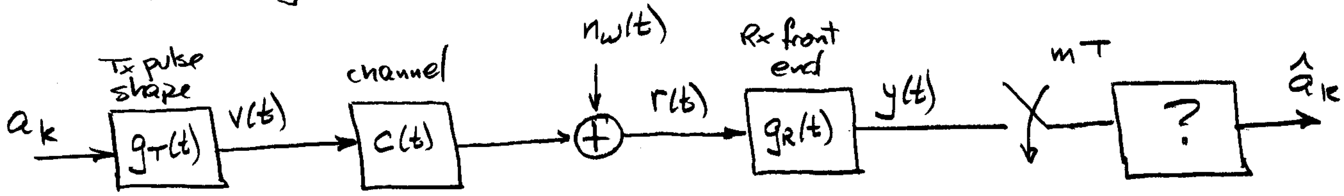
# 6.6 Dealing With ISI – the Equalizer

- In our discussion so far, we have acknowledged the existence of ISI, but we have avoided situations where it happens (except for the "controlled ISI" of duobinary, AMI, etc). Now it's time to face it.
- In 3½ decades of equalizers, hundreds of patents, thousands of journal papers. Some contain advances, most split hairs — but it shows there is much to discuss and lots of interest.
- Our treatment will be superficial, but will establish a few principles and sketch the main classes of solution. You will see
  - general properties of the problem
  - the linear equalizer, both zero forcing and mmse
  - the decision feedback equalizer
  - the Viterbi equalizer

## General Properties of the Problem

- Model, using P+S notation




filter $h(t) = g_T(t) \circledast c(t)$

filter $x(t) = h(t) \circledast g_R(t)$

$$v(t) = \sum_k a_k\, g_T(t-nT) \qquad r(t) = \sum_k a_k\, h(t-kT) + n_w(t)$$

$$y(t) = \sum_k a_k\, x(t-kT) + \nu(t)$$

- If $c(t)$ is known, then can jointly optimize $g_T(t)$ and $g_R(t)$ for max SNR and zero ISI $\boxed{\text{P+S 8.4.1}}$. Not very realistic, since $c(t)$ is not usually known a priori, and it's hard to get Tx to adapt.

- If $c(t)$ and $g_T(t)$ are specified, then we can optimize $g_R(t)$ for white noise (max SNR, sufficient stats) by matching it to $h(t)$.

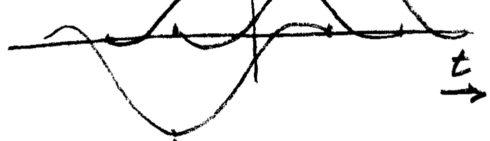
$$g_R(t) = g_T(-t) \circledast c(-t) \qquad G_R(f) = G_T^*(f)\, C^*(f)$$

so $\quad x(t) = R_{g_T}(t) \circledast R_c(t) \qquad X(f) = |G_T(f)|^2\, |C(f)|^2$

A little more realistic, since Rx has some chance to adapt.

Notes: – composite pulse $x(t)$ is even, so symmetric interference from past and future symbols

noise free $y(0) = a_{-1}\, x(T) + a_0\, x(0) + a_1\, x(-T)$



– noise samples are coloured by $g_R(t)$

$$R_v(iT) = \frac{N_0}{2}\int_{-\infty}^{\infty} |G_R(f)|^2 e^{j2\pi f iT}\,df \;=\; \frac{N_0}{2}\int_{-\infty}^{\infty} X(f)\, e^{j2\pi f iT}\,df$$

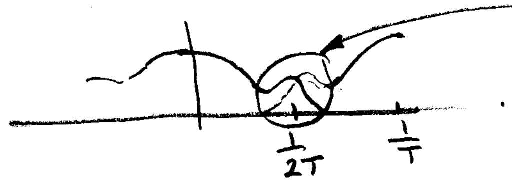
– one sample per sym is sufficient if taken at the right time.

• More commonly, we don't know $c(t)$, so we match $g_R(t)$ to $g_T(t)$. This makes

$$x(t) = R_{g_T}(t) \circledast c(t) \qquad \text{asymmetric, in general}$$

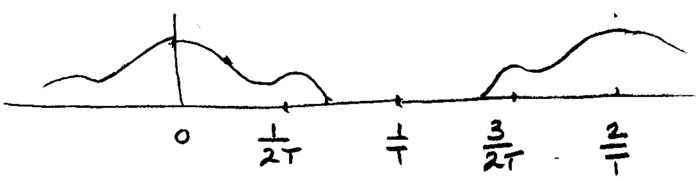
- Noise is coloured by $|G_R(f)|^2 = |G_T(f)|^2$, so symbol-spaced samples are uncorrelated if pulse satisfies Nyquist criterion.

– But symbol-spaced samples may not be sufficient stats. Sampling at rate $1/T$ produces



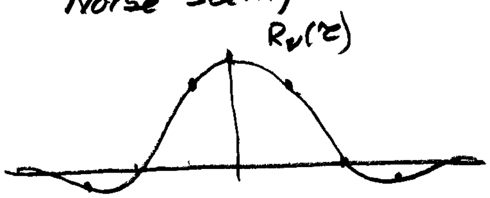

asymmetric $x(t)$, so phase not zero and we may get nulls (cancellation) at some frequencies in overlap.

– Usual remedy: sample more frequently (at Nyquist rate), typically $2/T$ or even faster. ("FT" or "FS")



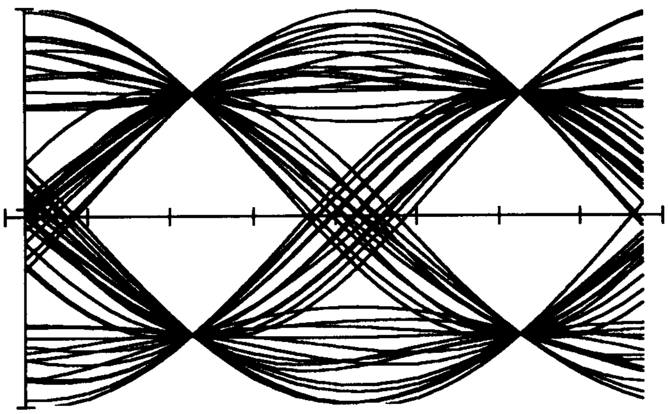

In principle, we can reconstruct the signal from the samples – sufficient stats again if noise 6.1.

Noise samples are correlated.




$$R_v(iT/2) = \frac{N_0}{2}\int_{-\infty}^{\infty} |G_R(f)|^2 e^{j2\pi f iT/2}\,df$$

- The ISI is not Gaussian.
  - For perfect channel filter and a Nyquist pulse, symbol-spaced samples have zero ISI, but FT samples have ISI



rolloff coefficient

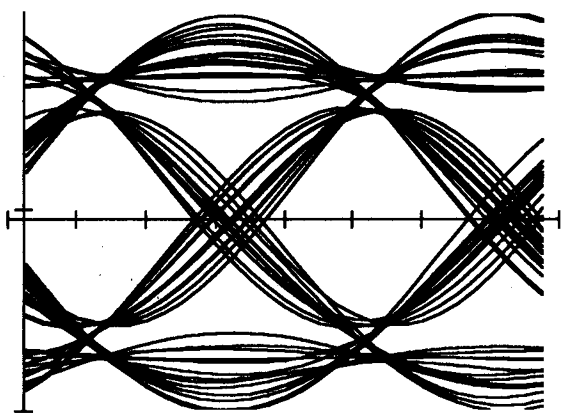
$\alpha = 0.35$

channel filter time const

$\tau = \tau' \cdot T$

$\tau' = 0$

  - For 1st order LPF channel filter $c(t) = \frac{1}{\tau} e^{-t/\tau}$, $C(f) = \frac{1}{1 + j2\pi f \tau}$ :



rolloff coefficient

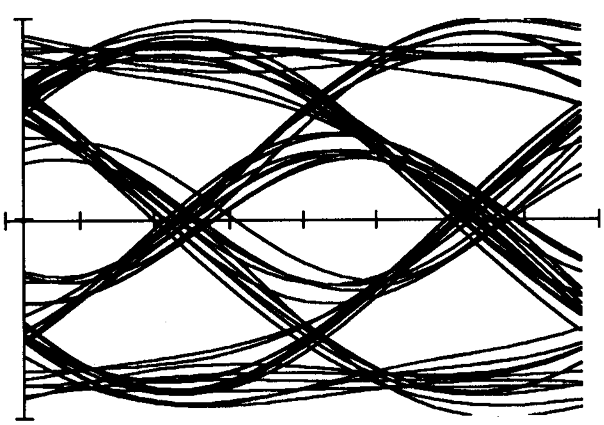
$\alpha = 0.35$

channel filter time const

$\tau = \tau' \cdot T$

$\tau' = 0.3$

- ISI has discrete values, unlike Gaussian noise.
- Some trajectories suffer a significant loss of noise margin.
- $y_m = a_m x_0 + \sum_{i \neq m} a_i x_{m-i}$



rolloff coefficient

$\alpha = 0.35$

channel filter time const

$\tau = \tau' \cdot T$

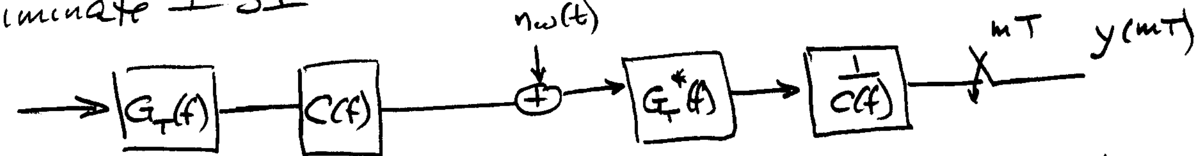
$\tau' = 0.7$

- Where to sample?
- BER:

$$P_b = \sum_{\underline{a}'} Q\left(\frac{1 - ISI(\underline{a}')}{\sigma_v}\right) Pr[\underline{a}]$$

# The Linear Equalizer - Zero Forcing  $\boxed{\text{P+S } 8.4.2}$

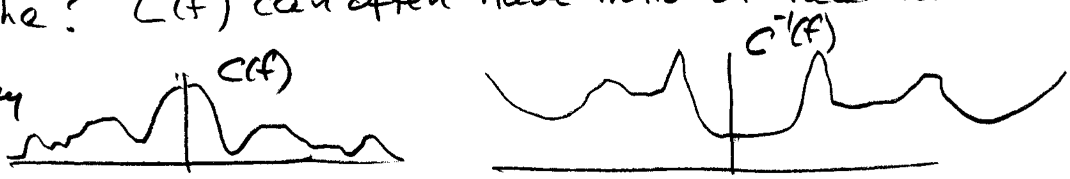
- If we know $C(f)$, it is tempting to insert its inverse to eliminate ISI




Assuming $G_T(f)$ is a root-Nyquist shape, we now have

$y_m = a_m + \nu_m$   problem solved (?) - forced zero ISI

- The gotcha? $C(f)$ can often have nulls or near nulls in frequency




This causes the noise power to be exaggerated, so you lose SNR margin.

- Recall from SNR maximization weights are $\frac{m_i^*}{\sigma_i^2}$
  so MF is $G_T^*(f)\,C^*(f)$. Less weight where signal is weak, not more weight.

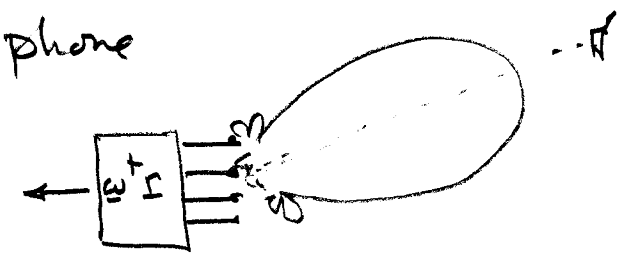
- A related problem. Suppose there is a vector $\underline{a}$ which we observe through a linear transformation in noise, so

$$\underline{y} = X\underline{a} + \underline{\nu}$$

- If $X = I$, then no problem $y_i = a_i + \nu_i$, perhaps $\hat{a}_i = y_i$
- If $X$ is orthogonal $X^TX = I$, so $X^T = X^{-1}$
  then $X^{-1}\underline{y} = \underline{a} + X^{-1}\underline{\nu}$ and noise cov $X^{-1}\underline{\nu}\,\nu^TX^{-T} = \frac{N_0}{2}I$
- If $X$ is ill conditioned (eigen vals $|\lambda_{max}| \gg |\lambda_{min}|$) then
  $X^{-1}$ is large: $X^{-1}\underline{y} = \underline{a} + X^{-1}\underline{\nu}$   "noise enhancement"

- Another related problem — an array of antennas.


Each antenna receives the signal, with a phase shift from one antenna to the next. Independent noise on each antenna.
A weighted sum to maximize SNR forms a beam toward the cell phone

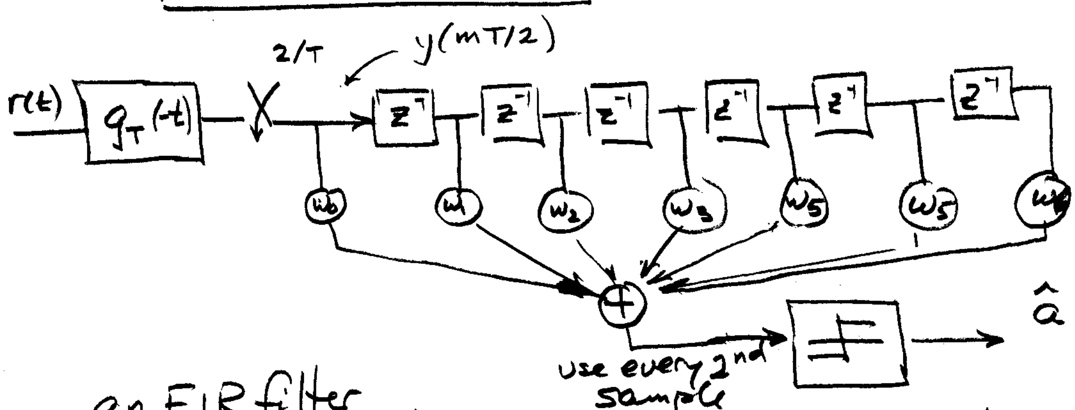



lots of gain toward cell phone

Now if interfering cell phones, must pick $\underline{w}$ to create nulls in those directions. Each such constraint reduces by one the number of degrees of freedom to max SNR.

Less gain available toward desired phone, so less SNR.

- Read $\boxed{\text{P+S pp 581–583}}$ on the ZF equalizer.




an FIR filter
- cancels interference from neighbouring pulses
- tries to add samples of desired pulse constructively