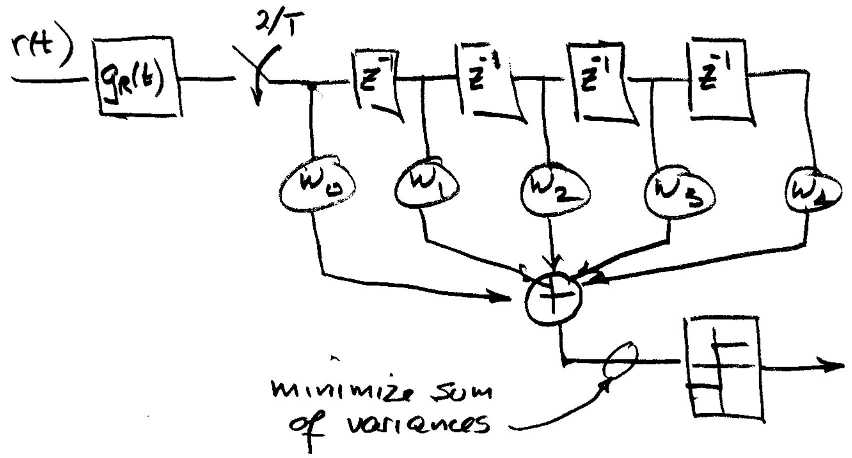
The Linear Equalizer — MMSE

- Insistence on zero interference is costly in terms of noise. Why not allow a little interference in exchange for a big reduction in noise?
  In fact why not minimize the sum of the ISI and noise variances?



- Form a MMSE estimate of (say) a_0 as a LC of samples $\underbrace{y(-N\tfrac{T}{2}),\ y(-(N-1)\tfrac{T}{2}),\ \cdots,\ y(-\tfrac{T}{2}),\ y(0),\ y(\tfrac{T}{2}),\ \cdots y(N\tfrac{T}{2})}_{\underline{y}^T}$

We know how to do this already!

$$\hat{a}_0 = \underline{w}^T \underline{y} \qquad \underline{w}_{opt} = R_y^{-1} \underline{p}_{ay}$$

$$R_y = E[\underline{y}\,\underline{y}^T] \qquad \underline{p} = E[a_0 \underline{y}]$$

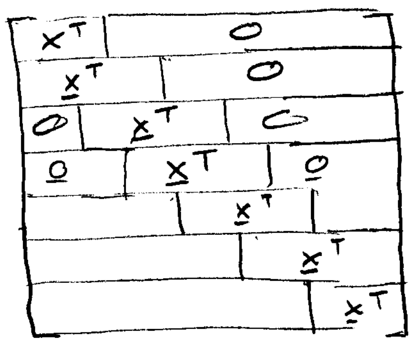
- Look at the formation of the arrays. We have

$$y(t) = \sum_k a_k\, x(t-kT) + v(t), \quad y\left(m\tfrac{T}{2}\right) = \sum_k a_k\, x\left(m\tfrac{T}{2} - kT\right) + v(mT/2)$$

Suppose span of impulse response is $2L+1$ samples

$$\underbrace{x\left(-L\tfrac{T}{2}\right), x\left(-(L-1)\tfrac{T}{2}\right), \ldots, x\left(-\tfrac{T}{2}\right), x(0), \ldots, x(LT/2)}_{\underline{x}^T}$$

Then

$\underline{y} =$ 

$\underline{a} + \underline{v}$

$\underline{x}^T$ slipped by 1 sample, row to row

X, $(2N+1) \times ($[illegible]$)$

and $\underline{y} = X\underline{a} + \underline{v}$

so $R_y = \overline{\underline{y}\,\underline{y}^T} = X \underbrace{\overline{\underline{a}\,\underline{a}^T}}_{=\sigma_a^2 I \text{ if iid data}} X^T + \overline{\underline{v}\,\underline{v}^T} = \sigma_a^2 X X^T + R_v$

and $\underline{p} = \overline{a_0 \underline{y}} = \sigma_a^2\, \tilde{\underline{x}}$ (time reversed $\underline{x}$, zero padded if necessary, to make length $2N+1$)

since $\overline{a_0\, y\left(m\tfrac{T}{2}\right)} = \sum_k \overline{a_0 a_k}\, x\left(m\tfrac{T}{2} - kT\right) + \overline{a_0\, v(mT/2)}$

$$= \sigma_a^2\, x\left(m\tfrac{T}{2}\right)$$

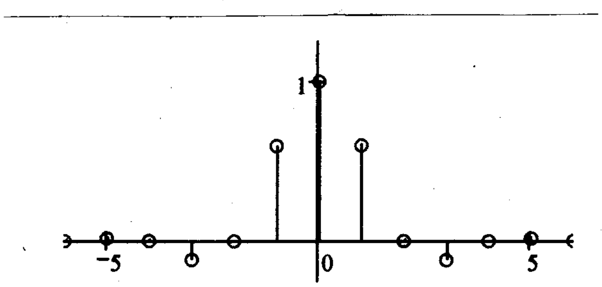
- performance is <u>much</u> better than ZF in low SNR.

The Decision Feedback Equalizer

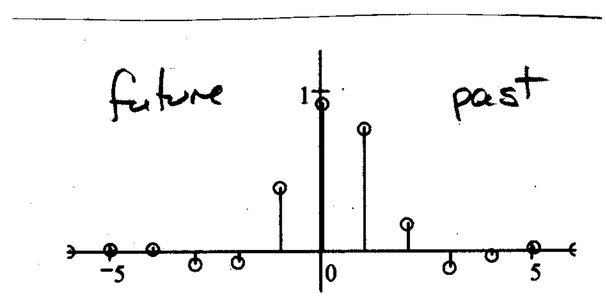
- This is the most widely used structure. It's easy and it works better than the linear equalizer.

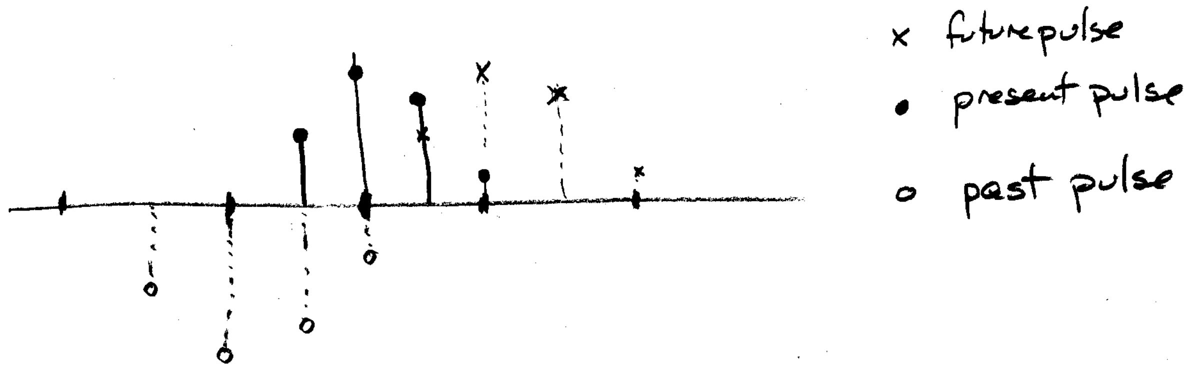
- Recall that ISI contributions are typically from the past and future. This is root raised cos pulse and MF, with 1^{st} order LPF channel

$f_c T = 100$

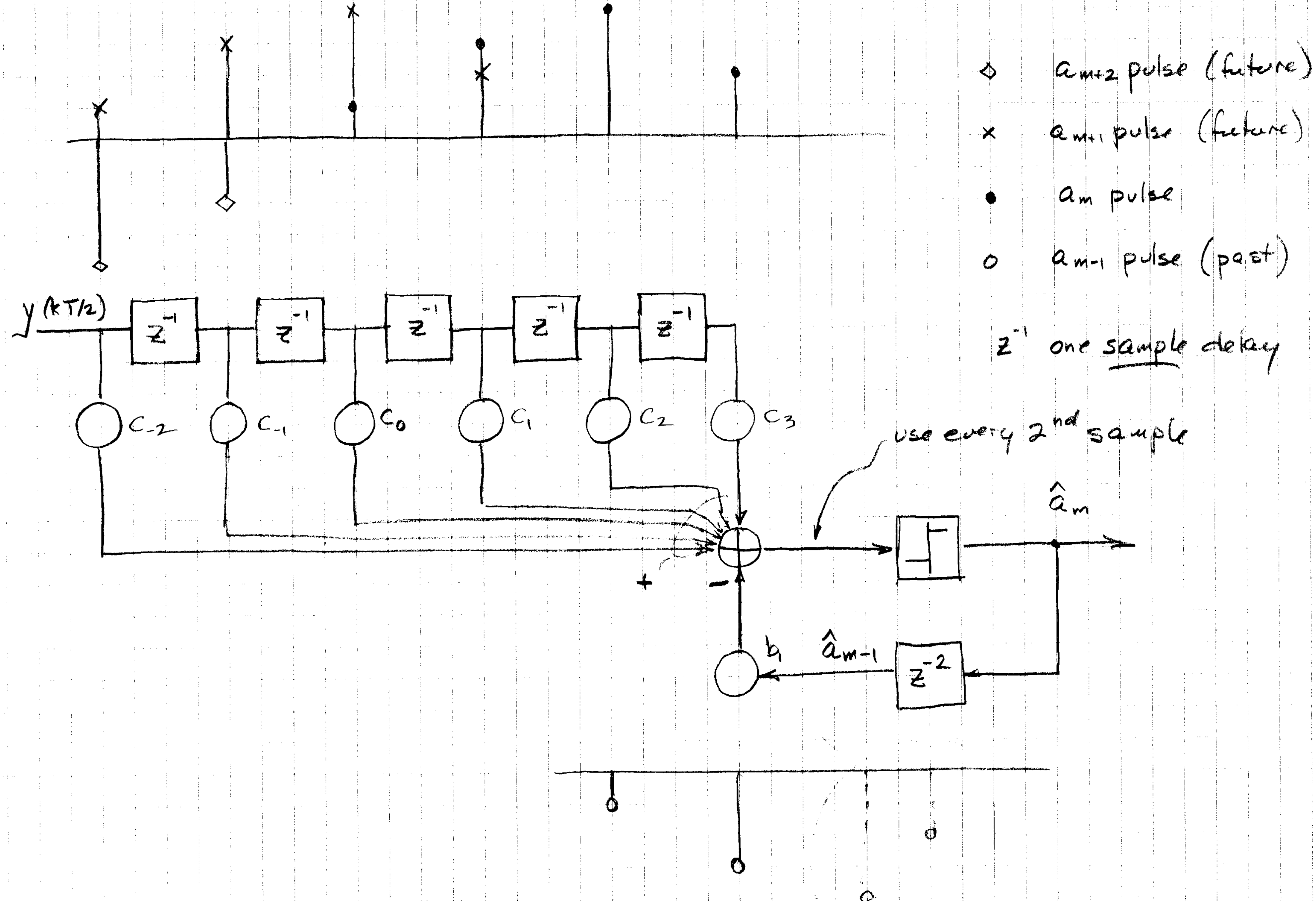


$f_c T = 1$





- If we happen to know the values of all neighbouring bits a_k, $k \neq m$, then we could subtract their contributions, leaving only $a_m x_{i-m}$ pulses and we could make a decision with only noise to cause errors.



a_{m+2} pulse (future)
a_{m+1} pulse (future)
a_m pulse
a_{m-1} pulse (past)
z^{-1} one sample delay
$y(kT/2)$
z^{-1}
c_{-2}
c_{-1}
c_0
c_1
c_2
c_3
use every 2^{nd} sample
$\hat{a}_m$
+
−
b_1
$\hat{a}_{m-1}$
z^{-2}

We do have past decisions. Proceed as if they are correct, and cancel their ISI.

- Each decision has its own weight to cancel the tail of its contribution
- The feedback section is symbol spaced
- The forward section is FT and coefficients selected by ZF or MMSE.

Channels A, B defined in text. A has roughly 10dB amplitude variation B has deep null.

In DFE, does a fed back error make another one more likely, leading to collapse?

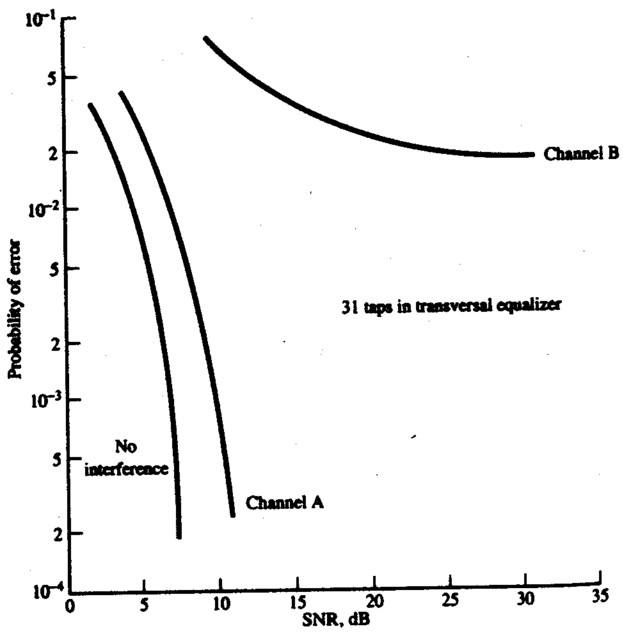


FIGURE 8.28. Error rate performance of linear MSE equalizer.

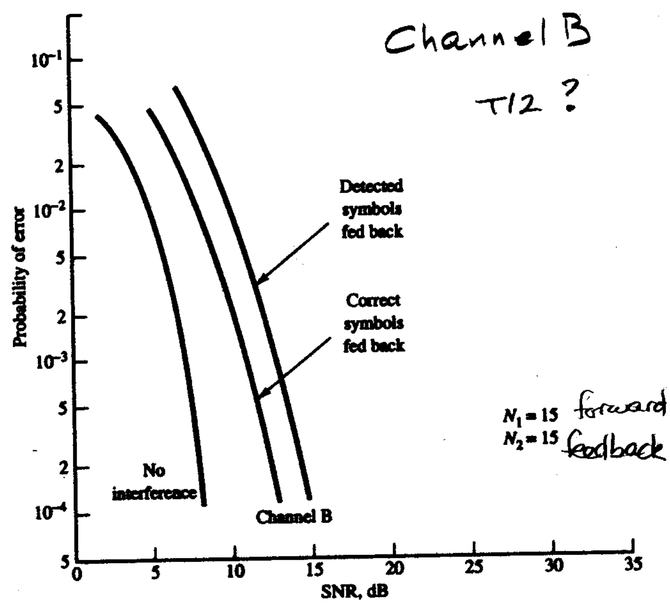


FIGURE 8.31. Performance of DFE with and without error propagation.

A DFE is particularly effective when there are deep nulls.