6.7 Dealing With ISI – the Viterbi Equalizer

- We could handle ISI most effectively if we made sequence decisions. If the composite pulse response is known, then use of basic principles tells us:

  - For each of the many sequences $\underline{a}_i$, we can convolve with $x(t)$ and sample to get the signal we would see in the absence of noise $\underline{a}_i \xrightarrow{c(t)} \underline{s}_i$.

    Each $\underline{s}_i$ is a point in signal space (and an array of MF output samples).

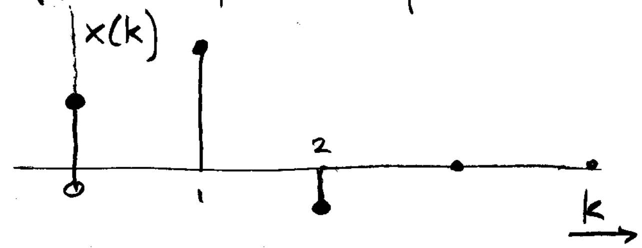
  - We calculate the minimum metric

$$\arg\min_i \left(\|\underline{r} - \underline{s}_i\|^2\right)$$

page 5.37, equiprobable seq's

  - It's not quite right if samples ν_k are correlated, but return to fix this later.

- If the noise-free pulse response is limited to a span of L symbols, then $\underline{s}_i$ can be generated by a trellis.
  - Suppose symbol-spaced to start, and



  - Noise free $$y_m = \sum_{k=0}^{L-1} a_{m-k} x_k$$

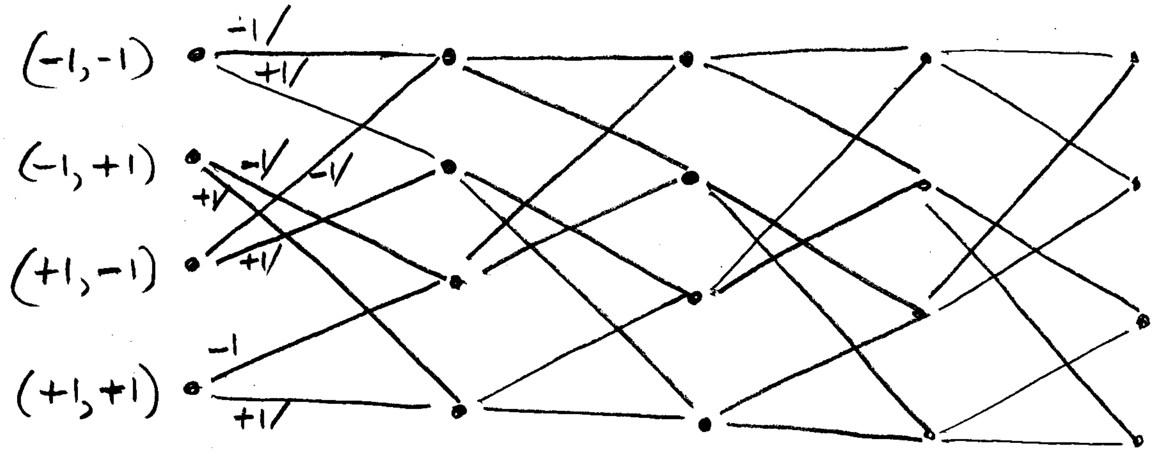
    It's determined by L adjacent symbols

    $$a_{m-L+1},\ a_{m-L+2}, \cdots, a_{m-1}, a_m$$

  - Define "state" as $\sigma_m = (a_{m-L+2}, \cdots, a_{m-1}, a_m)$, L-1 symbols

    Then state transition determines noise-free signal

    $$y_m = f(\sigma_{m-1}, \sigma_m) = a_{m-L+1} x_{m-L+1} + \cdots + a_{m-1} x_{m-1} + a_m x_0$$
    $$= g(a_{m-L+1} \cdots, a_{m-1}, a_m)$$



- The sequence metric for some $\underline{\sigma}_N = (\sigma_0, \sigma_1, \ldots, \sigma_N)$ is the sum of the branch metrics because the noise is white.

$$p_{\underline{r}|\underline{s}_i} = \frac{1}{(\pi N_0)^{N/2}} \exp\left(-\frac{1}{N_0}(\underline{r}-\underline{s}_i)^T C_{\underline{n}}^{-1}(\underline{r}-\underline{s}_i)\right), \quad C_{\underline{n}} = E[\underline{n}\,\underline{n}^T] = \frac{N_0}{2} I$$

$$= \frac{1}{(\pi N_0)^{N/2}} \exp\left(-\frac{1}{N_0} \sum_{m=1}^{N} (r_m - s_{im})^2\right)$$

Sequence metric $M(\underline{\sigma}_N) = \sum_{m=1}^{N} \mu(r_m, \sigma_{m-1}, \sigma_m)$

where $\mu(r_m, \sigma_{m-1}, \sigma_m) = \left(r_m - f(\sigma_{m-1}, \sigma_m)\right)^2$

- This is a perfect formulation for the VA. The result is called the Viterbi Equalizer, and is the most effective way to deal with ISI.

- Computation? Still linear in N (the good news). It's also proportional to the number of states, and that is 2^{L-1}. Long impulse responses can bring it to its knees. Moore's law?

- Constraint length L is the number of received symbols directly affected by a single transmitted symbol.

- What if more than one sample per symbol? $\underline{K}$

  Assuming that samples of ν are still white, we have the conditional prob

$$\frac{1}{(\pi N_0)^{NK/2}} \exp\left(-\frac{1}{N_0}\sum_{m'=1}^{NK}\left(r_{m'} - s_{im'}\right)^2\right)$$ where m' indexes samples

Sequence metric $M(\underline{\sigma}_N) = \sum_{m=1}^{N} \mu(r_{Km}, \cdots, r_{Km+K-1}, \sigma_{m-1}, \sigma_m)$

where the branch metric is

$$\mu(r_{Km}, \cdots, r_{Km+K-1}, \sigma_{m-1}, \sigma_m) = \sum_{\ell=0}^{K-1}\left(r_{Km+\ell} - f_\ell(\sigma_{m-1}, \sigma_m)\right)^2$$

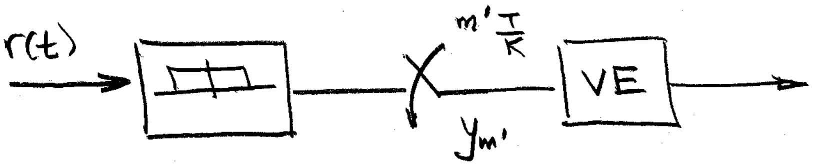
and $f_\ell(\quad)$ uses the ℓ^{th} decimation of the pulse response $x_{m'}$

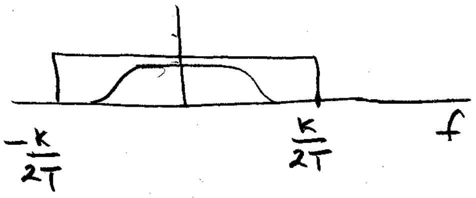
$\ell=0$ $(x_0 \quad x_K \quad x_{2K} \quad \cdots \quad)$

$\ell=1$ $(x_1 \quad x_{K+1} \quad x_{2K+1} \quad \cdots \quad)$

$\ell=K-1$ $(x_{K-1} \quad x_{2K-1} \quad x_{3K-1} \quad \cdots \quad)$

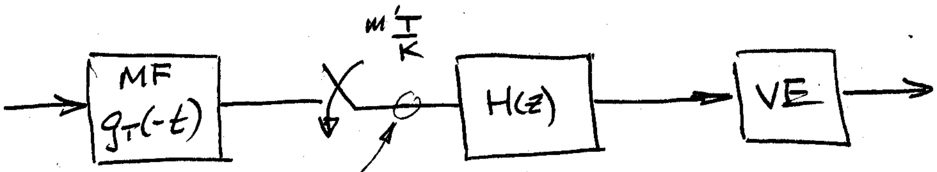
- What if the noise is not white?
  - Solution 1: Don't use a MF – use a rectangular LPF





Rely on the ML action of the VE to perform MF implicitly: $\sum_{m'} (r_{m'} - s_{m'})^2 = \sum_{m'} r_{m'}^2 - 2\sum_{m'} r_{m'} s_{m'} + \sum_{m'} s_{m'}^2$

  - Solution 2: Whiten the noise in discrete time



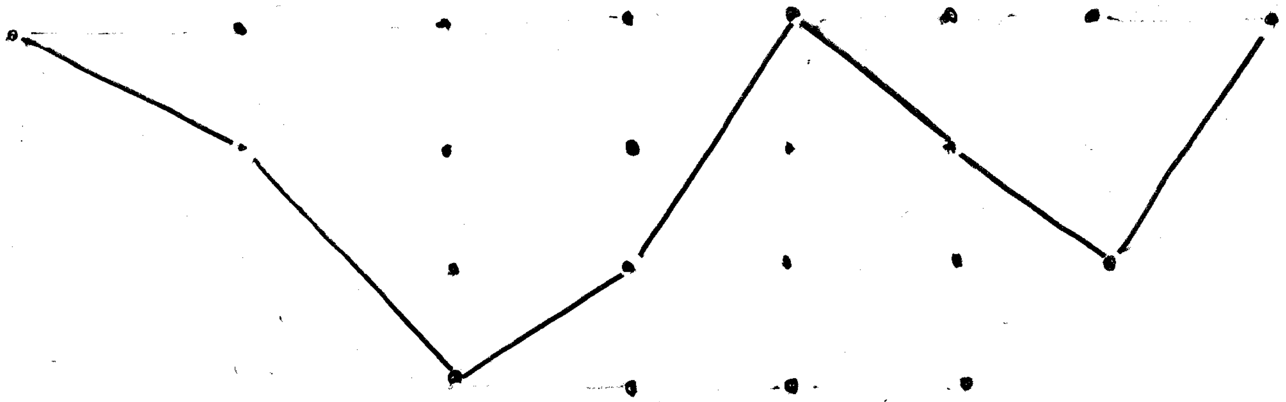
noise PSD

$S_\nu(z) = \mathcal{Z}\left[R_\nu(k)\right]$

$R_\nu(k) = E\left[\nu_{m'} \nu_{m'-k}\right]$

$H(z) = \frac{1}{\sqrt{S_\nu(z)}}$

- Performance?



- The best path, correct or not, has the min metric and the VA finds it.
- Local departures from the correct path are error events. For this trellis, shortest event has length 3.
- Union bound: sum over different lengths as before.

  Prob of error event of length M is

  average over all possible transmitted seqs of the $\underbrace{\text{sum of pairwise probs for each one}}_{\text{another union bound.}}$

$$\frac{1}{2^M} \sum_{\underline{\sigma}_M} \sum_{\underline{\sigma}_M'(\underline{\sigma}_M)} P_2(\underline{\sigma}_M, \underline{\sigma}_M')$$

- Finding dominant events (smallest Euclidean distance) is tricky.

- On the difficult channel B, we have a comparison of MLSE and DFE

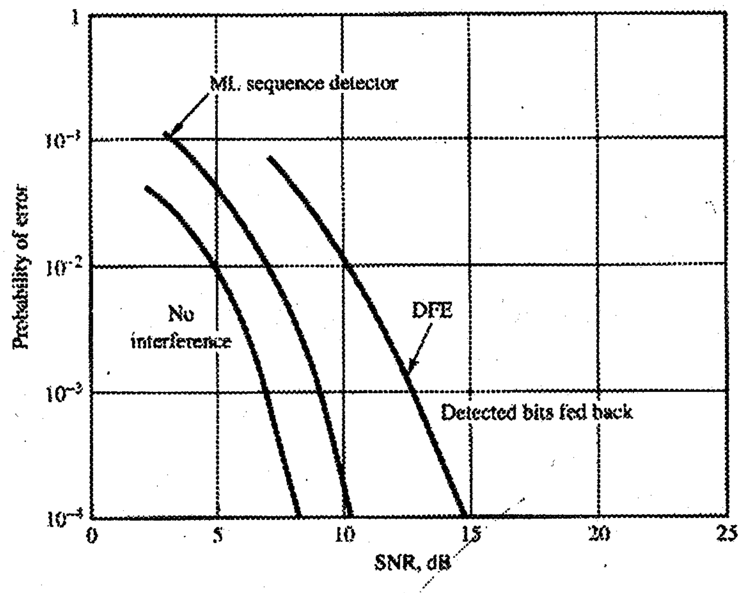


FIGURE 8.32. Performance of Viterbi detector and DFE for channel B.

- One interpretation of the Viterbi equalizer is that it keeps several alternative contexts alive (i.e. tentative immediate past and future neighbouring symbols), so can make use of full pulse energies (in x_k) while accounting nonlinearly (i.e. $g(a_{m-L+1}, \cdots a_m)$, a_i discrete) for ISI.

  In contrast:
  - DFE uses hard decisions from the past, not tentative ones, although it does so nonlinearly, but has only linear protection from the future symbols
  - LE is linear in past and future.