7 BANDPASS SIGNALS AND DIGITAL CARRIER MODULATION [P+S 9]

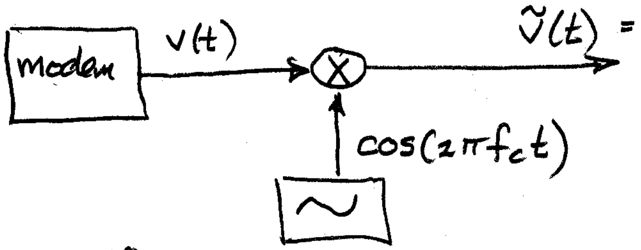
- Although we have studied detection of lowpass signals, it is more common to transmit bandpass signals, ones that result from modulating a carrier. Why??
  - Move the signals away from dc and centre them in the sweet spot of the transmission characteristic (e.g., telephone line modems)
  - Need to use radio or ultrasound etc for the link
  - More bandwidth is available at higher frequencies.

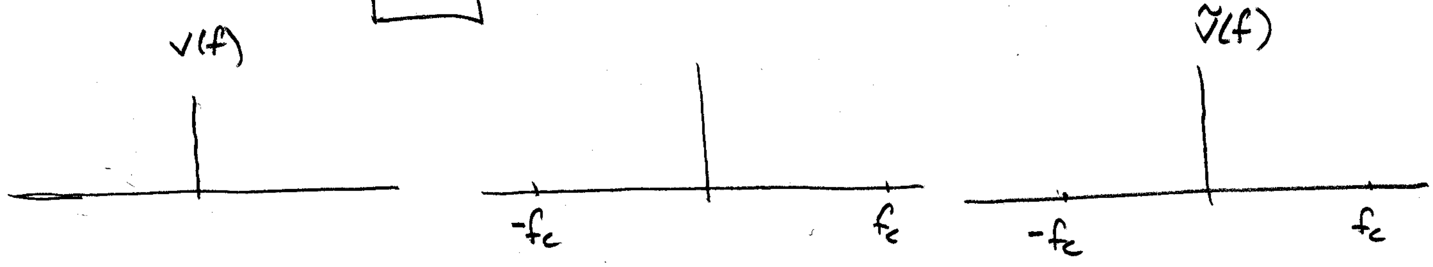
    1% b.w. (W/f_c) is 10 kHz at 1 MHz
    10 MHz at 1 GHz
    280 MHz at 28 GHz

- Special techniques for
  - generation
  - recovery
  - received carrier phase estimation

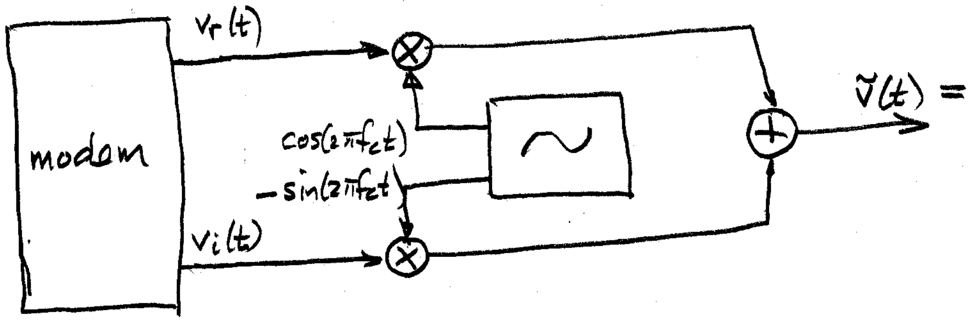
7.1 Narrowband Signals and the Complex Envelope

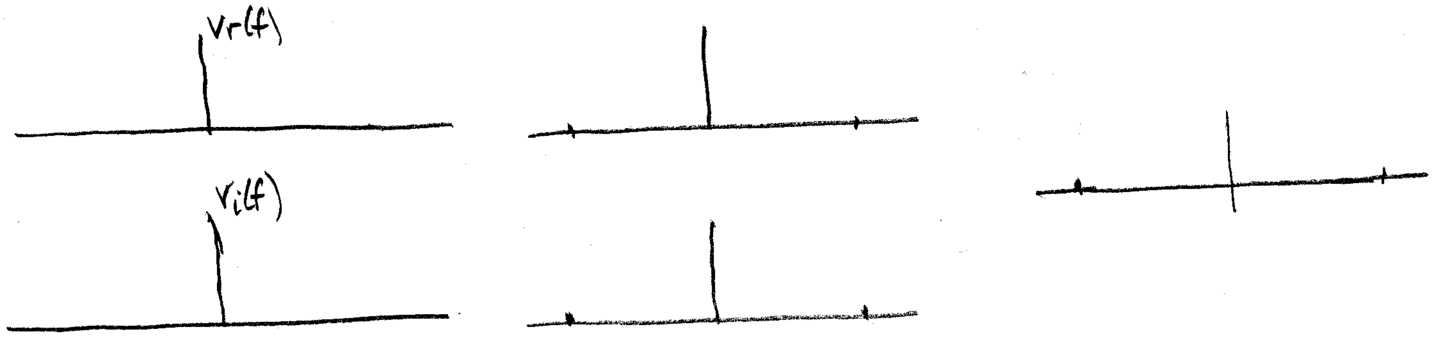
- Here's a basic model of modulation:





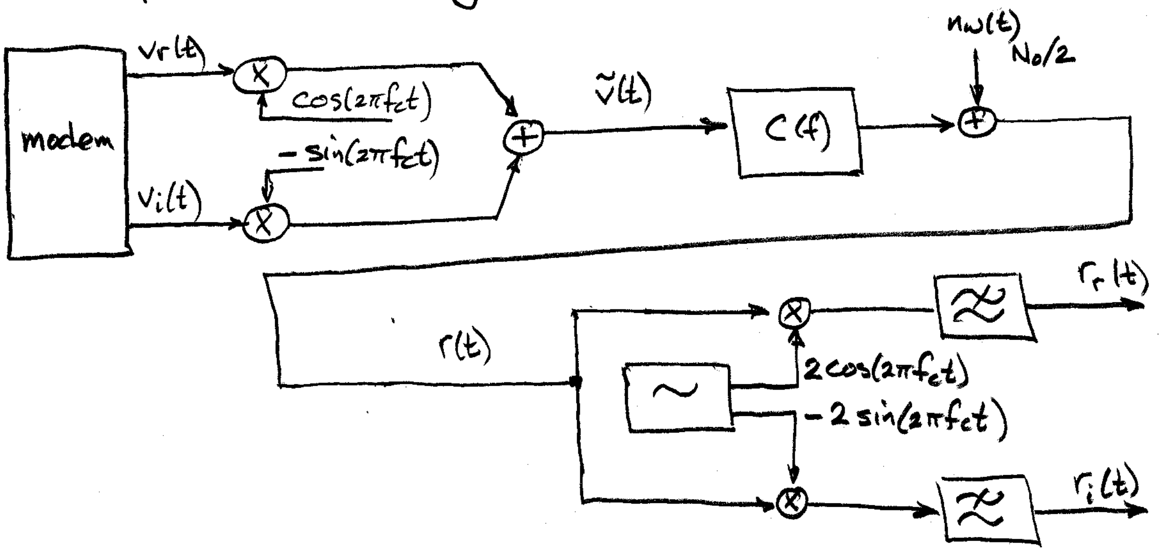
- Why be content with just one signal in that bandwidth? It is twice as wide as original. So pack in a second signal, on a sine carrier





"quadrature multiplexing"

- The quadrature multiplex is good, provided we can separate the signals at the receiver. Can we?



– Ignore channel filter and noise: $C(f) = 1$, $\frac{N_0}{2} = 0$

Then

$$r(t) = \tilde{v}(t) = v_r(t)\cos(2\pi f_c t) - v_i(t)\sin(2\pi f_c t)$$

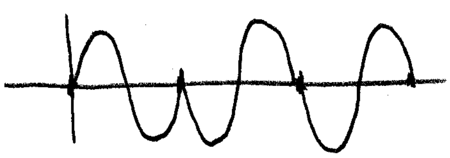
$$2\cos(2\pi f_c t)\, r(t) = 2 v_r(t)\cos^2(2\pi f_c t) - 2 v_i(t)\cos(2\pi f_c t)\sin(2\pi f_c t)$$

lowpassing,

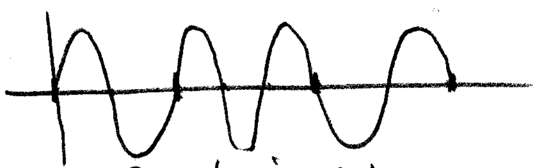
$r_r(t) = v_r(t)$. Similarly, $r_i(t) = v_i(t)$

A clean separation, since cos, sin are orthogonal.

– Note symbol timing, carrier phase not linked. Thus signals like

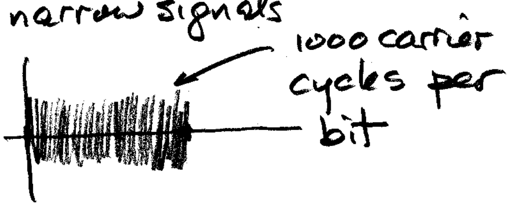


and



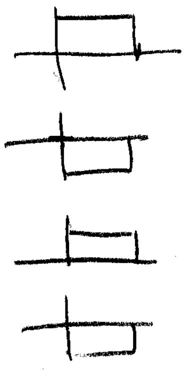
are anomalous, because large fractional b.w.

For narrow signals



a little timing error is a big phase error — so consider them to be separate.

• Since cos, sin are orthogonal dimensions, we can use all our theory from Sections 5 & 6. For example

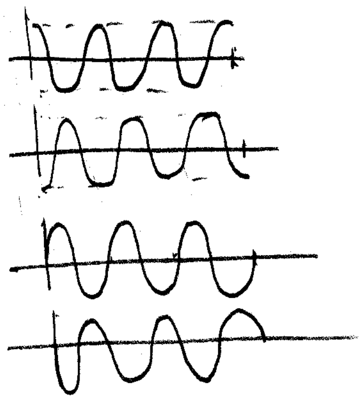


$\times \cos(2\pi f_c t)$

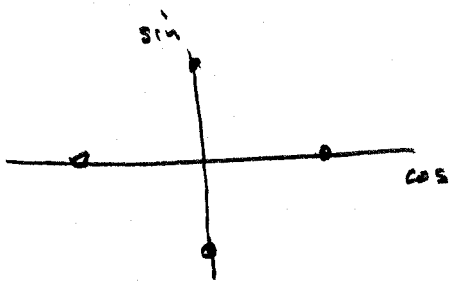
$\times \cos(2\pi f_c t)$

$\times \sin(2\pi f_c t)$

$\times \sin(2\pi f_c t)$



forms



$\underline{r} = \begin{bmatrix} r_r \\ r_i \end{bmatrix}$

So you don't have to learn anything new.

However... it's easier to use complex notation

$v(t) = v_r(t) + j\, v_i(t)$ $\qquad \underline{v}(t) = \begin{bmatrix} v_r(t) \\ v_i(t) \end{bmatrix}$

$v'(t) = e^{j\theta} v(t)$ $\qquad \underline{v}'(t) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \underline{v}(t)$

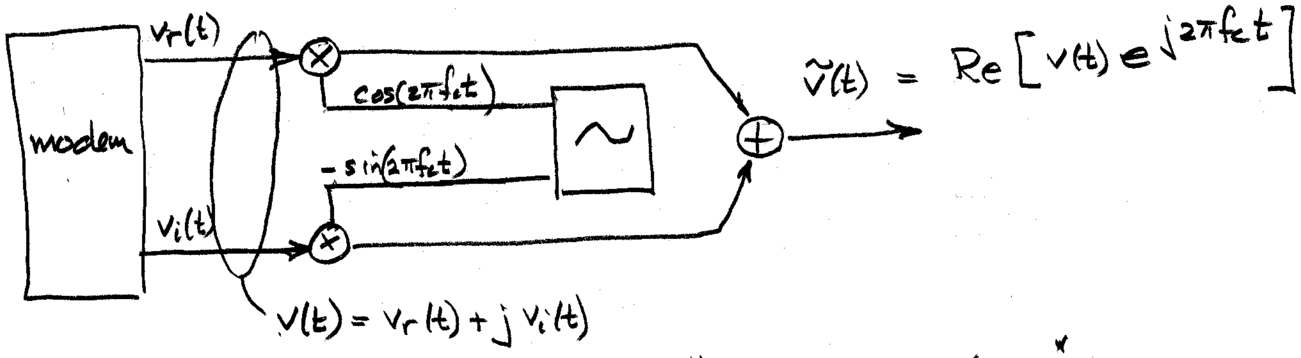
$|v(t)|^2$ $\qquad \underline{v}^T(t)\, \underline{v}(t)$

$v^*(t)$ $\qquad \begin{bmatrix} v_r(t) \\ -v_i(t) \end{bmatrix}$

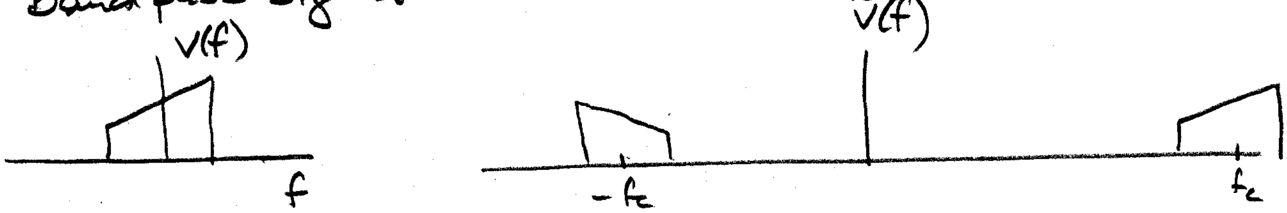
$v w$ $\qquad \begin{bmatrix} v_r & -v_i \\ v_i & v_r \end{bmatrix} \begin{bmatrix} w_r \\ w_i \end{bmatrix}$

$v^* \underline{w}$

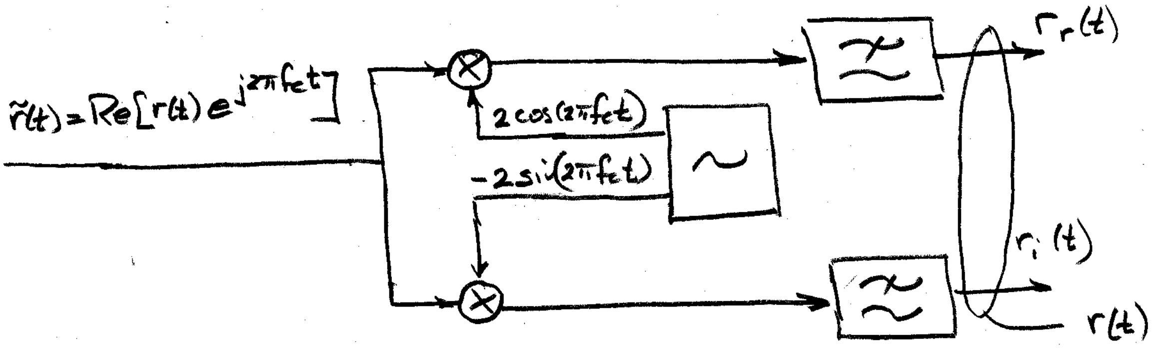
- Modulation and demodulation in complex terms



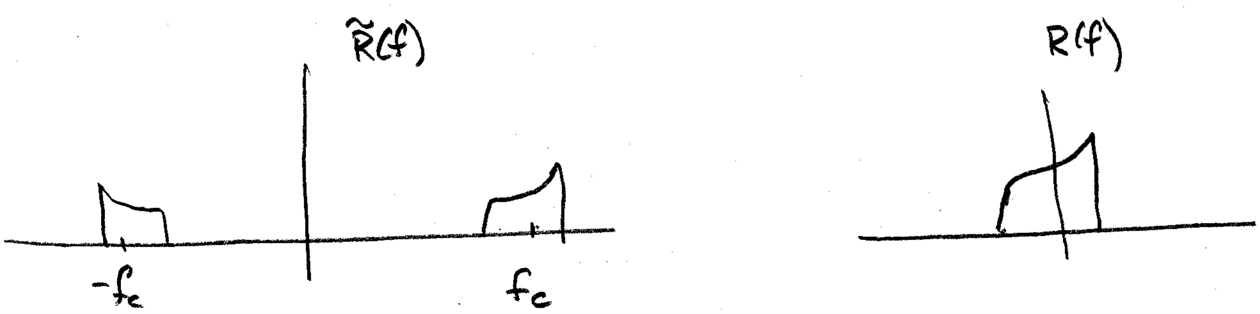
The complex signal $v(t)$ is the "complex envelope".
The quadrature modulator produces the corresponding bandpass signal



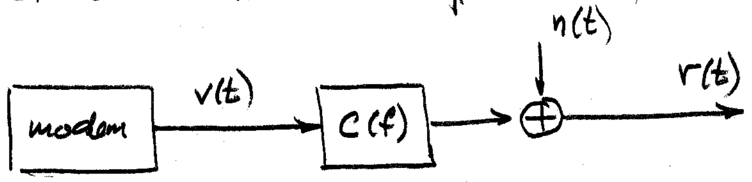
Since $v(t)$ is complex, its transform need not be conjugate symm.

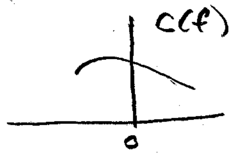


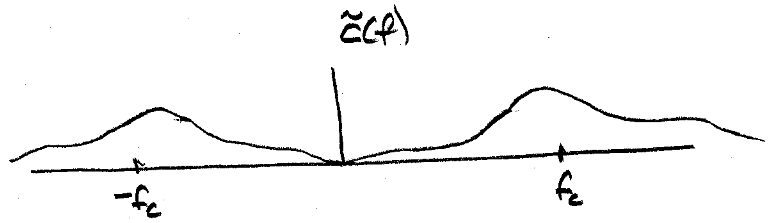
The quadrature demodulator recovers the complex envelope.



- We usually omit the quadrature mod and demod and just show the envelope







- Power relationships:

$$P_{\tilde{v}} = E\left[\tilde{v}(t)^2\right] = E\left[Re\left[v(t)e^{j2\pi f_c t}\right] Re\left[v(t)e^{j2\pi f_c t}\right]\right]$$

$$= \frac{1}{2} E\left[|v(t)|^2\right] + \frac{1}{2} E\left[Re\left[v^2(t)\, e^{j4\pi f_c t}\right]\right]$$

As a time average

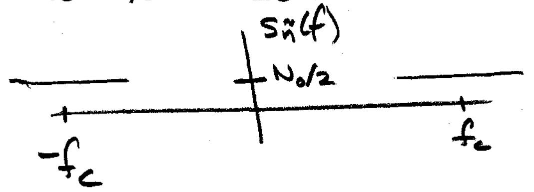
$$P_{\tilde{v}} = \frac{1}{2} P_v$$

Hence the convention when using complex envelopes

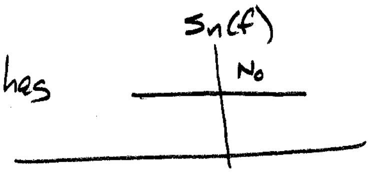
$$P_v = \frac{1}{2} E\left[|v(t)|^2\right] \qquad R_x(\tau) = \frac{1}{2} E\left[x(t)\, x^*(t-\tau)\right]$$

$$\mathcal{E}_s = \frac{1}{2}\int_0^T |s(t)|^2 dt \qquad R_{xy}(\tau) = \frac{1}{2} E\left[x(t)\, y^*(t-\tau)\right]$$

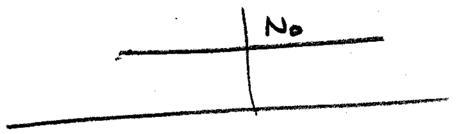
- White noise has a subtlety. Its PSD is $N_0/2$



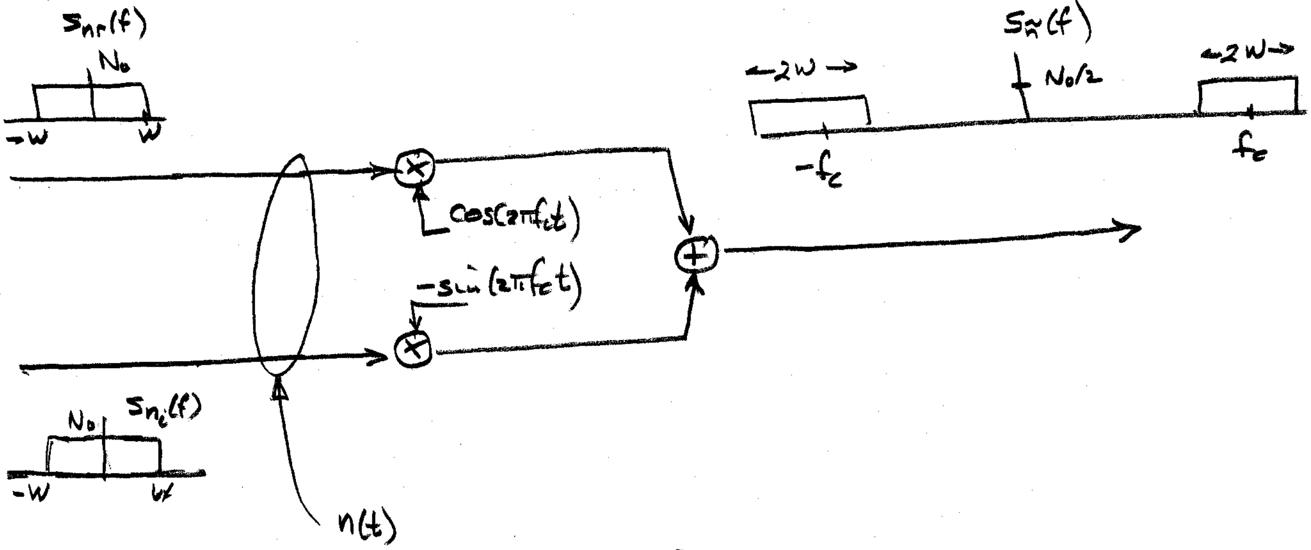
but its complex envelope has



and $S_{n_r}(f) = S_{n_i}(f) = N_0$



Here's why: suppose we modulate two independent noise processes, each of PSD N_o, b.w. W.



$$P_{nr} = \sigma_{nr}^2 = 2N_oW \qquad P_{n_i} = \sigma_{n_i}^2 = 2N_oW$$

Multiplication by cos or sin cuts power in half,

so power in the sum is $\frac{1}{2}\sigma_{nr}^2 + \frac{1}{2}\sigma_{n_i}^2 = 2N_oW.$

The bp signal has twice the b.w., so its PSD is $N_o/2$ in order to give correct power

$$\frac{N_o}{2} \cdot 2W + \frac{N_o}{2} 2W = 2N_oW$$

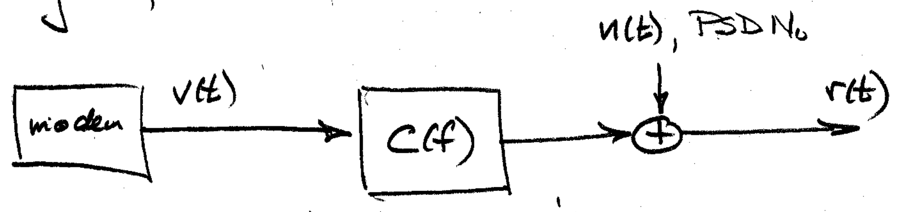
Hence $N_o/2$ in channel, N_o in each component of complex envelope

Also $R_n(\tau) = \frac{1}{2} E[n(t)n^*(\tau)] = \frac{1}{2}[R_{nr}(\tau) + R_{ni}(\tau)]$

$$S_n(f) = \mathcal{F}[R_n(\tau)] = \frac{1}{2}(S_{nr}(f) + S_{ni}(f)) = N_o \quad |f| \leq W$$

Hence N_o for the complex envelope as well as constituent processes

- Again, the link model



What if the channel filter is flat, but causes a phase shift θ?

$$C(f) = e^{j\theta} \qquad c(t) = e^{j\theta}\,\delta(t)$$

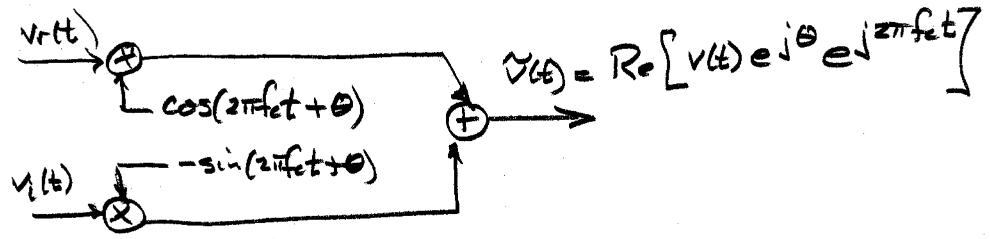
Then we recover the complex envelope (w/o noise)

$$r(t) = e^{j\theta}\, v(t)$$

$$r_r(t) = \cos\theta\; v_r(t) - \sin\theta\; v_i(t)$$

$$r_i(t) = \sin\theta\; v_r(t) + \cos\theta\; v_i(t)$$

oops – we don't have a clean separation of the two channels at the receiver. Same problem if Tx oscillator has a phase shift of θ wrt Rx osc.



The complex envelope is rotated by θ.

We'd better figure out some way that the Rx can track the phase shift!