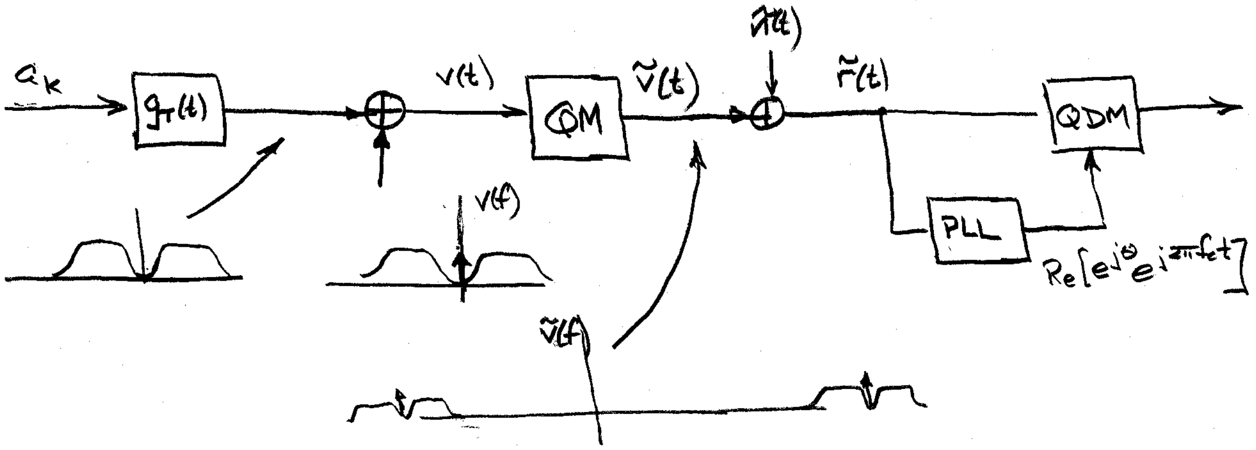
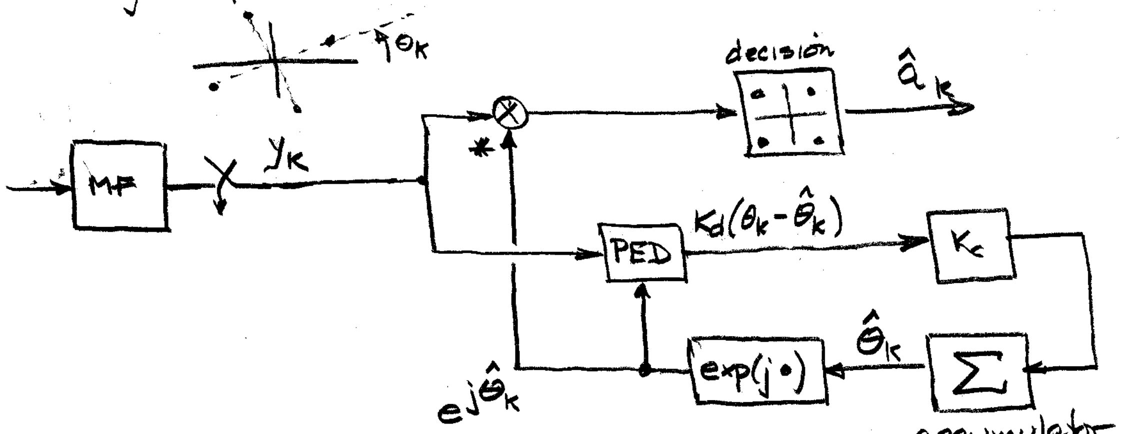
7.3 Synchronization II - Carrier Recovery

- If the pulse shape has a null at dc, and if we drop a low level carrier in it (a "pilot tone") then getting a phase reference is trivial.

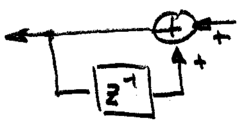


- Pilot tones are not often used. Also, we'd like a solution that operates once per symbol, post MF, so we can use DSP. We'll develop such solutions for linear modulation. FSK is tougher.
- Tracking loops look like this:



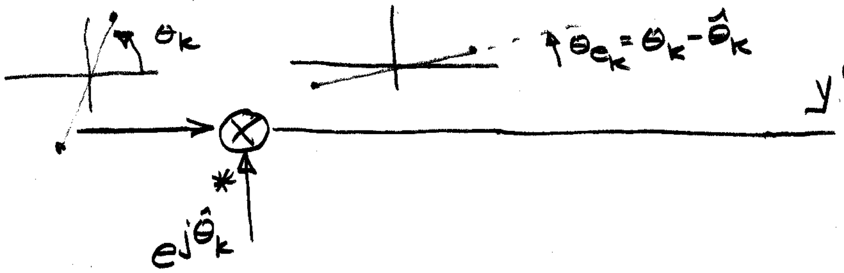
$\frac{1}{1-z^{-1}}$

The trick is getting a good PED



Squaring and n^{th}-Power Loops

- Suppose the modulation is BPSK. After derotation by $e^{j\hat{\theta}_k}$ we have



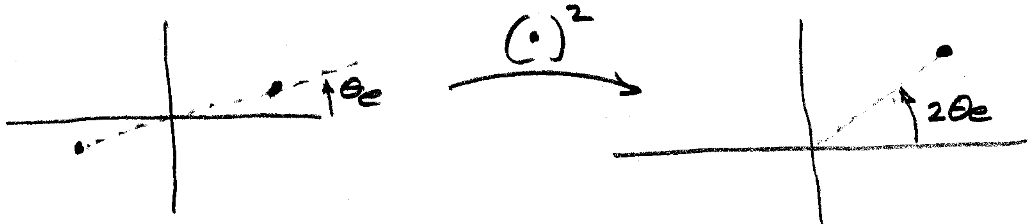
$$y' = y e^{-j\hat{\theta}_k} + \nu'$$

The error signal we want is linked to $\mathrm{Im}[y']$ or $\arg[y']$, but somehow we must get rid of the modulation.

- Inspiration! Square it — that kills the BPSK modulation. (It also doubles the error and increases the noise, but we'll live with them).

$$y'_k = \sqrt{2E_b}\, a_k\, e^{j\theta_{ek}} + \nu'_k \qquad \text{where } a_k = \pm 1$$

$$y'^2_k = 2E_b e^{j2\theta_{ek}} + 2\sqrt{2E_b}\, a_k e^{j\theta_{ek}} \nu'_k + \nu'^2_k$$ more than doubles noise



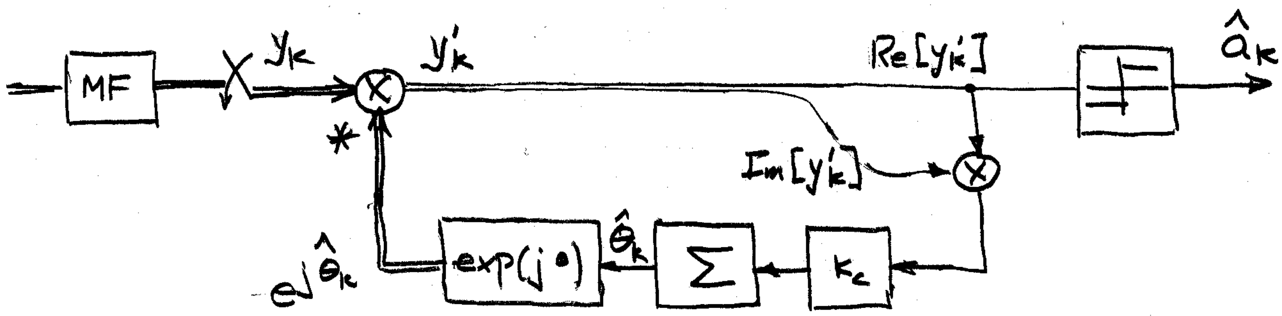
plus noise

no modulation

- Let's use $\mathrm{Im}[y_k'^2]$ to drive the tracker; we want it to be zero, it has the right sign to make phase corrections and its magnitude is monotonic in phase error θ_e (provided $|\theta_e| < \pi/2$).

$$\mathrm{Im}[y'^2] = 2\,\mathrm{Re}[y']\,\mathrm{Im}[y']$$

- Block diagram, with double lines to emphasize complex



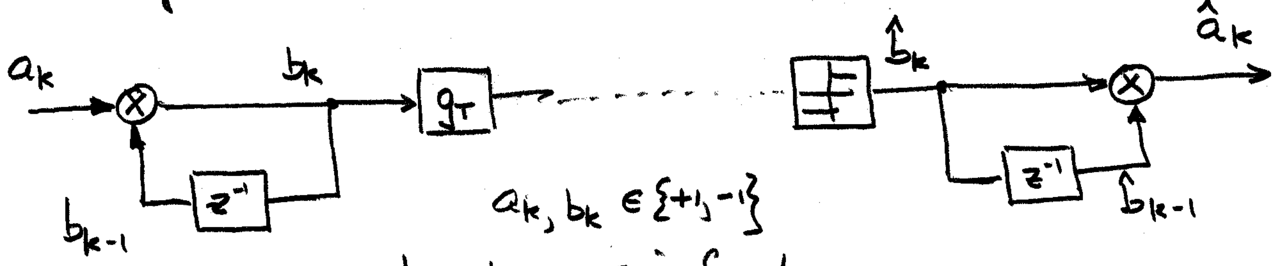
This is often called a Costas loop,

- It works for analog DSB-SC AM, as well

- Note phase ambiguity: data could be reversed (180°) and PED wouldn't notice; so loop can lock 180° in error, and all data is wrong

$$\hat{a}_k = -a_k$$

- Solution: differential encode, decode; information in the changes



$a_k, b_k \in \{+1, -1\}$

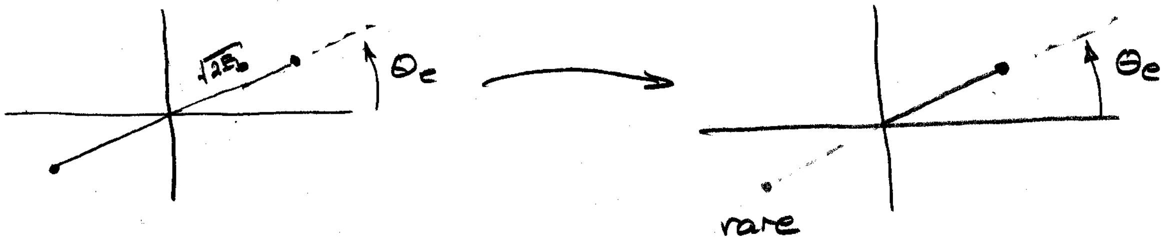
b_k changes sign from b_{k-1} iff $a_k = -1$

- For QPSK, use $\mathrm{Im}[y_k'^4]$, noise and all.

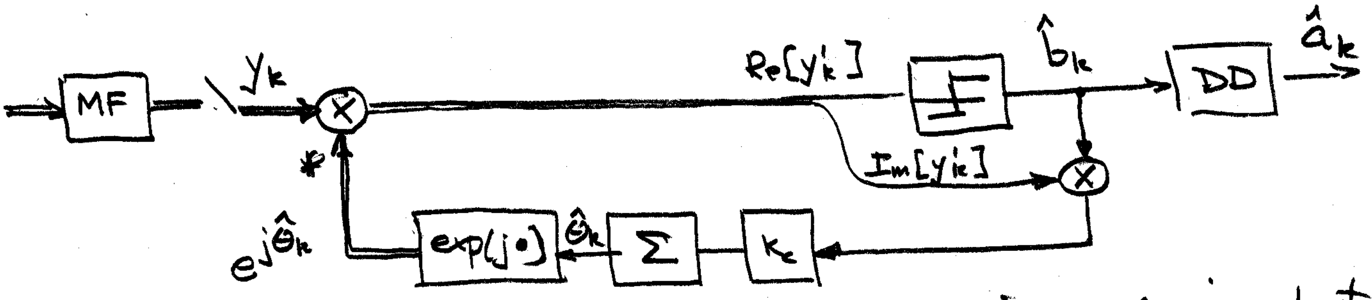
This produces a 4-fold phase ambiguity.

Maximum Likelihood and Decision Directed Loop

- Let's get this one in the obvious way, and reassure ourselves later with ML. Another way to remove the modulation is simply to multiply by the decision.



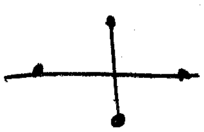
No noise enhancement



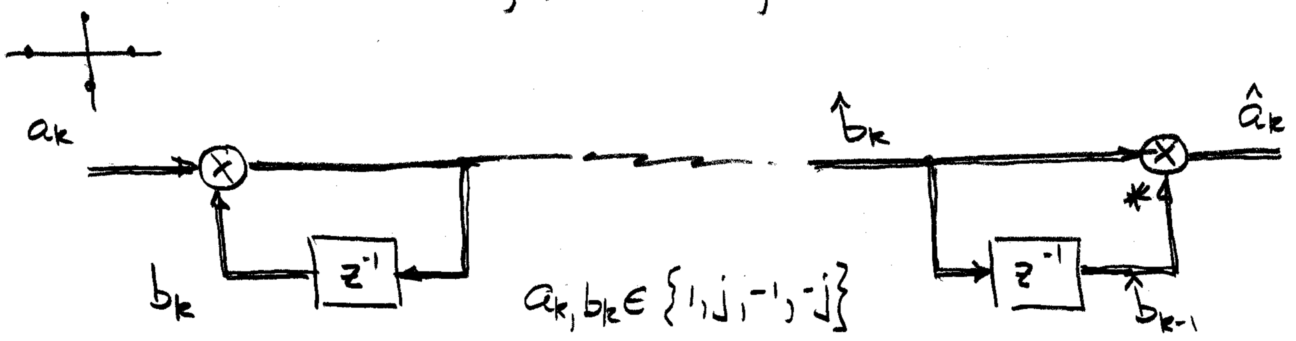
Differs from Costas only in placement of decision device, but much better performance

Still get two-fold phase ambiguity.

Easy generalization to 4-PSK: use $\mathrm{Im}[y_k' \hat{b}_k^*]$

where b_k is 

Note diff'l encoding, decoding for 4PSK is



$$a_k, b_k \in \{1, j, -1, -j\}$$

$$b_k = a_k b_{k-1}$$

$$a_k = b_k b_{k-1}^*$$

- Now to get a ML interpretation of our decision-directed tracker. We receive

$$y = v\, e^{j\theta} + \nu \qquad \text{where } v = a\sqrt{2E_b}$$

$$\sigma_\nu^2 = N_0$$

Assume we know a.

– Then choose $\hat{\theta} = \arg\max_{\theta} p_{y|\theta}(y|\theta)$

or $\arg\max_{\theta} p_{\underline{y}|\theta}(\underline{y}|\theta)$ if sequence $\underline{y} = (y_1, \cdots, y_N)$

$$p_{\underline{y}|\theta}(\underline{y}|\theta) = \frac{1}{(2\pi\sigma_\nu^2)^N} \exp\left(-\frac{1}{N_0}|\underline{y} - e^{j\theta}\underline{v}|^2\right)$$

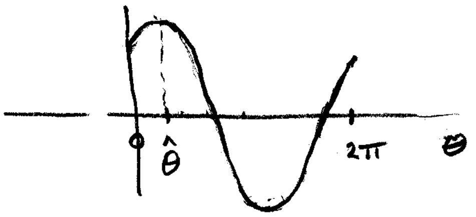
$$\hat{\theta} = \arg\min_{\theta} |\underline{y} - e^{j\theta}\underline{v}|^2 = \arg\min_{\theta}\left(\sum_k |y_k|^2 - 2\sum_k \mathrm{Re}[y_k v_k^* e^{-j\theta}] + \sum_k |v_k|^2\right)$$

$$= \arg\max_{\theta} \sum_k \mathrm{Re}[y_k a_k^* e^{-j\theta}]$$

Since a_k is assumed known, vary θ to find the max

Since $\sum_k \mathrm{Re}[y_k a_k^* e^{-j\theta}] = \mathrm{Re}[e^{-j\theta} \sum_k y_k a_k^*]$

the variation is sinusoidal



- Of course, we want the derivative. It is zero at the best value and, when nonzero, provides an error signal for a tracker. Call the function $\Lambda(\theta, y)$

$$\frac{\partial \Lambda}{\partial \theta} = \sum_k \mathrm{Re}[-j\, y_k a_k^* e^{-j\theta}] = \sum_k \mathrm{Im}[y_k a_k^* e^{-j\theta}]$$

- In a loop: – use $\hat{a}_k$ instead of a_k
  - actually, use $\hat{b}_k$ instead of $\hat{a}_k$ if diff'l encode
  - interpret θ in that expression as $\hat{\theta}_k$, our best guess so far.

Therefore the error signal at time k is

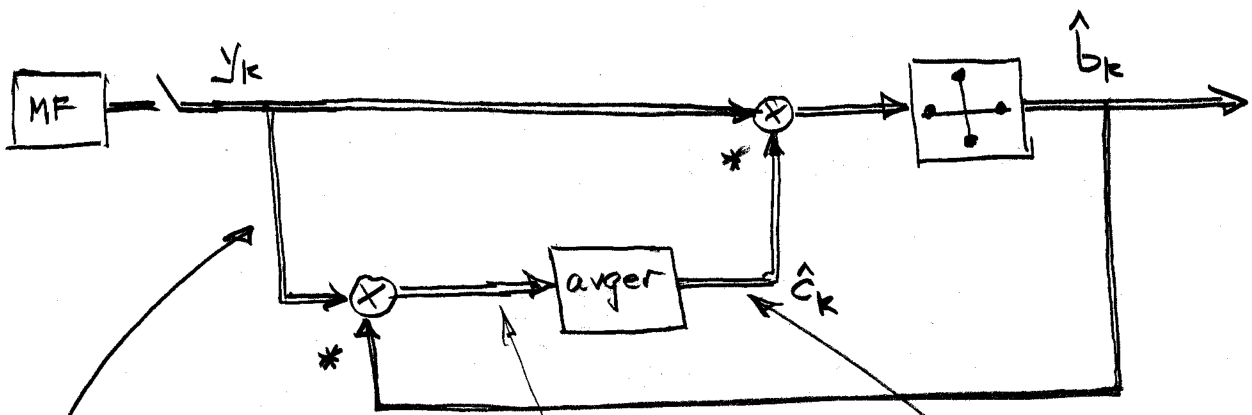
$$\mathrm{Im}[y_k \hat{b}_k^* e^{-j\hat{\theta}_k}] = \mathrm{Im}[y_k' \hat{b}_k^*]$$

so $\hat{\theta}_{k+1} = \hat{\theta}_k + K_c \mathrm{Im}[y_k' \hat{b}_k^*]$

and this is our DD loop.

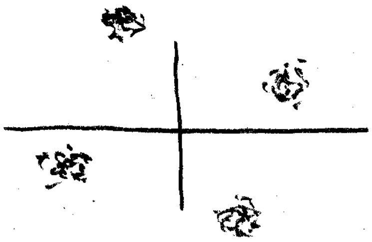
A Feedforward Approach

- Suppose you don't like loops. There's another good topology

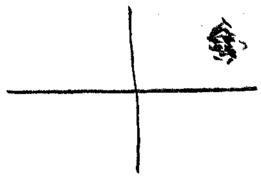


$y_k = c_k v_k + \nu_k$

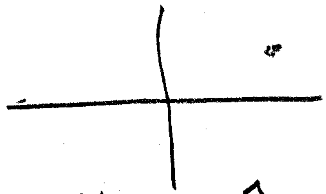
$c_k = e^{j\theta}$ the channel gain



$y_k \hat{b}_k^* = \sqrt{2E_b}\, c_k + \underbrace{\nu_k \hat{b}_k^*}_{\nu_k'}$

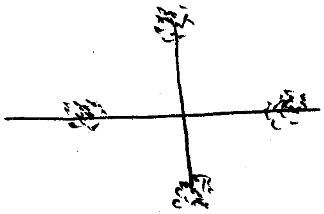


average reduces noise var



this is $\hat{c}_k$

Then $y_k \hat{c}_k^*$ is



ready for decision

This is feedforward phase correction, and it is also based on ML.