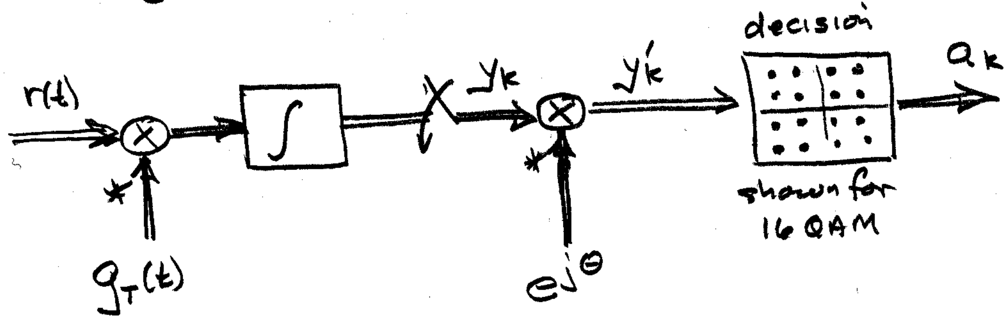
7.4 Coherent and Quasi Coherent Detection

- "coherent detection": you have a perfect phase ref $\hat{\theta} = \theta$ and you use it to de-rotate the received signal
- "quasi coherent detection": you have a reasonably good phase reference, and you decide to use it as if it were perfect and hope for the best.

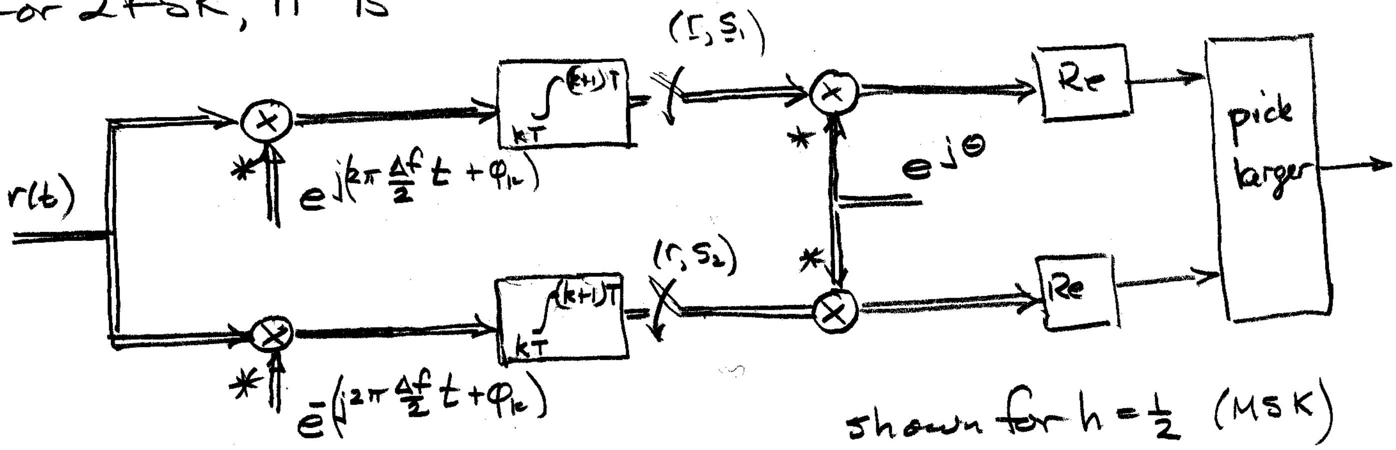
- Not too much to say here. Just:
  - structure of coherent receivers
  - performance of coherent receivers
  - justification of the quasi coherent approach

Coherent Receiver Structures

- For linear modulation it is just what we have been doing



- For 2FSK, it is



shown for $h = \frac{1}{2}$ (MSK)

- receive $r(t) = e^{j\theta} s_i(t) + n(t)$

  lump $e^{j\theta}$ with $s_i(t)$ to make modified signals $s_i'(t) = e^{j\theta} s_i(t)$

- with some orthonormal basis, we have

$$\underline{r} = \underline{s}_i' + \underline{n}$$

- equiprob signals, MAP detector has

$$\hat{i} = \arg\min_i |\underline{r} - \underline{s}_i'|^2 = \arg\min_i (\underline{r} - \underline{s}_i')^+ (\underline{r} - \underline{s}_i')$$

a reminder that things are complex

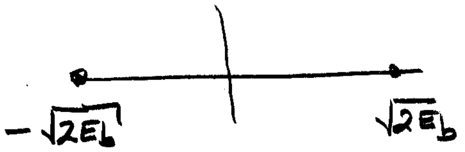
$$= \arg\min_i \left(|\underline{r}|^2 - 2\,\mathrm{Re}[\underline{s}_i'^+ \underline{r}] + |\underline{s}_i'|^2 \right)$$

$$= \arg\max_i \mathrm{Re}[(\underline{r}, \underline{s}_i')]$$

$$= \arg\max_i \mathrm{Re}\left[e^{-j\theta} \int r(t)\, s_i^*(t)\, dt \right]$$

Performance of Coherent Detection

- Coherent detectors eliminate phase offset, so performance is just what we derived in Sections 5 and 6
- Example: BPSK, a binary antipodal set



noise variance, each dimension

$\sigma_\nu^2 = N_0$

$$P_b = Q\left(\frac{\sqrt{2E_b}}{\sigma_\nu}\right) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = Q\left(\sqrt{2\gamma_b}\right)$$ as before.

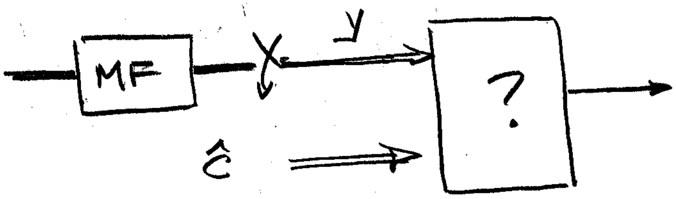
Justification of Quasi-Coherent Structure

- Suppose we follow a feedforward or pilot based approach and create an estimate of the gain $c = e^{j\theta}$

$\hat{c} = c + w$ $\quad$ w is Gaussian noise

var $\sigma_w^2 = \frac{1}{2} E[|w|^2]$

- We have for linear modulation:



– Received sample is $y = c s_i + v$ complex scalar

but $c = \hat{c} - w$, so

$$y = \hat{c} s_i \underbrace{- w s_i + v}_{u,\ \text{Gaussian}} = \hat{c} s_i + u$$

$$\sigma_{u_i}^2 = \sigma_w^2 E_i + v$$

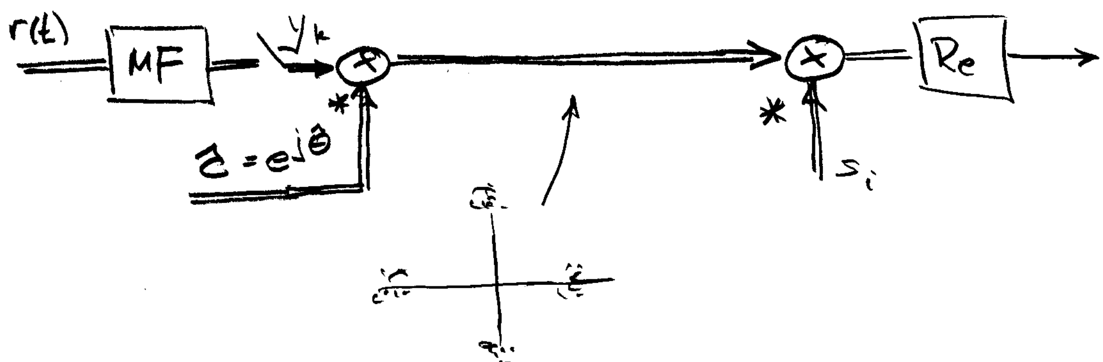
Since we have $\hat{c}$ at hand, we should do

$$\hat{i} = \arg\max_i \; p(y \mid s_i, \hat{c}) = \frac{1}{2\pi\sigma_{u_i}^2} \exp\left(-\frac{|y - \hat{c} s_i|^2}{2\sigma_{u_i}^2}\right)$$

$$= \arg\min_i \left(\frac{|y - \hat{c} s_i|^2}{2\sigma_{u_i}^2} + \ln(\sigma_{u_i}^2) \right) = \arg\min_i \left(|y - \hat{c} s_i|^2 + 2\sigma_{u_i}^2 \ln(\sigma_{v_i}^2) \right)$$

$$= \arg\min_i \left(|y|^2 - 2\mathrm{Re}\left[y s_i^* \hat{c}^* \right] + |\hat{c}|^2 |s_i|^2 + 2\sigma_{v_i}^2 \ln(\sigma_{v_i}^2) \right)$$

If equal energy $\hat{i} = \arg\max_i \mathrm{Re}\left[y s_i^* \hat{c}^* \right]$



– Performance? Difficult integrals of Bessel functions.
Ugh. Closed form in terms of Marcum Q-function.