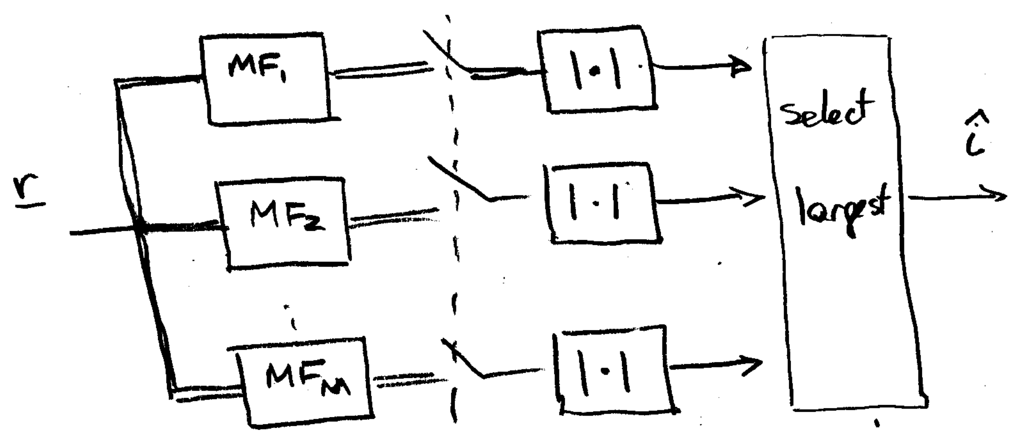
7.5 Incoherent Detection $\boxed{\text{P+S 9.4.2, 9.4.3}}$

- There are situations where no phase ref is available, and the phase has a uniform pdf over $[-\pi, \pi)$. Why?
  - Cost. It's cheaper not to do phase tracking and sometimes this justifies the increased SNR requirement
- The structure of the receiver is quite different. "Incoherent detection".

- The incoherent detector does not have phase, so it works with the magnitude of the correlations:



Proof next page

This is for equal energy signals

- Proof of incoherent structure

- Receive $\underline{r} = \underline{s}_i e^{j\theta} + \underline{n}$

  where the dimensions are due to various pulse shapes, not cos, sin (which are conveyed by the complex notation.

- We want $\hat{i} = \arg\max\, p(\underline{r}\,|\,\underline{s}_i)$, but we have $p(\underline{r}\,|\,\underline{s}_i, \theta)$. Remove the "nuisance parameter" θ by

$$p(\underline{r}\,|\,\underline{s}_i) = \int_0^{2\pi} p_{r|s\theta}(\underline{r}\,|\,\underline{s}_i, \theta)\, p_\theta(\theta)\, d\theta$$

and work with the marginal pdf.

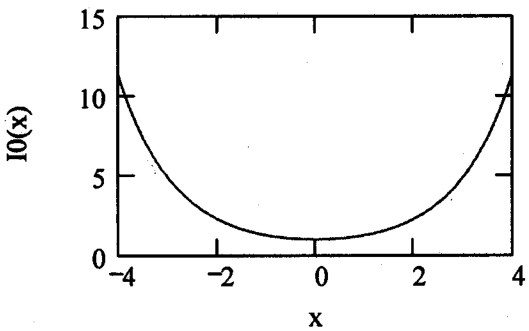
$$p_{\underline{r}|\underline{s}_i}(\underline{r}\,|\,\underline{s}_i) = \int_0^{2\pi} \frac{1}{(2\pi N_0)^N} \exp\left(-\frac{|\underline{r} - \underline{s}_i e^{j\theta}|^2}{2N_0}\right) \frac{1}{2\pi}\, d\theta$$

Now $|\underline{r} - \underline{s}_i e^{j\theta}|^2 = (\underline{r} - \underline{s}_i e^{j\theta})^{+}(\underline{r} - \underline{s}_i e^{j\theta})$

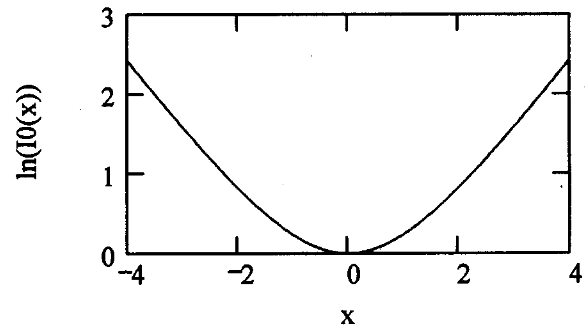
$$= |\underline{r}|^2 + |\underline{s}_i|^2 - 2\,\mathrm{Re}\left[e^{-j\theta}\, \underline{s}_i^{+} \underline{r}\right]$$

$$= |\underline{r}|^2 + |\underline{s}_i|^2 - 2\,|\underline{s}_i^{+}\underline{r}| \cos(\psi - \theta) \qquad \psi = \arg(\underline{s}_i^{+}\underline{r})$$

$$p(\underline{r}\,|\,\underline{s}_i) = \frac{\exp\left(\frac{|\underline{r}|^2 + |\underline{s}_i|^2}{2N_0}\right)}{(2\pi N_0)^N} \underbrace{\frac{1}{2\pi}\int_0^{2\pi} e^{-\frac{|\underline{s}_i^{+}\underline{r}| \cos(\psi - \theta)}{N_0}}\, d\theta}_{I_0\left(\frac{|\underline{s}_i^{+}\underline{r}|}{N_0}\right)}$$



Bessel Function I0(x)

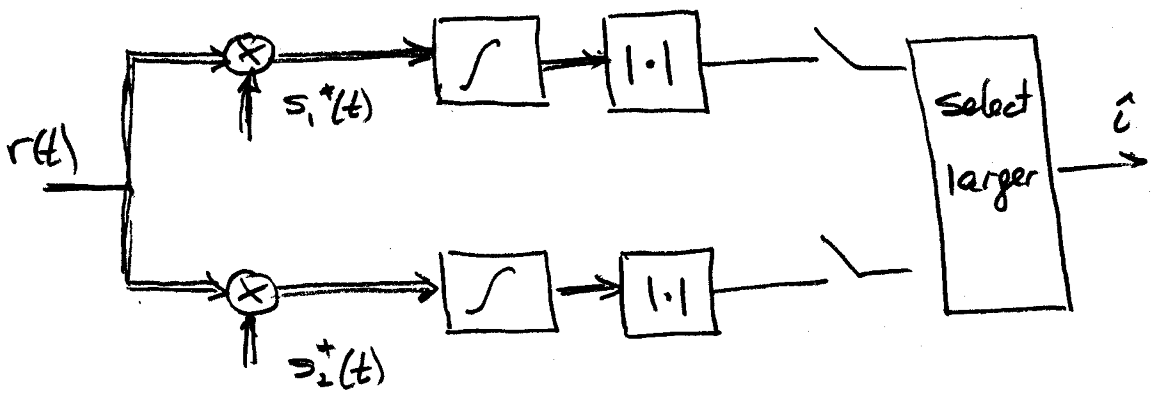


Natural Log of Bessel Function I0(x)

Since the Bessel function is monotonic in $|\underline{s}_i^{+}\underline{r}|$, for equal energy signals, the receiver is

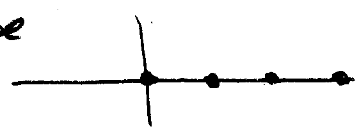
$$\hat{i} = \arg\max_i |\underline{s}_i^{+}\underline{r}|$$

which is realised by



- Corollary: Forget about PSK, QAM, two sided PAM.

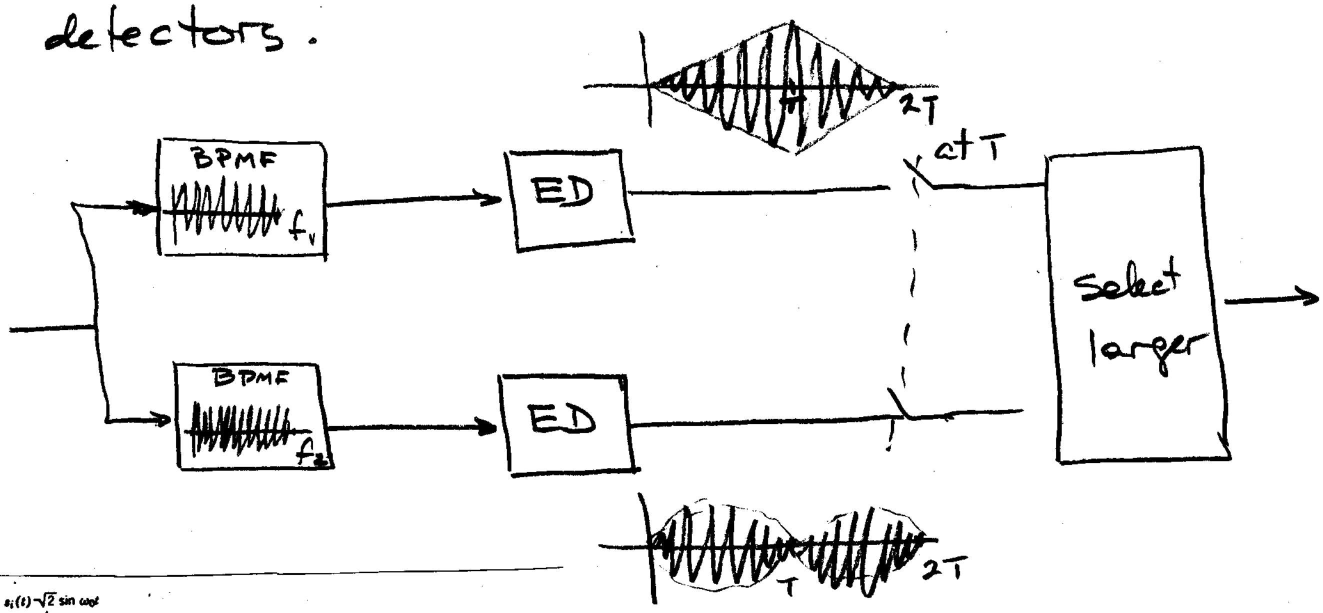
Use



or orthog $\int s_1(t)\, s_2^*(t)\, dt = 0$

or partly decorrelated $\int s_1(t)\, s_2^*(t)\, dt = \rho_{12}\sqrt{E_1 E_2}$, $|\rho| < 1$

- For FSK, there is a bandpass implementation suited for analog technology, that uses envelope detectors.



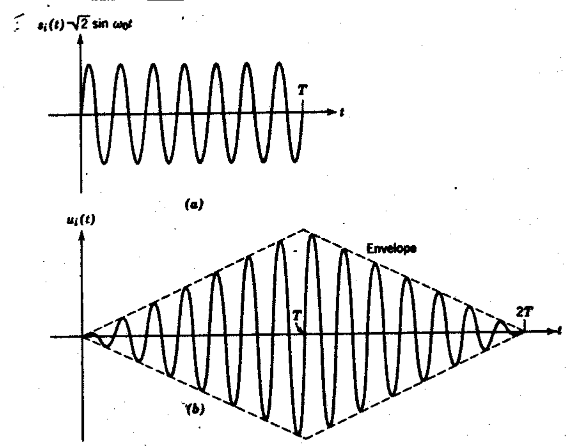
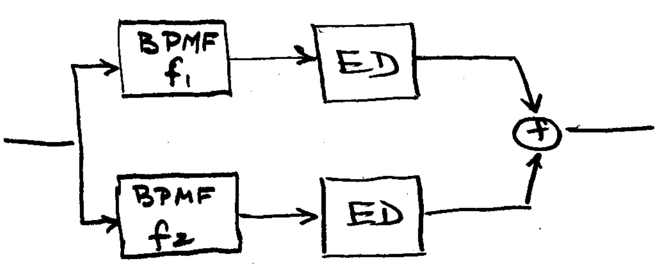


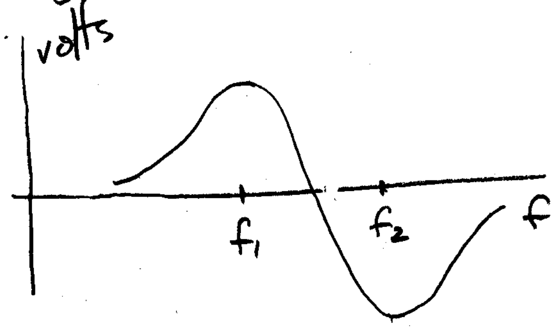
Figure 7.32 Envelope detection of (noiseless) pulsed sine wave.

If we know the phase, then we can sample at the peak (coherent) otherwise sample the envelope at its peak (incoherent)

Resembles discriminator, since freq resp of



is



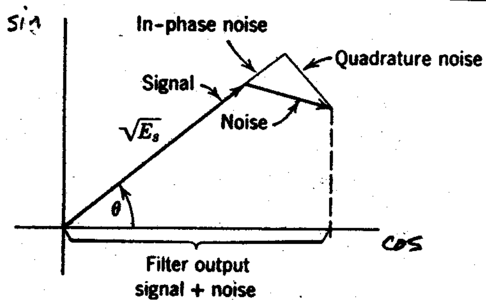


Figure 7.34 Phasor diagram for envelope detection.

for high SNR, not much difference between coherent & envelope detection. Here's why:

- conceptually, resolve noise into 2 components, one inphase with signal one in quadrature
- if we know signal phase θ (coherent) then discard quadr noise
- if we use envelope detection (incoherent) then use total length $\|\underline{r}\| = \|\underline{s} + \underline{n}\|$
- but if $\|\underline{n}\| \ll \|\underline{s}\|$ then $\|\underline{r}\|$ is affected mainly by n_I, not n_Q

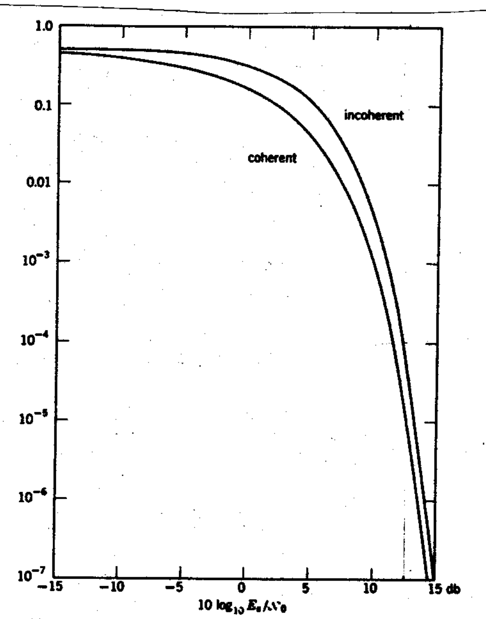


Figure 7.33 Probability of error for coherent and incoherent reception of two equally likely orthogonal signals of energy E_s.

Transmission of orthogonal signals confirms this.

Note orthogonal in phase indep sense:

$$\int e^{j\theta} s_1(t)\, s_2^*(t)\, dt = 0 \quad \text{any } \theta$$

or $\int s_1(t)\, s_2^*(t)\, dt = 0$

e.g. FSK if $\Delta f = k/T$

e.g. not if 90° sep'n sine bursts.

The near equivalence of coherent & incoherent for orthog signals doesn't mean no gain from coherent. If coherent, then can use binary antipodal & save 3 dB.

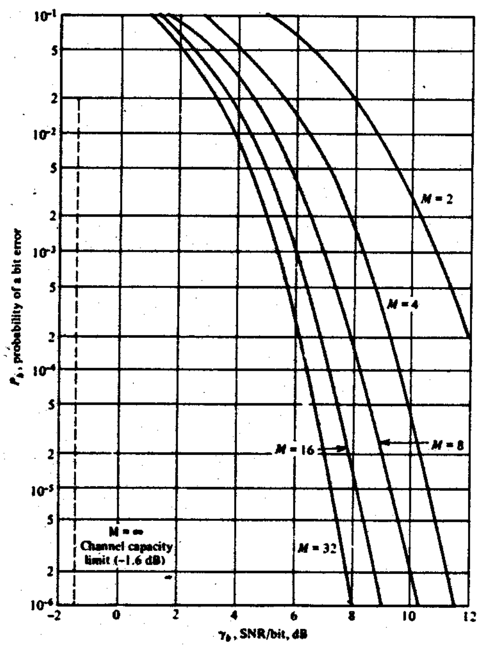


Figure 4.3.5 Probability of a bit error for noncoherent detection of orthogonal signals.

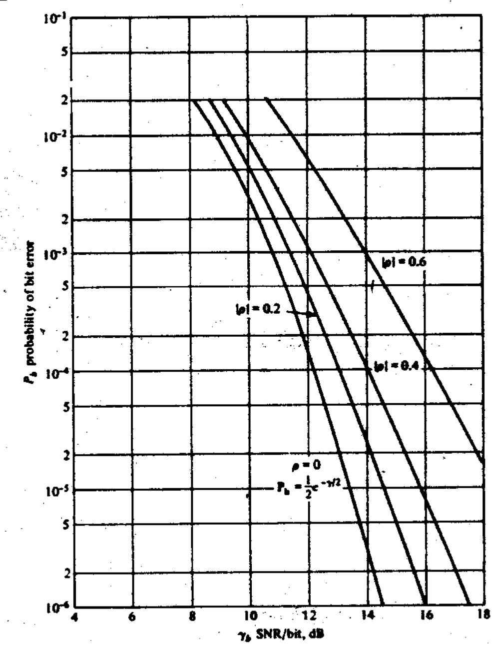


Figure 4.3.2 Probability of error for noncoherent detection.

for non orthog signals define correlation coeff

$$\rho = \frac{\int s_1(t)\, s_2^*(t)}{\sqrt{\int |s_1(t)|^2 dt \int |s_2(t)|^2 dt}}$$

For binary orthogonal signals:

$$P_e = \frac{1}{2} e^{-E_b/2N_0}$$