

7.6 Differential Detection

P+5 9.2.4

7.6.1

- The discussion on incoherent detection of single pulses shows that we pay 3dB (since orthog instead of bin. antip), plus a little for incoherent instead of coherent.
- We can save the 3dB, though, by using differential detection. The idea? Use the previous pulse as the phase ref, with data encoded into the changes (diff'l encoding)

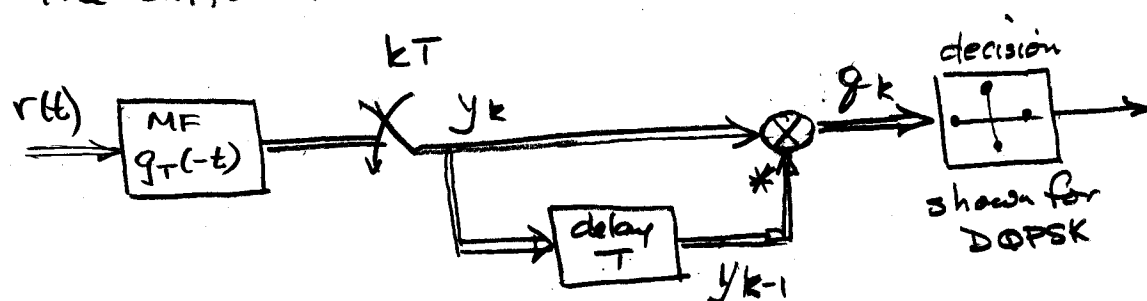
send $v(t) = \sum_k b_k g_T(t - kT)$

DPSK $b_k = b_{k-1} a_k$ $a_k, b_k \in \{-1, 1\}$

DQPSK $b_k = b_{k-1} a_k$ $a_k, b_k \in \{1, j, -1, -j\}$

so $a_k = b_k b_{k-1}^*$

- The differential detector is

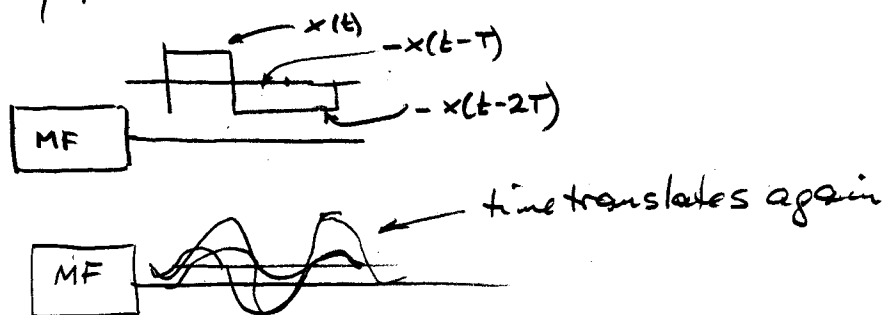


g_{k-1} takes the role of \hat{c} , p. 7.4.4

$$\begin{aligned}
 g_k &= y_k y_{k-1}^* = (e^{j\theta} \sqrt{2E_s} b_k + v_k) (e^{j\theta} \sqrt{2E_s} b_{k-1} + v_{k-1})^* \\
 &= \underbrace{2E_s b_k b_{k-1}^*}_{2E_s a_k} + \underbrace{v_k e^{-j\theta} b_{k-1}^* \sqrt{2E_s} + v_{k-1}^* e^{j\theta} b_k \sqrt{2E_s}}_{\text{zero mean Gaussian}} + \underbrace{v_k v_{k-1}^*}_{\text{zero mean non Gaussian}}
 \end{aligned}$$

Error Performance P+S 9.2.5

- A trick relates it to incoherent detection, orthog pulses.
- Consider binary DPSK... Note successive pulses are orthogonal due to time separation, so they form a basis. (assumes Nyquist property)



- Think of sending

$$a_k = +1 \text{ as } e^{j\theta} x(t - (k-1)T), e^{j\theta} x(t - kT)$$

$$a_k = -1 \text{ as } e^{j\theta} x(t - (k-1)T), -e^{j\theta} x(t - kT)$$

These composite signals are orthogonal, so use pair of successive samples of MF output for decision.

$$\underline{y} = \begin{bmatrix} y_{k-1} \\ y_k \end{bmatrix} = \underline{s}_i + \underline{n} \quad i \text{ indexes alternatives, not time}$$

$$\text{where } \underline{s}_i = e^{j\theta} \sqrt{2E_b} \begin{bmatrix} +1 \\ +1 \end{bmatrix} \text{ for } a_k = +1$$

$$= e^{j\theta} \sqrt{2E_b} \begin{bmatrix} +1 \\ -1 \end{bmatrix} \text{ for } a_k = -1$$

note $\underline{s}_0^\dagger \underline{s}_1 = 0$, so orthogonal.

- Incoherent detection of the orthogonal composite pulses does this:

$$|\underline{s}_0^\dagger \underline{y}| \geq |\underline{s}_1^\dagger \underline{y}|$$

$$|e^{j\theta} \sqrt{2E_b} [1, 1] \underline{y}| \geq |e^{j\theta} \sqrt{2E_b} [1, -1] \underline{y}|$$

$$|y_{k-1} + y_k| \geq |y_{k-1} - y_k|$$

$$|y_{k-1} + y_k|^2 \geq |y_{k-1} - y_k|^2$$

$$|y_{k-1}|^2 + 2\operatorname{Re}[y_k y_{k-1}^*] + |y_k|^2 \geq |y_{k-1}|^2 - 2\operatorname{Re}[y_k y_{k-1}^*] + |y_k|^2$$

$$\operatorname{Re}[y_k y_{k-1}^*] \geq 0$$

and this is the differential detector for BPSK

- So differential detection has the same BER as incoherent detection of orthogonal pulses of duration $2T$; that is, with energy $2E_b$

From p. 7.5.6, we obtain

$$P_b = \frac{1}{2} e^{-(2E_b)/2N_0} = \frac{1}{2} e^{-\gamma_b}$$

- We have just recovered the 3 dB lost when we abandoned binary antipodal for orthogonal in order to work without a phase reference.