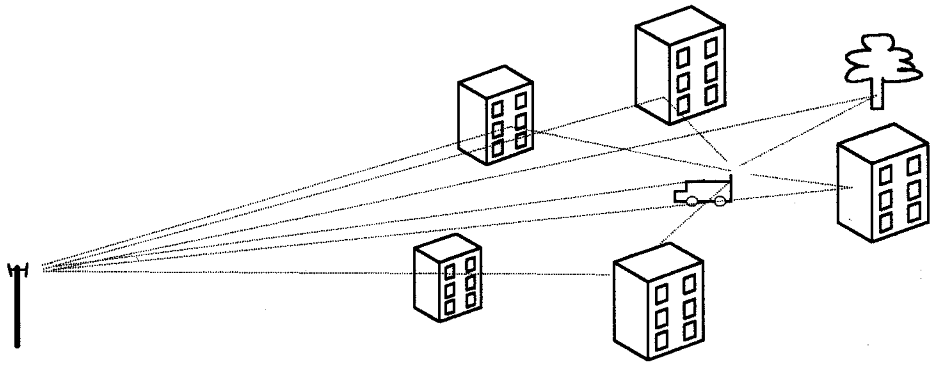
9.2 Scattering and Fading

• Zoom in on fading...

The sketch below shows a typical link between mobile and base station antennas. There are several reflectors - buildings, hills, other vehicles, etc. - around the mobile, but few or none near the base station, because it is usually mounted high above its surroundings. These reflectors are known generically as scatterers. Communication between the base and mobile takes place over many paths, each of which experiences one or more reflections, and the receiver picks up the sum of all the path signals. Note that this applies to either direction of transmission, to or from the mobile.



We can infer much of what happens just from consideration of the sketch.

* Since the individual paths are linear (i.e., they satisfy the superposition requirements), the overall multipath channel is linear.

* Each path has its own delay and gain/phase shift, so the aggregate of paths can be described by its impulse response or frequency response. Therefore different carrier frequencies will experience different gains and phase shifts. ("Gain" is used in a general sense here, since the paths really experience attenuation.)

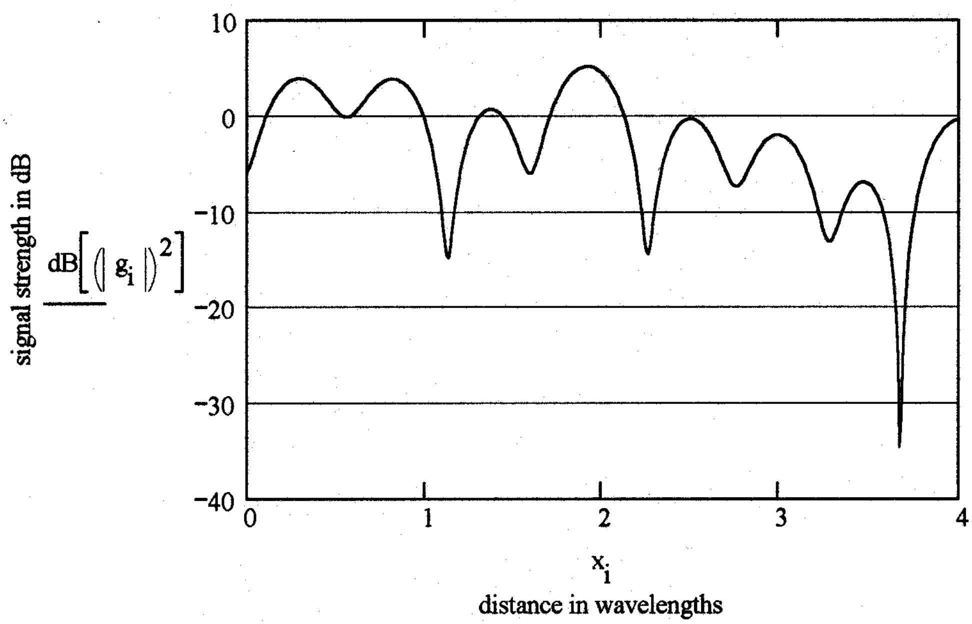
* Whether the range of delays (the "delay spread") has a significant effect on the modulation of the carrier depends on the time scale of the modulation (roughly, the reciprocal of its bandwidth). This implies that the dimensionless product of channel delay spread and signal bandwidth is an important measure.

* If the mobile changes position, the paths all change length in varying amounts Since a change in path length of just one wavelength produces 2π radians of phase shift, a displacement of a fraction of a wavelength in any direction causes a large change in the aggregate gain and phase shift, as the sum of the paths shifts between reinforcement and cancellation.

* When the mobile moves through this two-dimensional standing wave pattern, the impulse response and frequency response change with time, so the channel is a time-varying linear filter. The time variant nature of the net gain is termed "fading" and the fastest rate of change is the "Doppler frequency".

* Whether the time-varying nature of fading has a significant effect on the modulation of the carrier depends on the time span of the required receiver processing (e.g., differential detection over two symbols, equalization over many symbols, etc.). The dimensionless product of this time span and the Doppler frequency is another important parameter.

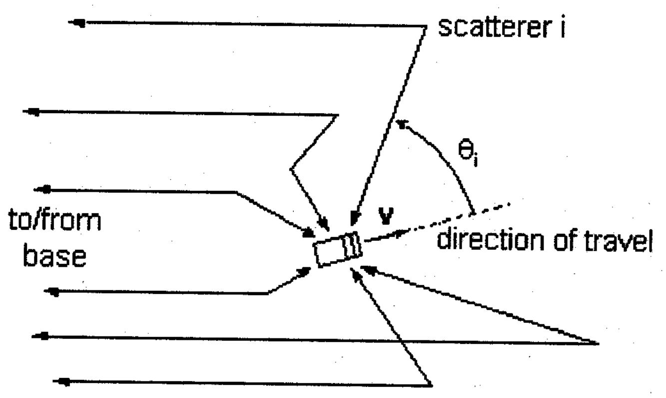
- A common picture is the signal strength in dB along an arbitrary direction for an unmodulated carrier.



Signal Strength as Function of Position

The graph shows tens of dB variation over a fraction of a wavelength. It also shows a rough periodicity in the received signal strength, with a roughly $\lambda/2$ spacing.

- Next, we need a tractable model that describes what happens to the transmitted signal.



– Bandpass signals

$$\tilde{y}(t) = \sum_i a_i \tilde{s}\left(t - \frac{x_i}{c}\right)$$

a_i reflection coeff
x_i path length

$$= \sum_i a_i \, Re\left[s\left(t - \frac{x_i}{c}\right) e^{j2\pi f_c \left(t - \frac{x_i}{c}\right)}\right]$$

$$= Re\left[\underbrace{\left(\sum_i a_i e^{-j2\pi x_i/\lambda} s\left(t - \frac{x_i}{c}\right)\right)}_{y(t)} e^{j2\pi f_c t}\right]$$

Complex envelopes

$$y(t) = \sum_i a_i e^{-j2\pi x_i/\lambda} s\left(t - \frac{x_i}{c}\right)$$

an FIR filter, like $y(t) = \sum_i h_i \, s(t - \tau_i)$

Simplest model - Flat Fading

If:
- variations among the path lengths x_i are negligible compared with the 15 km symbols
- equivalently, variations among the $\tau_i = x_i/c$ (the delay spread) are negligible compared with symbol (or chip) duration.

then
$$y(t) = \sum_i a_i e^{-j2\pi x_i/\lambda} s(t - \tfrac{x_i}{c}) \approx s(t) \underbrace{\sum_i a_i e^{-j2\pi x_i/\lambda}}_{g}$$
$$= g\, s(t)$$

- The net complex gain is random (though position dependent), and
  - $|g|$ is the amplitude gain
  - $\measuredangle g$ is the phase shift

- Since no frequency dependence, this is termed "flat fading"

- If mobile moves, $\Delta x_i = -v \cos(\theta_i) t$ (see sketch)

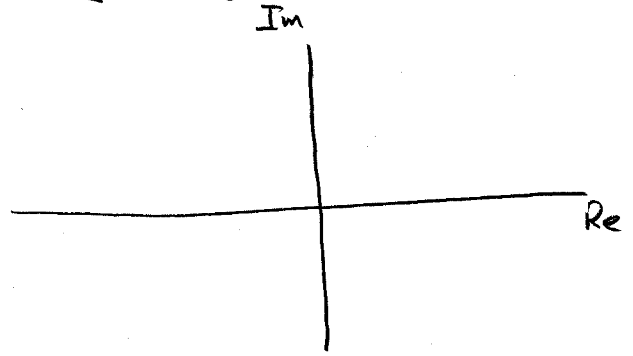
$$y(t) \approx s(t) \sum_i a_i e^{-j2\pi x_i/\lambda} e^{j2\pi \frac{v}{\lambda} \cos(\theta_i) t}$$

$$y(t) = g(t)\, s(t) \qquad \frac{v}{\lambda} = f_D \text{ Doppler freq}$$

Complex gain $g(t)$ varies slowly compared with $s(t)$.

$f_D = 100$ Hz for $f_c = 1$ GHz, $v = 108$ km/h

- The gain $g(t)$ varies randomly



Passing near the origin produces
- a fade
- a phase hit of about 180°

Challenges! Think what it does to a PLL.

Rayleigh Fading

- With enough paths, central limit theorem makes $g(t)$ a complex Gaussian process.

$$p_g(g) = \frac{1}{2\pi\sigma_g^2} e^{-\frac{|g|^2}{2\sigma_g^2}} \qquad \sigma_g^2 = \frac{1}{2} E[|g|^2]$$

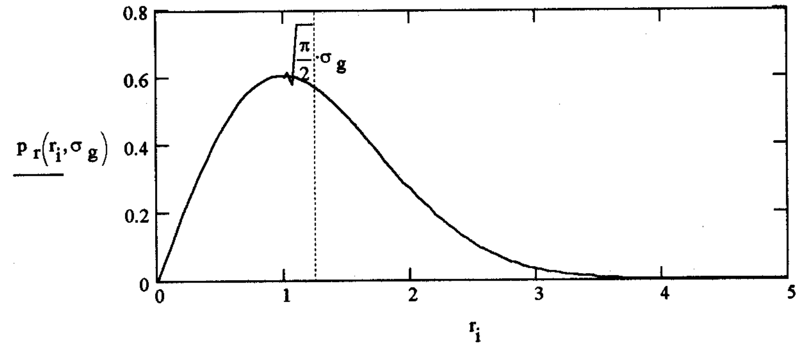
- Change to polars $\quad g = r e^{j\theta}$

$$p_{r,\theta}(r,\theta) = \frac{r}{2\pi\sigma_g^2} e^{-r^2/2\sigma_g^2} \qquad r \geq 0$$

or

$$p_r(r) = \frac{r}{\sigma_g^2} e^{-r^2/2\sigma_g^2}, \quad r \geq 0 \qquad p_\theta(\theta) = \frac{1}{2\pi}, \quad 0 \leq \theta \leq 2\pi$$

Note magnitude $r = |g|$, phase $\theta = \arg(g)$ are independent.



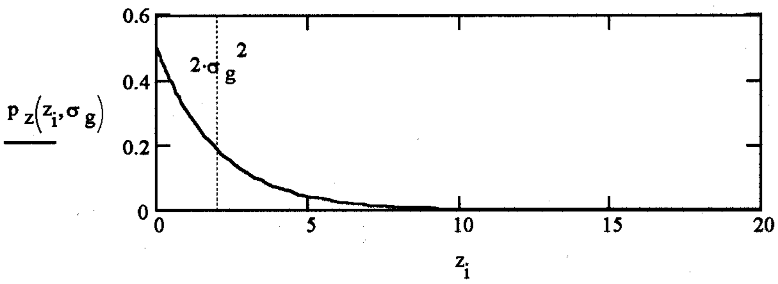
Rayleigh distribution of amplitude

shown for

$\sigma_g = 1$

mean	mode	standard deviation
$\mu_r = \sqrt{\frac{\pi}{2}}\cdot\sigma_g$	σ_g	$\sigma_r = \sqrt{2-\frac{\pi}{2}}\cdot\sigma_g$

The squared magnitude $r^2 = |g|^2 = g_r^2 + g_i^2$ has an exponential distribution.



$\sigma_g^2 = 1$

$z = r^2$

$$p_z(z) = \frac{1}{2\sigma_g^2} e^{-z/2\sigma_g^2}$$

Exponentially distributed squared magnitude

The mean equals the decay constant	and so does the standard deviation
$\mu_z = 2\cdot\sigma_g^2$	$\sigma_z = 2\cdot\sigma_g^2$

also known as χ^2 with 2 degrees of freedom.

More Difficult Model – Frequency Selective Fading

- Back to transmission model p 9.2.3

$$y(t) = \sum_i a_i e^{-j2\pi x_i/\lambda} s(t-\tau_i)$$

If delay spread is enough that the $s(t-\tau_i)$ look different i.e. if delay spread no longer negligible wrt time scale of signal, then channel is a filter

typical $\tau_d \sim 5\mu s$

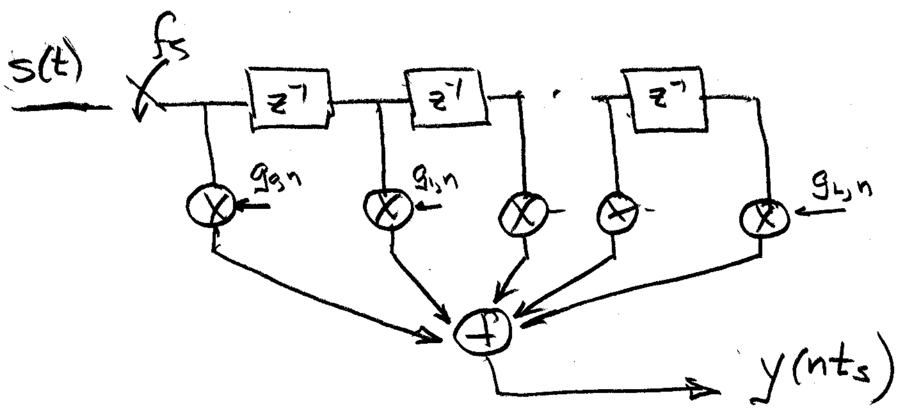
"frequency selective fading"

- Often model with sampling rate f_s, $t_s = \frac{1}{f_s}$ (over Nyquist rate)

$$y(t) = \sum_k g_k s(t-kt_s) \quad \text{or} \quad y(nt_s) = \sum_k g_k s((n-k)t_s)$$

and if mobile moves, the coeffs are $g_k(t) = g_k(nt_s)$

$$y(nt_s) = \sum_k g_{k,n} s((n-k)t_s)$$



a linear time varying filter.