DEMONSTRATION OF ONE-SHOT MODEM PROCESSING

This note illustrates two equivalent forms of receive modem for one-shot (i.e., isolated) pulses demonstrates that they make identical decisions.

Define the Signals

As a first step, we'll define the transmitted signals. In order to mimic the DSP-based modem view of the world, assume that the signals are discrete-time. In this example, we'll use $K := 16$ samples across the pulse; that's more than enough, but it does make it easier to visualize.

To keep things manageable, keep the number of dimensions to $M := 2$ and, for convenience, assume that the basis functions are cosine and sine:

$$k := 0 .. K-1$$

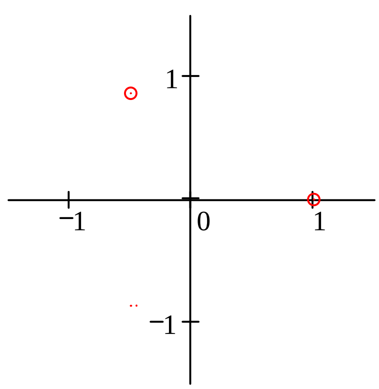
$$\psi_{0_k} := \frac{1}{\sqrt{8}} \cdot \cos\left(2 \cdot \pi \cdot \frac{k}{K}\right) \qquad \psi_{1_k} := \frac{1}{\sqrt{8}} \cdot \sin\left(2 \cdot \pi \cdot \frac{k}{K}\right)$$

Are they orthonormal? A check shows that they are:

$$\psi_0^T \cdot \psi_0 = (1) \qquad \psi_1^T \cdot \psi_1 = (1) \qquad \psi_1^T \cdot \psi_0 = (0)$$

For the signals themselves, we'll choose the $M := 3$ simplex set: $m := 0 .. M-1$

$$E_s := 1 \qquad \mathbf{s_0} := \sqrt{E_s} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \mathbf{s_1} := \sqrt{E_s} \cdot \begin{pmatrix} \frac{-1}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix} \qquad \mathbf{s_2} := \sqrt{E_s} \cdot \begin{pmatrix} \frac{-1}{2} \\ \frac{-\sqrt{3}}{2} \end{pmatrix}$$



is is the constellation.

For convenience in generation and detection, pack the signals together as columns of a matrix:

$\mathbf{S}^{\langle 0 \rangle} := \mathbf{s_0}$ $\mathbf{S}^{\langle 1 \rangle} := \mathbf{s_1}$ $\mathbf{S}^{\langle 2 \rangle} := \mathbf{s_2}$ **S** is *K*x*M*

and generate the signal index *m* (the information) $\mathrm{data3}(x) := \mathrm{floor}(\mathrm{rnd}(3))$ (*x* is a dummy argument)

Define the Channel

We'll work with the AWGN channel. Selecting the SNR defines the noise variance:

$\gamma_s := 5$ $N_o := \frac{E_s}{\gamma_s}$

This generates a Gaussian noise sample of the right variance

$$\mathrm{noise}(x) := \sqrt{\frac{N_o}{2}} \cdot \sqrt{-2 \cdot \ln(\mathrm{rnd}(1))} \cdot \cos(\mathrm{rnd}(2 \cdot \pi))$$

(*x* is a dummy argument)

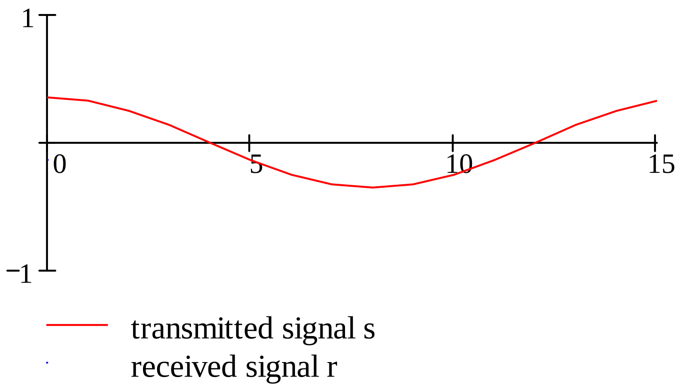
and this generates the vector of noise samples:

$$\mathrm{noisevec}(x) := \left| \begin{array}{l} \text{for } i \in 0..K-1 \\ \quad n_i \leftarrow \mathrm{noise}(x) \\ n \end{array} \right.$$

The Received Signal

Now to generate the received signal:

get index	get signal components	get signal waveform	get noise samples	form received signal
$m := \mathrm{data3}(1)$	$\mathbf{s} := \mathbf{S}^{\langle m \rangle}$	$s := \mathbf{s}_0 \cdot \psi_0 + \mathbf{s}_1 \cdot \psi_1$	$n := \mathrm{noisevec}(1)$	$r := s + n$
	length 2	length *K*	length *K*	length *K*



To see other possible received waveforms, park the cursor on one of the highlighted regions and press F9.

Receiver Processing

We'll do two forms of receiver processing. However, they both need a "select largest" or argr function. Define it here:

$$\text{argmax}(x) := \left| \begin{array}{l} M \leftarrow \text{length}(x) \\ \text{for } m \in 0..M-1 \\ \quad \text{index}_m \leftarrow m \\ x' \leftarrow \text{augment}(x, \text{index}) \\ x' \leftarrow \text{csort}(x', 0) \\ x'_{M-1,1} \end{array} \right.$$

csort rearranges the rows of a matrix into ascending order on the values of the identified column

Correlate Against Signals

In this form of the receiver, we calculate the inner products of the received waveform against possible signals, subtract a bias and select the largest. The bias is zero, since we have equiprobab equal-energy signals.

Form a matrix with columns equal to our signal waveforms. Column inc $j := 0..M-1$

$$S^{\langle j \rangle} := \mathbf{S}_{0,j} \cdot \psi_0 + \mathbf{S}_{1,j} \cdot \psi_1$$

This *K*x*M* matrix contains our reference waveforms.

Now get the inner products as a length-M vector of metrics **c**:

$$\mathbf{c} := S^T \cdot r \qquad \mathbf{c} = \begin{pmatrix} 0.983 \\ -0.284 \\ -0.699 \end{pmatrix}$$

and the decision is $m_{hat1} := \text{argmax}(\mathbf{c})$ But is it right? $m = 0$ $m_{hat1} = 0$

Correlate Against Basis Waveforms

In the second form of receiver, we vectorize the received signal to get its components in the ψ basis

$$\mathbf{\Psi} := \text{augment}(\psi_0, \psi_1) \qquad \mathbf{r} := \mathbf{\Psi}^T \cdot r \qquad \mathbf{r} = \begin{pmatrix} 0.983 \\ 0.239 \end{pmatrix}$$

then do the inner products with the signal vectors expressed in the same basis to get the metrics:

$$\mathbf{c} := \mathbf{S}^T \cdot \mathbf{r} \qquad \mathbf{c} = \begin{pmatrix} 0.983 \\ -0.284 \\ -0.699 \end{pmatrix} \qquad m_{hat2} := \text{argmax}(\mathbf{c}) \qquad m_{hat2} = 0$$

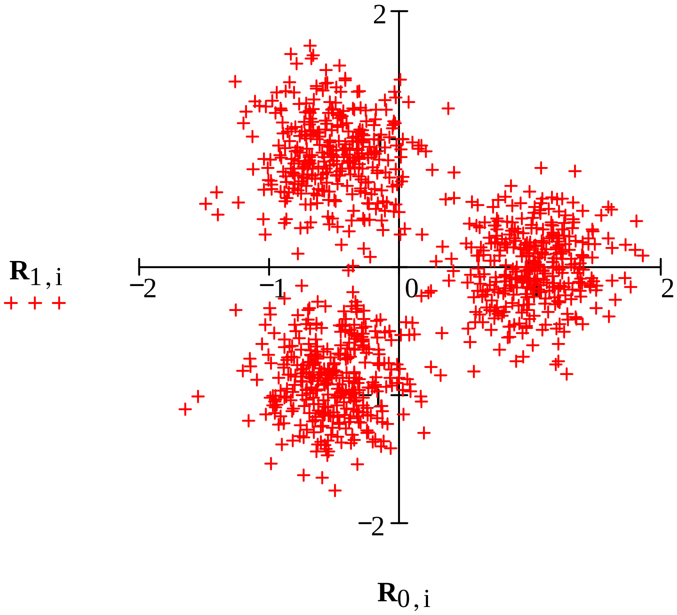
Not surprisingly, the results are the same.

What the Receiver Sees

It is interesting to visualize the variability of the received signal over many, many trials. Set up a simulation and plot the results.

$$\text{Rxsigs}(N_{sim}) := \left| \begin{array}{l} \text{for } i \in 0 .. N_{sim} - 1 \\ \quad \left| \begin{array}{l} m \leftarrow \text{data3}(i) \\ r \leftarrow S^{\langle m \rangle} + \text{noisevec}(i) \\ \mathbf{r} \leftarrow \mathbf{\Psi}^T \cdot r \\ \mathbf{R}^{\langle i \rangle} \leftarrow \mathbf{r} \end{array} \right. \\ \mathbf{R} \end{array} \right.$$

$N_{sim} := 1000$ $\mathbf{R} := \text{Rxsigs}(N_{sim})$ $i := 0 .. N_{sim} - 1$



To see others, park the cursor on the highlighted equation an press F9.

To see the effect of other SNR values, go back and change γ_s.