MULTIUSER DETECTION

IN A MULTICELL ENVIRONMENT

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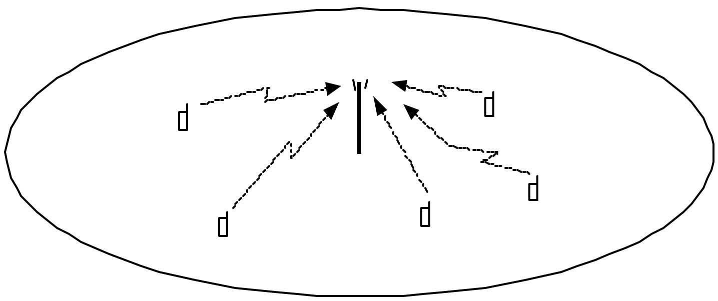
- “With multiuser detection, it’s as if the interferers were gone – you get single-user performance.”
- MUD may be able to increase wireless system capacity.
- Most analyses consider a single cell.
- When applied system-wide, with multiple cells, some conclusions shift.
- We’ll see how selected MUD methods play out in the transition from single cell to multiple cells.

1. MUD FOR SINGLE CELLS

Consider uplink only.



Mutually interfering signals in same time or frequency slot

MUD tries to separate them

$$r(t) = g_1 s_1(t) + \ldots + g_K s_K(t) + n(t)$$

Objective: more users per cell-slot:

- more than 1 for narrowband (TDMA)
- more than about 10%-15% of processing gain for CDMA (about 20 for current systems)

MUD candidates:

- Linear: With one or more antennas, form linear combinations of projections of $r(t)$ onto subspaces to obtain an *analog* estimate of each user signal *separately*, then slice for decisions.

- Hybrid (ML wannabees): SIC (successive interference cancellation) is typical; *linearly* estimate strongest user; make decision; subtract that *reconstructed* signal; continue with the rest; repeat if desired.

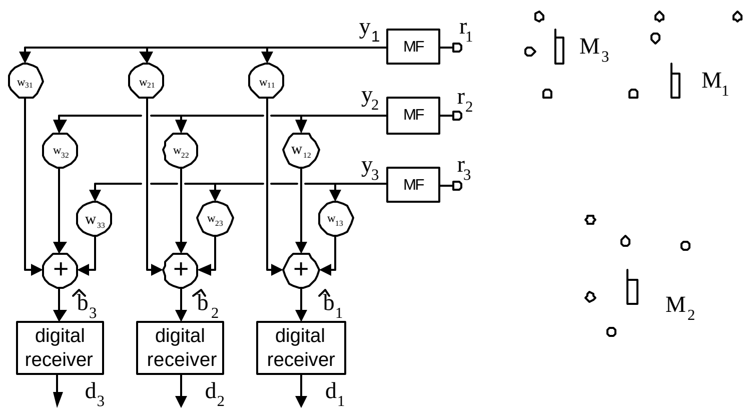
- ML: Run down a list of all possible combinations of users' data, see which is the best match to $r(t)$. Inherently *digital*, with a *joint decision*.

Type:	Effectiveness:	Complexity:
linear	moderate	moderate
hybrid	better	high
maximum likelihood	high	very high*

* exponential in number of users – but remember Moore's Law!

2. MUD FOR NARROWBAND SYSTEMS

- A linear receiver uses antennas to separate the signals



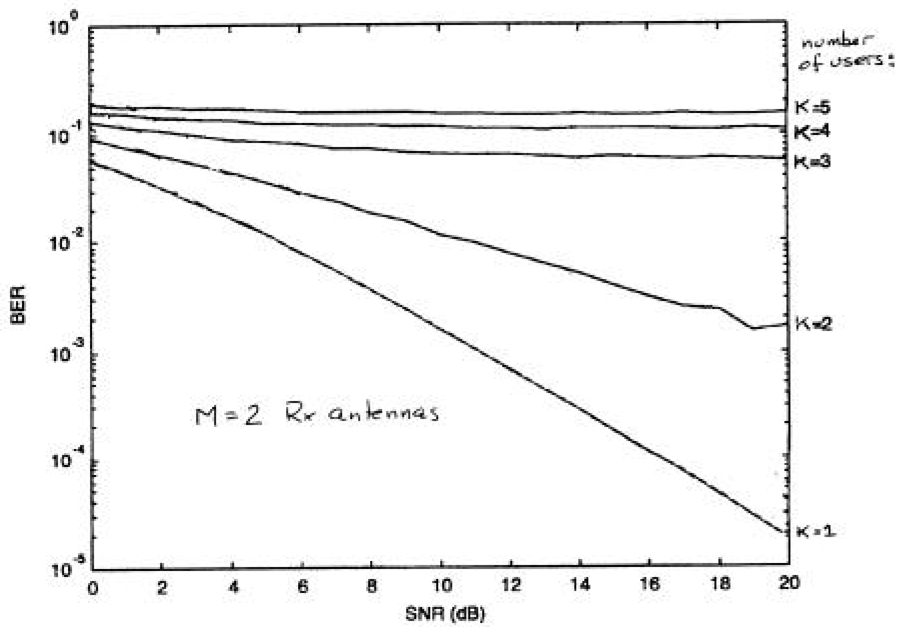
Select the user-*i* weight vector to be orthogonal to gain vectors of the other users, and they *disappear* – so no interference. Support up to *M* intracell users with *M* antennas.

Variation: compromise between interference and noise with MMSE combining, instead of ZF.

Either way, computation is proportional to M^3 to invert the gains matrix.

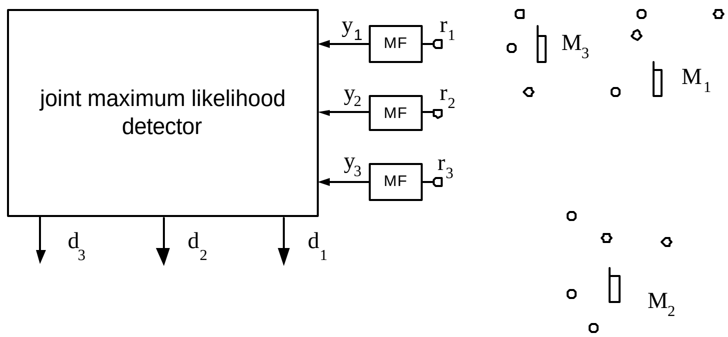
The cost? Lose one degree of diversity for each interferer cancelled. In the three-antenna sketch, BER changes from SNR^{-3}, SNR^{-2}, SNR^{-1}.

The plot below is for M=2 antennas and MMSE combining.



- The two antennas can support up to two users.

- An ML detector finds which of the 2^K multiuser bit patterns is the best match to the signal observed at the *M* antennas.



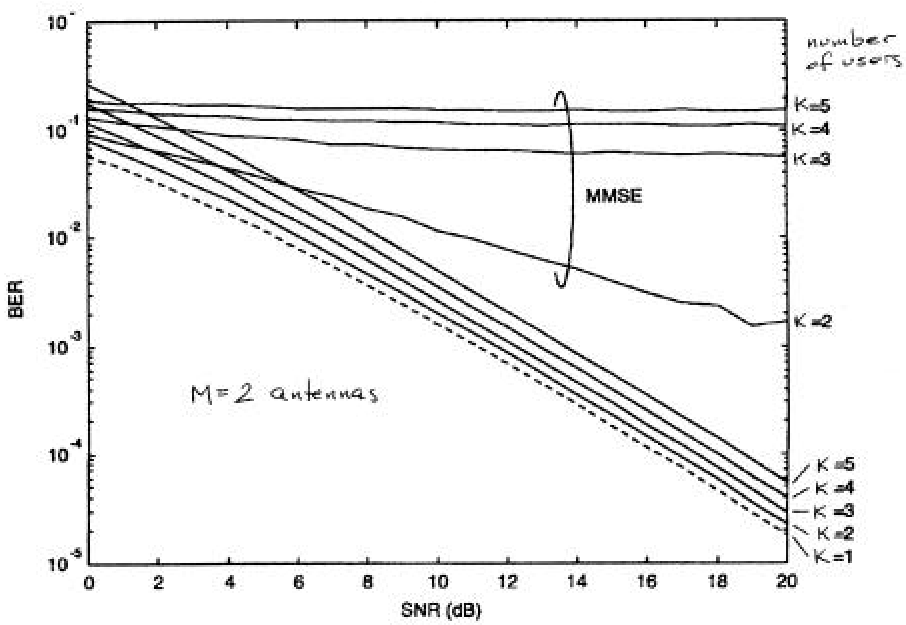
The other users don't quite disappear, but their effect is minimal.

All users experience *M*-fold diversity – no diversity loss!

No hard link between number of users and number of antennas, so *M* antennas can support *as many users as you like*.

The cost? Computation is proportional to $M2^K$ for binary (!)

Comparison of the linear MMSE combining and ML detection shows quite different variation with SNR.
[Grant & Cavers '98]

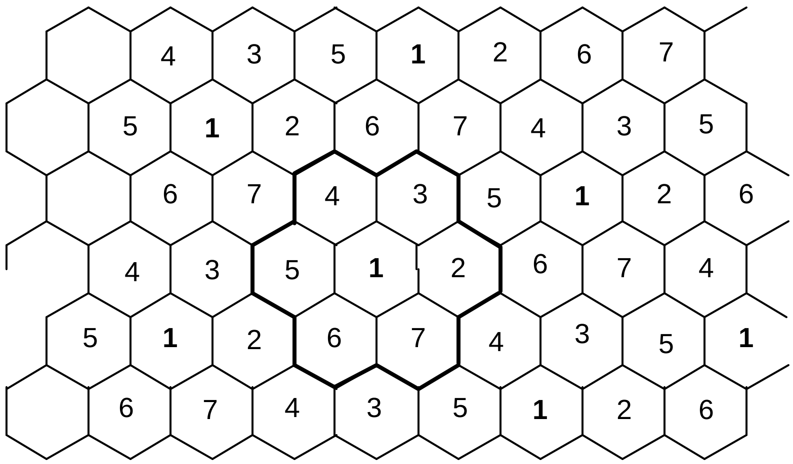


The dB shift of the ML curves becomes very small as the number of antennas increases.

- Looks as though both linear and ML methods can increase the capacity by a large or very large factor, depending on the investment in antennas and/or computation.

But…

- … a system has more than one cell. Narrowband systems are usually organised into clusters – here's an idealised view:



And here's the catch:

⇒ if you take advantage of MUD to increase the number of users in a cell, so will all the cochannel cells;

⇒ the level of noise-like unmodelled interference therefore rises in direct proportion;

⇒ and you have to increase the reuse distance and cluster size to compensate;

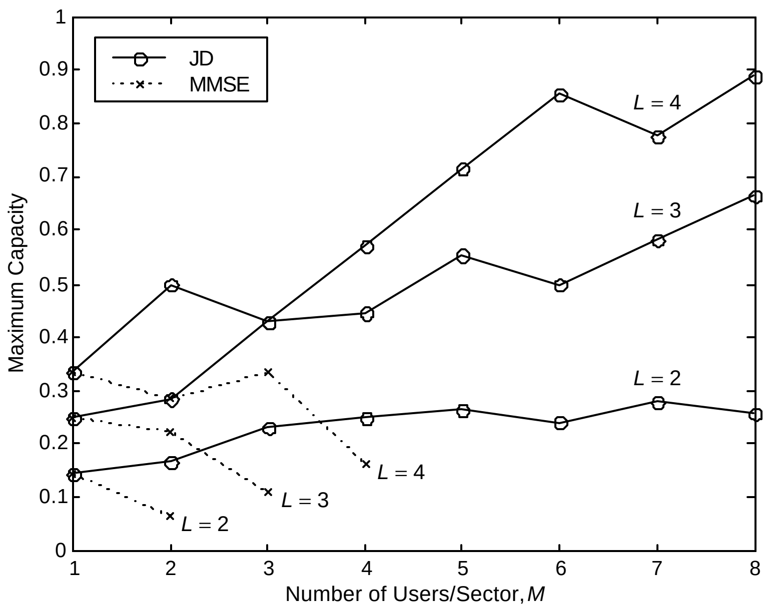
⇒ which reduces capacity.

Which effect wins?

Comparison: MUD by MMSE and MUD by ML

Define max capacity as # intracell users/cluster size. If it equals 1, then effective reuse factor of unity, like CDMA.

[Grant & Cavers '00]



L antennas

M users/cell

120^O sectors

inverse 4^{th}, fading, shadowing

designed for 1% outage

⇒ Use of ML (denoted JD here) to increase intracell users causes system capacity to *increase*, approaching unity and beyond.

⇒ Even with ML, increasing intracell users M allows a less-than-proportional increase in the system's max capacity (but faster with more antennas).

⇒ Use of MMSE combining to increase intracell users actually *reduces* system capacity!

⇒ Increasing the number of antennas benefits both systems, but benefits ML much more.

- For the record, the algorithms are

ML $$\mathbf{d} = \arg\min_{\mathbf{b}} \left(\left\| \mathbf{y} - \mathbf{C}\mathbf{b} \right\|^2 \right)$$

ZF, MMSE $$\mathbf{d} = \operatorname{sgn}(\mathbf{W}\mathbf{y})$$

where

$\mathbf{y}$ is vector of MF outputs

$\mathbf{C}$ is array of complex channel gains

$\mathbf{b}$ is vector of transmitted symbols

$\mathbf{W}$ is pseudoinverse (or its MMSE counterpart) of $\mathbf{C}$

3. MUD FOR CDMA SYSTEMS

- The shift from single-cell MUD to multicell MUD is even odder in CDMA…

- CDMA systems distinguish users by different pulse shapes (in contrast to narrowband). At antenna m in a symbol-synchronous, flat-fading system

$$r_m(t) = c_{m,1} b_1 p_1(t) + \ldots + c_{m,K} b_K p_K(t) + n(t)$$

- Filters matched to the pulse shapes produce sufficient statistics. At antenna m

$$\mathbf{y}_m = \mathbf{R}\mathbf{C}_m \mathbf{b} + \mathrm{n}_m$$

  where $\mathbf{C}$ is diagonal matrix of gains and $\mathbf{R}$ is a strongly diagonal matrix of pulse correlations.

- The conventional receiver combines antennas for sufficient statistics, and slices to get decisions

$$\hat{\mathbf{b}} = \sum_{m=1}^{M} \mathbf{C}^H \mathbf{y}_m = \sum_{m=1}^{M} \mathbf{C}_m^H \mathbf{R}\mathbf{C}_m \mathbf{b} + \mathrm{a} \qquad\qquad \mathbf{d} = \mathrm{sgn}\left(\hat{\mathbf{b}}\right)$$

- The micro-view of single-cell MUD:

$$\hat{\mathbf{b}} = \sum_{m=1}^{M} \mathbf{C}^{H} \mathbf{y}_{m} = \sum_{m=1}^{M} \mathbf{C}_{m}^{H} \mathbf{R} \mathbf{C}_{m} \mathbf{b} + \text{a}$$

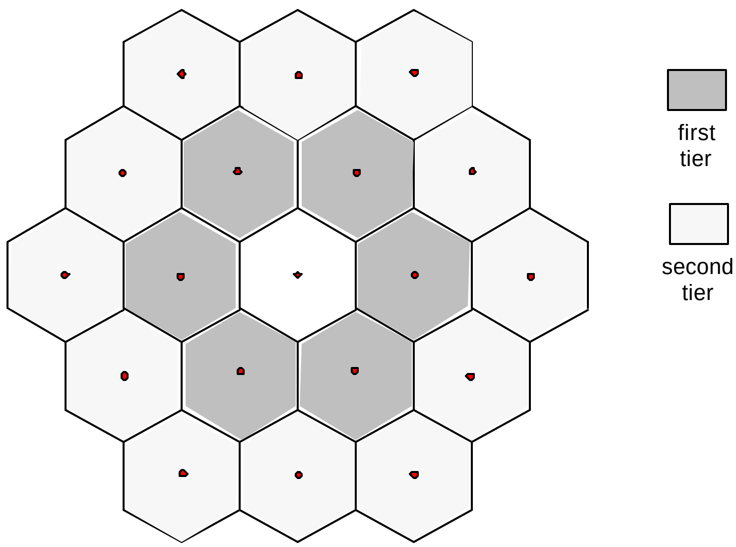
Off-diagonal elements of **R** produce residual interference among bits – small individually, but significant in aggregate. They limit the number of simultaneous users.

- Many MUD approaches to suppress or eliminate residual interference. The optimal ML detector is out of its depth – too many users! Only suboptimal methods are taken seriously.

- But in this presentation… pretend we can find a *perfect* MUD, so that intracell interference vanishes, as if by magic.
  ⇒ Even better than optimal ML.
  ⇒ Better than anything ever devised.
  ⇒ Equivalent to $\mathbf{R} = \mathbf{I}$.
  ⇒ All users enjoy order-*M* diversity, with no interference.
  ⇒ Have as many users as you like.

- What benefit do we gain from magic MUD when we move to a multicell environment?

- Again, the cochannel cells (all cells, in CDMA) increase their own user populations, causing other-cell interference to increase in direct proportion.



Now what happens??

- Use a simple, classical model [Gilhousen '91]:
  ⇒Each signal experiences path loss, shadowing and fading.
  ⇒Spatially white shadowing.
  ⇒Mobiles are connected to the nearest base.
  ⇒Perfect power control compensates for path loss, shadowing between mobile and its base.
  ⇒Speech activity factor reduces interference.
  ⇒Sectorisation.

- Central result: SNR per bit $\left(E_b / N_o\right)$ is a random variable

$$g_b = \frac{W/R}{\underbrace{\sum_{k=1}^{N_s-1} c_k}_{\text{intracell interference}} + \underbrace{\frac{I}{S}}_{\text{other cell interference}} + \underbrace{\frac{h}{S}}_{\text{additive noise}}}$$

[Gilhousen '91]

W is bandwidth, *R* is bit rate, *W*/*R* is processing gain

$\sum_{k=1}^{N_s-1} c_k$ is voice activity, binomial, probability a

N_s users per sector

S is own-signal power seen at base after perfect power control

I is other-cell interference power, mean $0.247N_s$, variance $0.078N_s$, model as Gaussian

h is thermal noise power in bandwidth *W*, h/S a design parameter (about 1.25)

- So we have

$$g_b = \frac{W/R}{\underbrace{\sum_{k=1}^{N_s-1} c_k}_{\substack{\text{intracell}\\ \text{interference}}} + \underbrace{\frac{I}{S}}_{\substack{\text{other cell}\\ \text{interference}}} + \underbrace{\frac{h}{S}}_{\substack{\text{additive}\\ \text{noise}}}} \qquad \text{conventional detector}$$

$$g_b = \frac{W/R}{\underbrace{\frac{I}{S}}_{\substack{\text{other cell}\\ \text{interference}}} + \underbrace{\frac{h}{S}}_{\substack{\text{additive}\\ \text{noise}}}} \qquad \text{magic MUD}$$

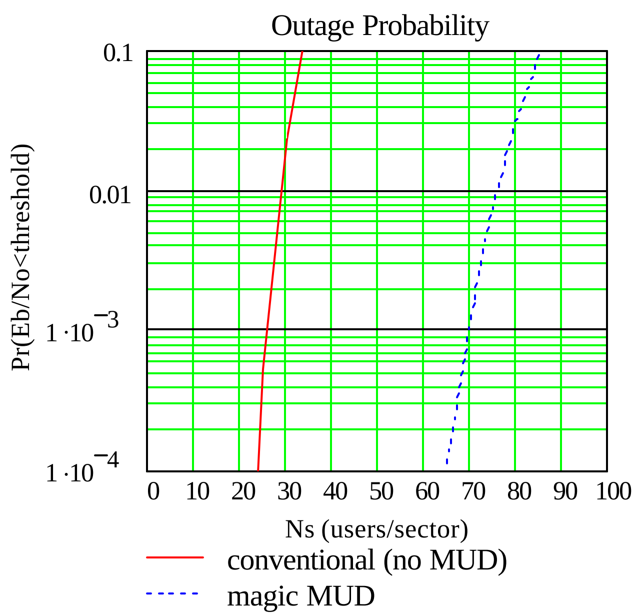
both with pdf as a mixture of Gaussian pdfs.

- Set up system on the basis of tolerable “outage probability” – the probability that g_b falls below some threshold g_t. As the number of users per sector N_s increases, so does the outage probability. This limits capacity.

- The SNR threshold for a BER of 10^{-3} depends on the code, and the number of receive antennas. For rate ½ convolutional code, dual diversity (two receive antennas), the threshold is

$$g_t = 5 \quad (7\text{ dB})$$

and here is the corresponding outage probability:



threshold:

$dB(\gamma_t) = 6.99$

processing gain:

$\frac{W}{R} = 128.125$

speech activity factor:

$\alpha = 0.375$

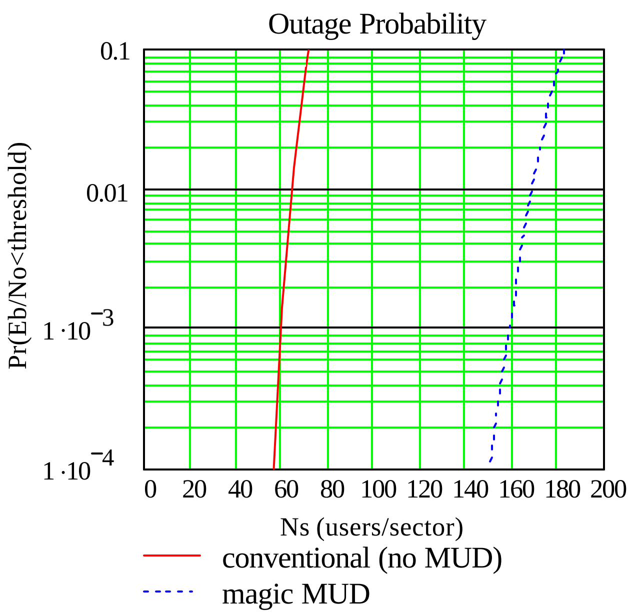
noise power/user power

$\frac{\eta}{S} = 1.25$

Applying magic MUD increases the number of users by a factor of only 2.5 to 2.7 ! ? !

(Ignore the absolute numbers – the model ignores correlated shadowing, imperfect power control, connection to best (not closest) base, soft handoff, irregular geometry, etc.)

Processing gain doesn't change the basic comparison. Double it:



threshold:

$dB(\gamma_t) = 6.99$

processing gain:

$\frac{W}{R} = 256.25$

speech activity factor:

$\alpha = 0.375$

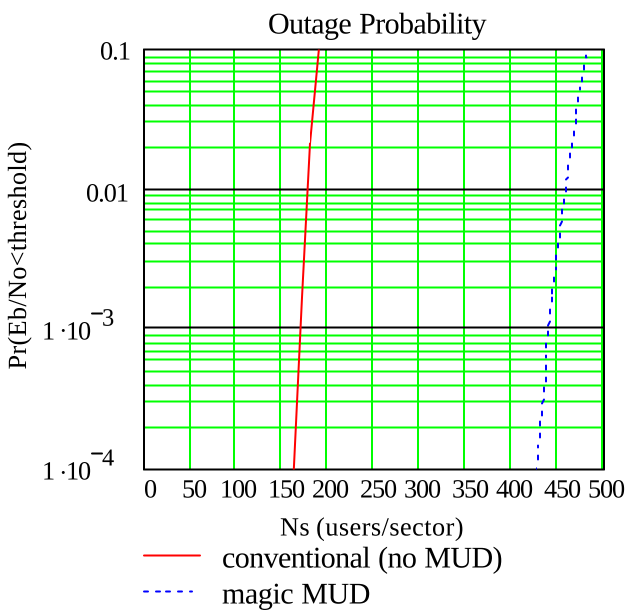
noise power/user power

$\frac{\eta}{S} = 1.25$

Again, magic MUD increases the number of users by a factor of only 2.5 to 2.7.

But we gave narrowband schemes the luxury of 4 base station antennas. If a CDMA base station has 4 antennas, will that change the comparison?

New threshold for order-4 diversity, rate ½ : $g_t = 1$ (0 dB)



threshold:

$dB(\gamma_t) = 0$

processing gain:

$\frac{W}{R} = 128.125$

speech activity factor:

$\alpha = 0.375$

noise power/user power

$\frac{\eta}{S} = 1.25$

The number of users increases, but the effect of magic MUD is still only a little over a factor of 2 ½ increase in number of users.

Even with magic MUD, there is a “2 ½” barrier on the increase in capacity in CDMA.

MUD methods developed to date, ones that can cope with tens of users, are *not* magic, so the increase will be less.

4. MACRODIVERSITY

- It is clear that unmodelled interference – interference from other cells – is the culprit in restricting improvement due to MUD. Why not include it in the MUD, as well?

- Make every base station detect every user bit in the system? No:

  ⇒ Too much computation

  ⇒ MUD benefits the weak, impedes the strong.

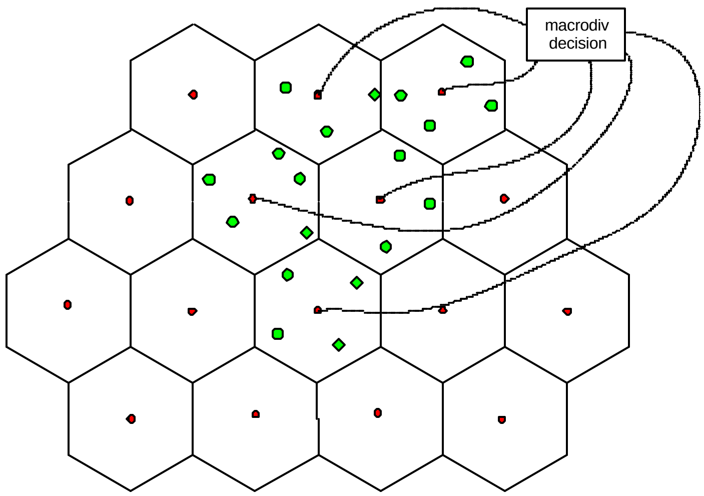
- Pool base station signals, then detect every bit? Yes:

  ⇒ Base stations act as multiple sensors, so strong signals not impeded.

  ⇒ But still too much computation, it appears.

  This is macrodiversity.

- Look at the macrodiversity computation question more closely.



If ML, we might attempt the familiar $\mathbf{d} = \arg\min_{\mathbf{b}} \left(\|\mathbf{y} - \mathbf{C}\mathbf{b}\|^2 \right)$ and realise that exponential growth has beaten us.

However,

$\Rightarrow$ the users "seen" by each antenna fall into different, but overlapping, subsets;

$\Rightarrow$ and there is a "micro-optimal" solution for each antenna, if treated individually.

Tailor-made for dynamic programming solutions.

- The conditional metric merge (CMM) algorithm [Welburn '01]:

  ⇒ is a spatio-temporal extension of the Viterbi algorithm, developed from dynamic programming principles;

  ⇒ allows recursive calculation of the globally optimal ML solution from locally optimal tentative solutions;

  ⇒ keeps computation to a minimum;

  ⇒ allows most of the computation to be distributed among base stations.

A paradigm for space-time multiuser detection?

5. CONCLUSIONS

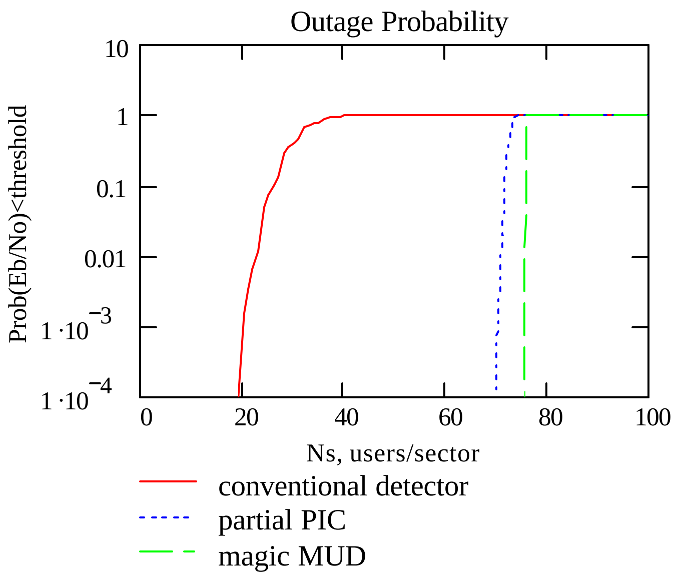
- MUD methods give significant capacity increases when applied to a single cell.

- Other-cell interference significantly reduces capacity increase in a multicell environment:
  - ⇒ Narrowband ML MUD capacity increases more slowly than linearly with number of users/cell.
  - ⇒ Narrowband MMSE MUD actually *decreases* capacity.
  - ⇒ CDMA MUD hits a 2 ½ barrier, even with a "magic MUD."

- The remedy is macrodiversity. Expensive computationally, but mitigated by the new CMM algorithm.

Appendix

The best CDMA MUD so far is partial parallel interference cancellation [Divsalar, Simon, Raphaeli '98], reported only for equipower, no fading, single cell. Adapt to multicell without fading or shadowing, as test bed.



processing gain

$N_{cs} = 100$

SNR threshold

$\gamma_t = 5$ $\quad dB(\gamma_t) = 6.99$

PPIC with 3 iterations

Looks very good! Caveat: absence of 2 ½ barrier shows the test bed is inadequate. Need study of PPIC in fading.