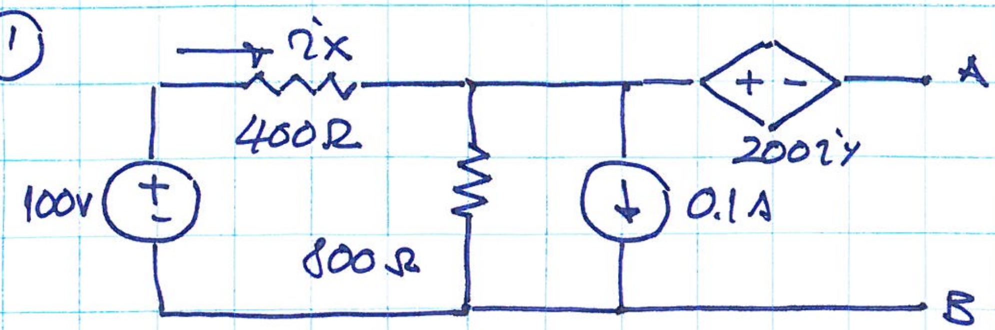
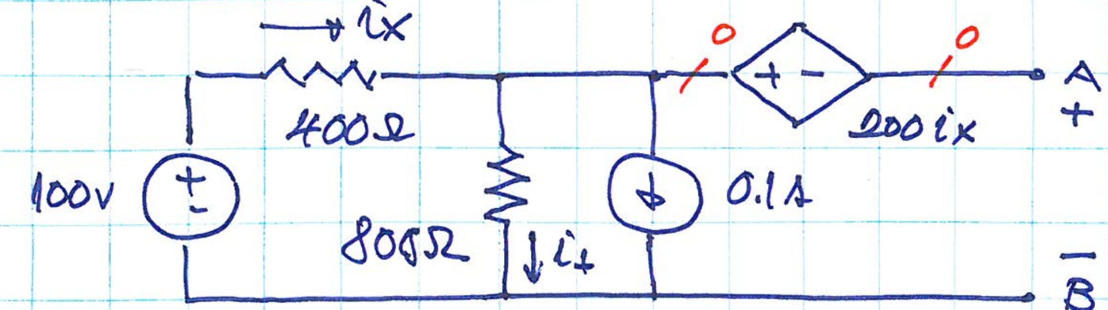
(1)




(a) Thévenin equivalent:

$V_{Th}$: Terminals A-B: open




KCL: $i_x = i_1 + 0.1$ $\qquad V_{Th} = V_{AB}$

KVL:

$$100 - 400 \cdot i_x - 800 \cdot i_1 = 0$$

$$V_{AB} + 200 \cdot i_x - 800 \cdot i_1 = 0$$

or:

$$400 \cdot i_x + 800 \cdot (i_x - 0.1) = 100 \quad /:100$$

$$4 i_x + 8 i_x - 0.8 = 1 \; ; \; 12 i_x = 1.8$$

$$i_x = \frac{18}{120} \; ; \; i_x = \frac{3}{20}$$

Then: $V_{AB} = -200 \cdot i_x + 800 \cdot v_1$

$$V_{AB} = -200 \times i_x + 800 (i_x - 0.1)$$

$V_{AB} = 600 \cdot i_x - 80$

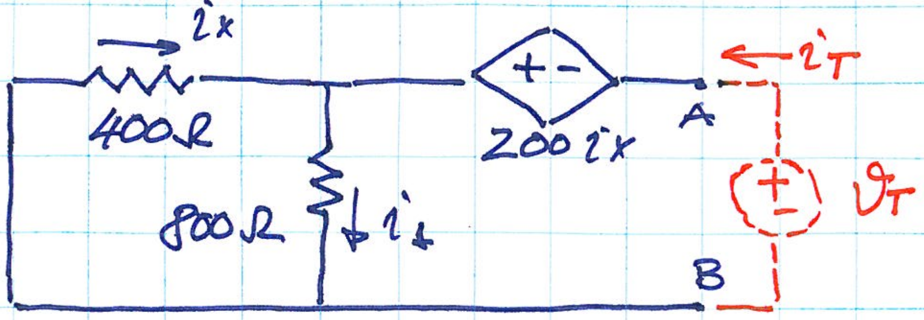
$V_{AB} = 600 \cdot \frac{3}{20} - 80$

$V_{AB} = 90 - 80$ ; $V_{AB} = 10V$

$\underline{V_{Th} = 10\,V}$

(b)

$R_{Th}$ : Disconnet the independent sources:




KVL: $400 \cdot i_x + 800 \cdot i_1 = 0 \Rightarrow i_x + 2i_1 = 0$

$i_1 = -\frac{1}{2} i_x$

KCL: $i_x + i_T = i_1$ ; $i_x = i_1 - i_T$

or

$i_x = -\frac{1}{2} i_x - i_T$

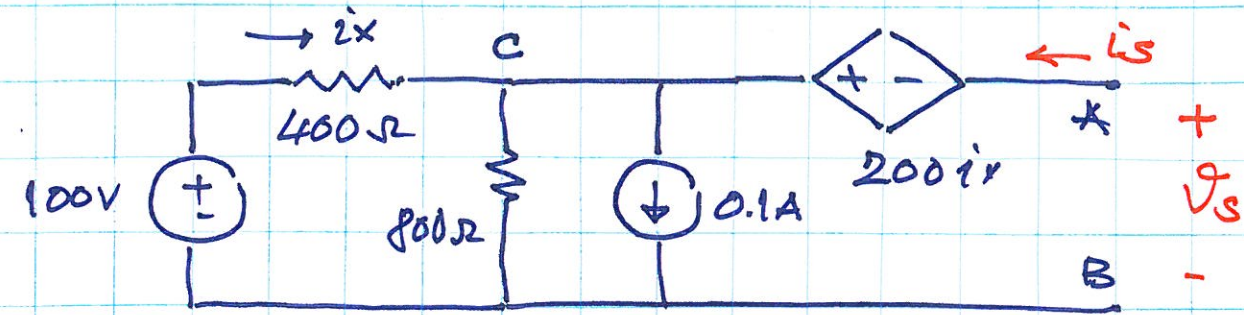
$\frac{3}{2} i_x = -i_T$ ; $i_x = -\frac{2}{3} \cdot i_T$

KVL: $V_T + 200 \cdot i_x + 400 \cdot i_x = 0$

$V_T = -600 \cdot i_x$ ; $V_T = -600 \times (-\frac{2}{3} i_T)$

$\frac{V_T}{i_T} = 400$ ; $R_{Th} = 400\,\Omega$

* One may find $V_{th}$ and $R_{th}$ in one step:




Node equation:

$$\frac{V_c-100}{400}+\frac{V_c}{800}+0.1-i_s=0 \quad /\times 800$$

$$2(V_c-100)+V_c+80-800\,i_s=0$$

$$3V_c-800\,i_s=120$$

$$V_c=40+\frac{800}{3}\cdot i_s \qquad (1)$$

KVL:

$$V_s+200\cdot i_x-V_c=0$$

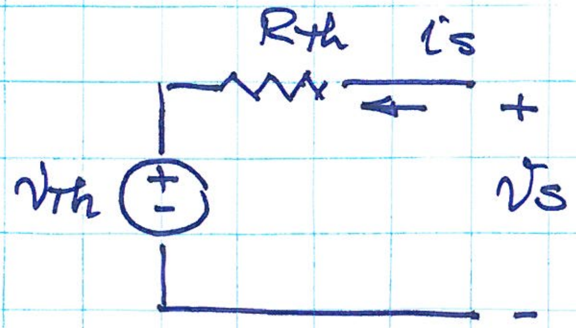
$$V_s=V_c-200\cdot i_x$$

Since $i_x=\dfrac{100-V_c}{400}$

$$V_s=V_c-200\cdot\frac{100-V_c}{400}\ ;\ V_s=\frac{3}{2}V_c-50 \qquad (2)$$

Using (1): $V_s=\frac{3}{2}\cdot\left(40+\frac{800}{3}\cdot i_s\right)-50$

$$V_s=10+400\cdot i_s$$




$$V_s - R_{th} \cdot i_s - V_{th} = 0$$

$$V_s = V_{th} + R_{th} \cdot i_s$$

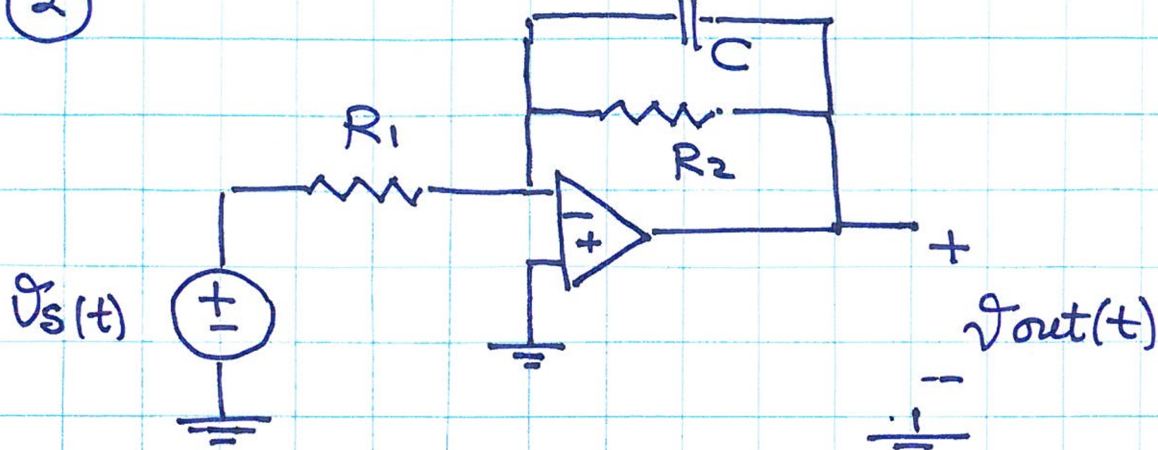
$$V_s = 10 + 400 \cdot i_s$$

Itena:

$$V_{th} = 10\ V$$

$$R_{th} = 400\,\Omega$$

(2)




$C = 1\mu F$

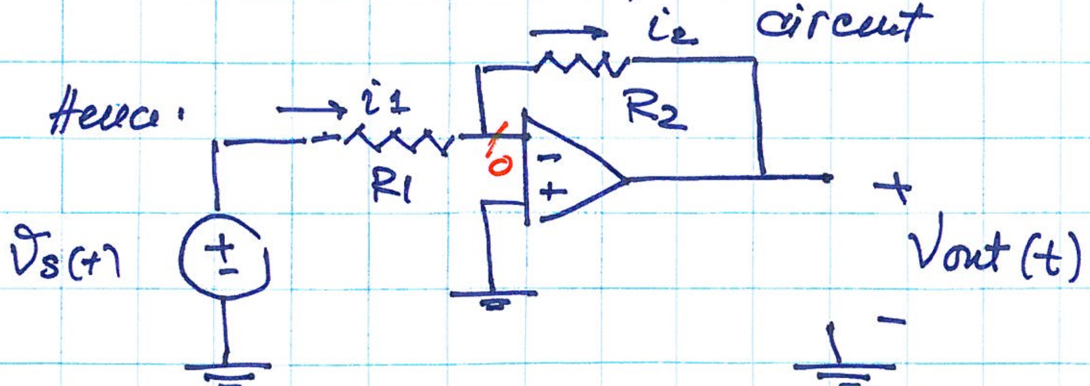
$R_2 = 10\ M\Omega$

$v_s(t) = -5u(t)$

(a) gain = ?

At $t \to \infty$, C behaves as an open circuit

Hence:




$v_- = v_+$ (ideal op-amp)

$i_- = 0$

$v_s - R_1 \cdot i_1 = 0$ ; $i_1 = \frac{v_s}{R_1}$ ; $i_1 = i_2$ (KCL)

$V_{out} + R_2 . i_2 = 0$ ; $V_{out} = -R_2 \cdot i_2$

$V_{out} = -R_2 \cdot i_1$ ;

$V_{out} = -R_2 \cdot v_s / R_1$

$gain = -\frac{R_2}{R_1}$

For gain to be equal to 10:

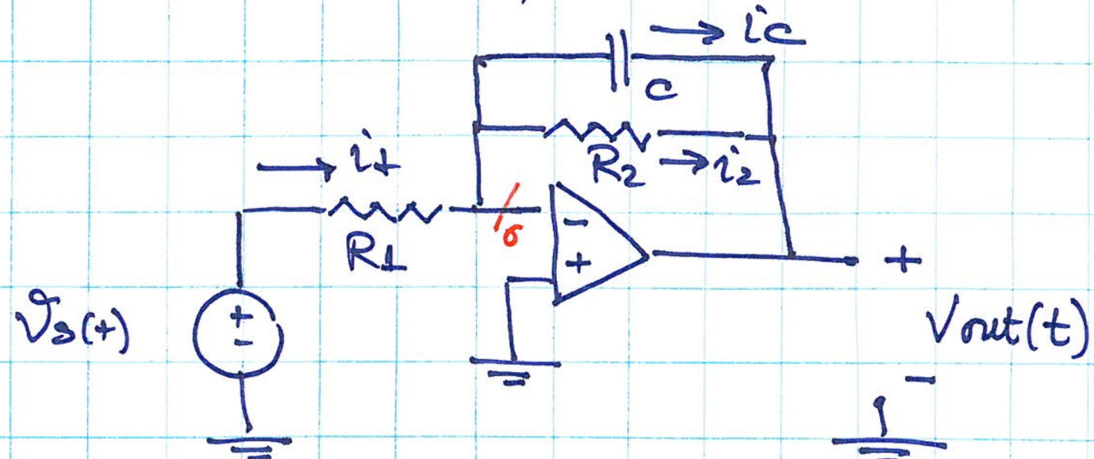
$$\frac{R_2}{R_1} = 10 \quad ; R_2 = 10 \times R_1$$

$$R_2 = 10\ M\Omega$$

Hence, $R_1 = 1\ M\Omega$

(b) Time domain response:

$$V_C(0-) = 0 \ ; \quad V_C(0+) = 0$$




$$v_s(t) - R_1 \cdot i_1 = 0$$

$$i_1 = i_2 + i_C \quad ; \quad i_C = C\frac{dv_c}{dt}$$

$$V_{out}(t) + R_2 \cdot i_2 = 0$$

Hence:

$$i_1 = \frac{v_s}{R_1} \qquad \frac{v_s}{R_1} = -\frac{V_{out}}{R_2} + C \cdot \frac{dv_c}{dt}$$

$$i_2 = -\frac{V_{out}}{R_2}$$

Since, $V_{out} + V_C = 0$ ; $V_{out} = -v_c$

Hence: $$C\frac{dv_c}{dt} + \frac{1}{R_2} \cdot V_C = \frac{1}{R_1} \cdot v_s$$ State equation

$$\frac{dv_C}{dt} + \frac{1}{R_2C} v_C = \frac{1}{R_1C} \cdot v_S \quad (+)$$

Recall : $v_S(t) = -5\,u(t)$

For $t \geq 0$ : $v_S(t) = -5$

$v_C(0+) = 0$

Characteristic equation:

$$s + \frac{1}{R_2C} = 0 \;;\; s = -\frac{1}{R_2C}$$

$$v_C(t) = K e^{-t/R_2C} + v_{cp}(t)$$

$v_{cp}(t) = A$ (constant, since $v_S(t)$ is a constant)

From (+) : $$\frac{d\,v_{cp}(t)}{dt} + \frac{1}{R_2C} \cdot v_{cp}(t) = \frac{1}{R_1C} \cdot v_S(t)$$

$$0 + \frac{1}{R_2C} \cdot A = \frac{1}{R_1C} \cdot (-5)$$

$$A = -\frac{R_2}{R_1} 5 \;;\quad A = -50$$

$R_2 = 10\ M\Omega$

$R_1 = 1\ M\Omega$

$$v_C(t) = K e^{-t/R_2C} - 50$$

$$v_C(0+) = 0 \Rightarrow K - 50 = 0 \;;\; K = 50$$

$$v_C(t) = 50\left(1 - e^{-t/R_2C}\right)$$

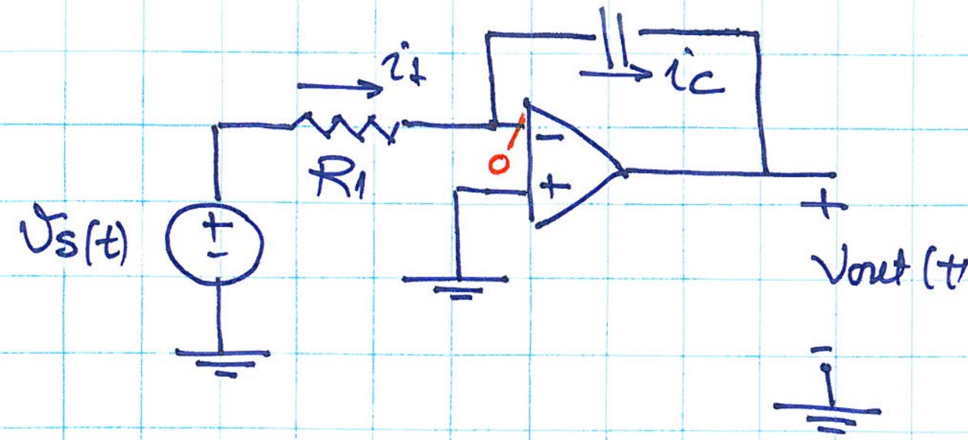
$R_2C = 10 \times 10^6 \times 10^{-6}$

$R_2C = 10$

$1/R_2C = 0.1$

$$v_C(t) = 50\left(1 - e^{-0.1t}\right) u(t)$$

(c) Ideal (non-leaky) integrater has ideal capacitr without the leakage represented by $R_2$.




$$V_s(t) - R_1 i_1 = 0$$

$$i_1 = \frac{V_s(t)}{R_1}$$

$$i_1 = i_C$$

$$i_1 = C\frac{dv_c}{dt} \quad ; \quad V_{out} = -v_c$$

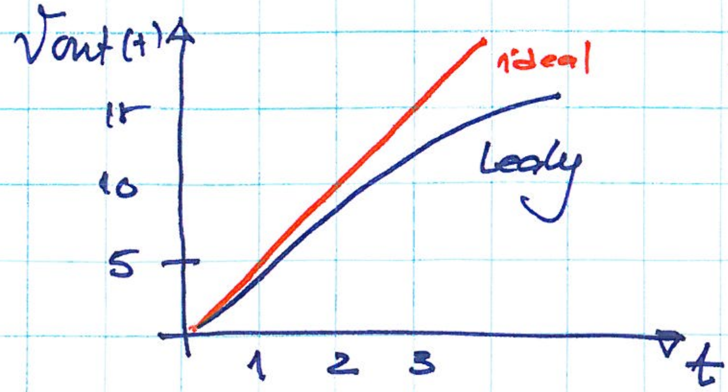
$$C\frac{dv_c}{dt} = \frac{1}{R_1}\cdot v_s(t)$$

$$\frac{dv_c}{dt} = \frac{1}{R_1C}\cdot(-5) \qquad R_1C = 1\times10^{+6}\times10^{-6}$$
$$R_1C = 1$$

$$\frac{dv_c}{dt} = -5 \quad ; \quad V_c = -5t\cdot u(t)$$

$$V_{out} + V_c = 0$$

$$V_{out}(t) = 5\cdot t\cdot u(t) \quad \text{ideal case}$$

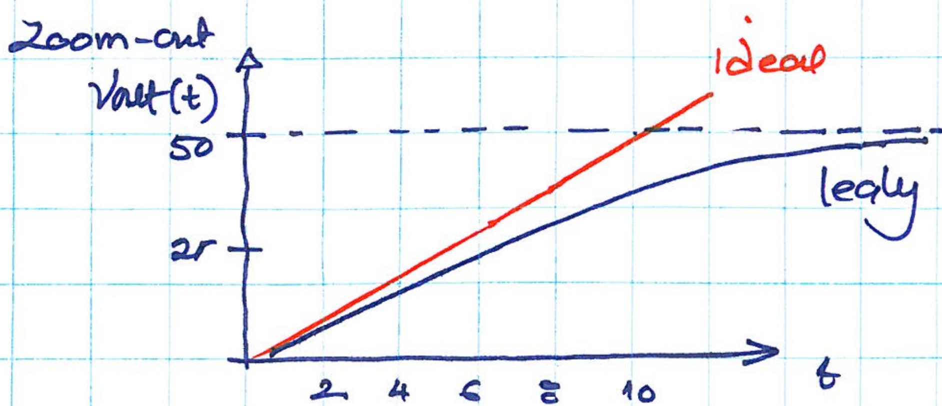



Ideal:

$$V_{out}(t) = 5t\,u(t)$$

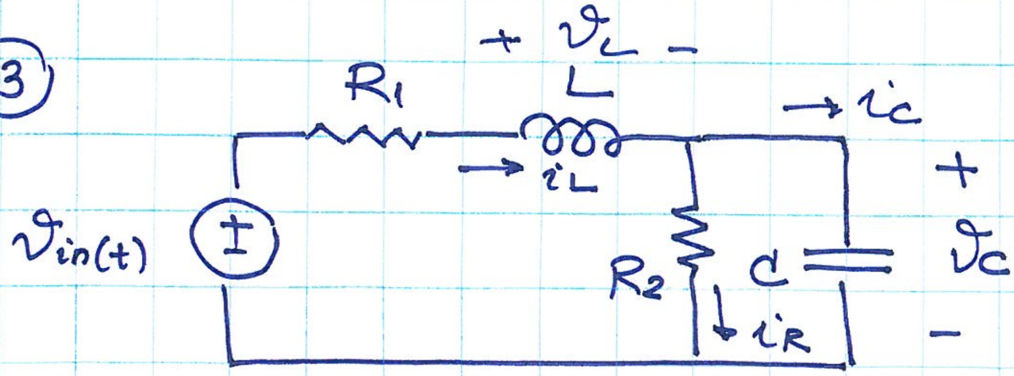
leaky:

$$V_{out}(t) = 50\left(1 - e^{-0.1t}\right)u(t)$$



(3)






$R_1 = 1\Omega$; $R_2 = 1\Omega$; $L = \sqrt{2}H$; $C = \sqrt{2}F$

(a) State equations : state variable $i_L$; $v_C$

KVL $\quad v_{in} - R_1 \cdot i_L - v_L - v_C = 0 \qquad (1)$

KCL $\quad i_L = i_R + i_C \qquad (2)$

$v_L = L\frac{di_L}{dt}$ constitutive relationships

$i_C = C\frac{dv_C}{dt}$

$i_R = \frac{v_C}{R_2}$ Ohm's Law

Hence:

$$R_1 \cdot i_L + L\frac{di_L}{dt} + v_C = v_{in}$$

$$i_L = \frac{v_C}{R_2} + C\frac{dv_C}{dt}$$

> only state variables appear

$$L\frac{di_L}{dt} + v_C + R_1 \cdot i_L = v_{in}$$

$$C\frac{dv_C}{dt} + \frac{1}{R_2} \cdot v_C - i_L = 0$$

} state equations

$$\left.\begin{aligned} \sqrt{2}\cdot\frac{di_L}{dt}+v_C+v_L&=v_{in}(t) && (3)\\ \sqrt{2}\cdot\frac{dv_C}{dt}+v_C-i_L&=0 && (4)\end{aligned}\right\}$$

(b) Second-order differential equati. in terms of $v_C$:

From (4):

$$i_L = C\frac{dv_C}{dt}+\frac{1}{R_2}\cdot v_C$$

use symbolic form in order to easily check algebra

Substituting into (3) gives:

$$L\cdot\frac{d}{dt}\cdot\left\{C\cdot\frac{dv_C}{dt}+\frac{1}{R_2}\cdot v_C\right\}+v_C+ R_1\cdot\left\{C\cdot\frac{dv_C}{dt}+\frac{1}{R_2}v_C\right\}=v_{in}$$

$$LC\cdot\frac{d^2v_C}{dt^2}+\frac{L}{R_2}\cdot\frac{dv_C}{dt}+v_C+R_1C\cdot\frac{dv_C}{dt}+\frac{R_1}{R_2}\cdot v_C=v_{in}$$

$$LC\cdot\frac{d^2v_C}{dt^2}+\left(\frac{L}{R_2}+R_1C\right)\cdot\frac{dv_C}{dt}+\left(1+\frac{R_1}{R_2}\right)v_C=v_{in}$$

or:

$$\frac{d^2v_C}{dt^2}+\left(\frac{1}{R_2C}+\frac{R_1}{L}\right)\frac{dv_C}{dt}+\frac{1}{LC}\left(1+\frac{R_1}{R_2}\right)v_C=\frac{1}{LC}\cdot v_{in}$$

Hence:

$$\frac{d^2v_C}{dt^2}+\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\right)\frac{dv_C}{dt}+\frac{1}{\sqrt{2}\cdot\sqrt{2}}\cdot(1+1)\cdot v_C=\frac{1}{\sqrt{2}\cdot\sqrt{2}}\cdot v_{in}$$

$$\frac{d^2v_C}{dt^2}+\sqrt{2}\cdot\frac{dv_C}{dt}+v_C=\frac{1}{2}\cdot v_{in} \qquad (5)$$

(c) Characteristic equation:

$$s^2 + \sqrt{2}s + 1 = 0$$

$$s_{1/2} = -\frac{\sqrt{2}}{2} \pm \sqrt{\left(\frac{\sqrt{2}}{2}\right)^2 - 1}$$

$$s_{1/2} = -\frac{\sqrt{2}}{2} \pm \sqrt{-1/2}$$

$$s_{1/2} = -\frac{\sqrt{2}}{2} \pm \frac{j}{\sqrt{2}} \; ; \qquad 1/\sqrt{2} = \frac{\sqrt{2}}{2}$$

$$s_{1/2} = \frac{-\sqrt{2} \pm j\sqrt{2}}{2}$$

or

$$s_{1/2} = -\frac{1 \pm 1}{\sqrt{2}}$$

(d) Two complex-conjugate roots: the system is underdamped

(e) Solutions:

$$v_C(t) = k_1 e^{s_1 t} + k_2 e^{s_2 t} + v_{Cp}(t)$$

$$i_L(t) = k_3 e^{s_1 t} + k_4 e^{s_2 t} + i_{Lp}(t)$$

Since $v_{in}(t) = u(t)$ ; $v_{in}(t) = 1$ for $t \geq 0$

$v_{Cp}(t) = A$ ; $A = ?$

from state equation (5):

$$0 + \sqrt{2} \cdot 0 + A = \frac{1}{2} \cdot 1$$

$$A = 1/2$$

$$v_C(t) = k_1 e^{s_1 t} + k_2 e^{s_2 t} + 1/2 \quad ; \quad k_1 + k_2 + 1/2 = 0 \qquad (6)$$

Note: $i_L(0-) = 0 \Rightarrow i_L(0+) = 0$

$v_C(0-) = 0 \Rightarrow v_C(0+) = 0$

Hence,
$$\left.\frac{dv_C}{dt}\right|_{0+} = k_1 s_1 + k_2 s_2$$

From state equations:
$$\sqrt{2}\cdot\frac{di_L}{dt} + v_C + i_L = 1$$
$$\sqrt{2}\cdot\frac{dv_C}{dt} + v_C - i_L = 0$$

At $t = 0+$:
$$\sqrt{2}\times(k_1 s_1 + k_2 s_2) + v_C(0-) - i_L(0+) = 0$$

$$k_1 s_1 + k_2 s_2 = 0 \qquad (7)$$

Equations:
$$\left.\begin{aligned} &k_1 + k_2 + 1/2 = 0 \\ &k_1\cdot\frac{-1+j}{\sqrt{2}} + k_2\cdot\frac{-1-j}{\sqrt{2}} = 0 \end{aligned}\right\}$$

$$\left.\begin{aligned} &k_1 + k_2 = -1/2 \\ &-(k_1+k_2) + j(k_1 - k_2) = 0 \end{aligned}\right\}$$

$$k_1 + k_2 = -1/2$$

Hence, $-(-1/2) + j(k_1 - k_2) = 0$

Then:
$$k_1 + k_2 = -1/2$$
$$k_1 - k_2 = j/2$$

$$-\frac{1}{2j} = -\frac{j}{2\times j^2} = j/2$$

$$k_1 + k_2 = -1/2$$
$$k_1 - k_2 = \frac{1}{2} \times j$$

$$2k_1 = \frac{-1+j}{2} \; ; \; k_1 = \frac{-1+j}{4}$$
$$k_2 = -\frac{1}{2} - \left(\frac{-1+j}{4}\right)$$
$$k_2 = \frac{-1-j}{4}$$

Hence.

Recall: $s_{1/2} = \frac{-1 \pm j}{\sqrt{2}}$

$$v_C(t) = \frac{-1+j}{4} e^{\frac{-1+j}{\sqrt{2}}t} + \frac{-1-j}{4} e^{\frac{-1-j}{\sqrt{2}}t} + 1/2$$

$$v_C(t) = \frac{1}{4} \cdot (-1+j) e^{-t/\sqrt{2}} \cdot \left(\cos\frac{t}{\sqrt{2}} + j \cdot \sin t/\sqrt{2}\right) +$$
$$\frac{1}{4}(-1-j) e^{-t/\sqrt{2}} \cdot \left(\cos t/\sqrt{2} - j \sin t/\sqrt{2}\right) + 1/2$$

$$v_C(t) = \frac{1}{4} e^{-t/\sqrt{2}} \cdot \left(-\cos t/\sqrt{2} - j \sin t/\sqrt{2} + j \cos t/\sqrt{2} - \sin t/\sqrt{2}\right.$$
$$\left. - \cos t/\sqrt{2} + j \sin t/\sqrt{2} - j \cos t/\sqrt{2} - \sin t/\sqrt{2}\right)$$
$$+ 1/2$$

$$v_C(t) = \frac{1}{4} e^{-t/\sqrt{2}} \cdot \times 2 \times \left(-\cos\frac{t}{\sqrt{2}} - \sin\frac{t}{\sqrt{2}}\right) + \frac{1}{2}$$

$$v_C(t) = \frac{1}{2} \cdot \left(1 - e^{-\frac{t}{\sqrt{2}}} \left(\cos t/\sqrt{2} + \sin t/\sqrt{2}\right)\right) u(t)$$

Note:

$$\cos t/\sqrt{2} + \sin t/\sqrt{2} = \sin\left(\frac{\pi}{2} - \frac{t}{\sqrt{2}}\right) + \sin\frac{t}{\sqrt{2}}$$

$$= 2\sin\frac{\pi}{4} \times \cos\left(\frac{\pi}{4} - \frac{t}{\sqrt{2}}\right)$$

$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cdot\cos\frac{\alpha-\beta}{2}$

$$= 2\times\frac{\sqrt{2}}{2}\cdot\cos\left(\frac{\pi}{4} - \frac{t}{\sqrt{2}}\right)$$

$$= \sqrt{2}\cos\left(\pi/4 - t/\sqrt{2}\right)$$

$$= \sqrt{2}\cos\left(t/\sqrt{2} - \pi/4\right)$$

$\cos(-\alpha) = \cos\alpha$

$$= \sqrt{2}\cdot\sin\left(\frac{\pi}{2} - \frac{t}{\sqrt{2}} + \pi/4\right)$$

$$= \sqrt{2}\cdot\sin\left(\frac{3\pi}{4} - t/\sqrt{2}\right)$$

$$= \sqrt{2}\cdot\sin\left(\frac{t}{\sqrt{2}} + \pi/4\right)$$

$\sin\alpha = \sin(\pi - \alpha)$

Hence,

$$v_C(t) = \frac{1}{2}\left(1 - e^{-t/\sqrt{2}} \times \sqrt{2} \times \cos\left(\frac{t}{\sqrt{2}} - \frac{\pi}{4}\right)\right)$$

$$= \frac{1}{2}\left(1 - e^{-t/\sqrt{2}} \times \sqrt{2} \times \sin\left(\frac{t}{\sqrt{2}} + \frac{\pi}{4}\right)\right)$$

*

Another approach:

$s = \frac{-1 \pm j}{\sqrt{2}}$

$$v_C(t) = e^{-t/\sqrt{2}}\cdot\left(A_1\sin\frac{t}{\sqrt{2}} + A_2\cos\frac{t}{\sqrt{2}}\right) + \frac{1}{2}$$

$v_C(0+) = 0$ ; $v_C(0+) = A_2 + 1/2$

Hence; $A_2 = -1/2$

$$\frac{dv_C}{dt} = -\frac{1}{\sqrt{2}}e^{-t/\sqrt{2}}\cdot\left(A_1\sin\frac{t}{\sqrt{2}} + A_2\cos\frac{t}{\sqrt{2}}\right) +$$
$$e^{-t/\sqrt{2}}\cdot\left(A_1\cdot\frac{1}{\sqrt{2}}\cdot\cos\frac{t}{\sqrt{2}} - A_2\cdot\frac{1}{\sqrt{2}}\sin\frac{t}{\sqrt{2}}\right)$$

$$\left.\frac{dv_c}{dt}\right|_{0+} = 0$$

Hence:

$$-\frac{1}{\sqrt{2}}A_2 + \frac{A_1}{\sqrt{2}} = 0 \; ; \; A_1 = A_2$$

$$A_2 = -1/2$$

$$v_{c(t)} = e^{-t/\sqrt{2}} \cdot \left(-\frac{1}{2} \cdot \sin t/\sqrt{2} - \frac{1}{2}\cos t/\sqrt{2}\right) + 1/2$$

As before:

$$v_{c(t)} = \frac{1}{2}\left(1 - e^{-t/\sqrt{2}} \cdot \left(\cos t/\sqrt{2} + \sin t/\sqrt{2}\right)\right) u(t)$$

**

Yet another approach:

$$v_{c(t)} = e^{-t/\sqrt{2}} \cdot A_3 \cdot \cos\left(t/\sqrt{2} - \theta\right) + 1/2$$

$$v_c(0+) = A_3 \cos(-\theta) + 1/2$$

$$A_3 \cos\theta + 1/2 = 0$$

$$A_3 \cos\theta = -1/2$$

$$\frac{dv_c}{dt} = -\frac{1}{\sqrt{2}} \cdot e^{-t/\sqrt{2}} \cdot A_3 \cos\left(\frac{t}{\sqrt{2}} - \theta\right) + e^{-\frac{t}{\sqrt{2}}} \cdot A_3 \cdot \frac{1}{\sqrt{2}} \cdot \left(-\sin\left(\frac{t}{\sqrt{2}} - \theta\right)\right)$$

$$\left.\frac{dv_c}{dt}\right|_{0+} = -\frac{1}{\sqrt{2}} \cdot A_3 \cos(-\theta) + \frac{1}{\sqrt{2}} A_3 \cdot \sin\theta$$

$$-\frac{1}{\sqrt{2}} A_3 \cos(-\theta) + \frac{1}{\sqrt{2}} A_3 \sin\theta = 0$$

$$\tan\theta = 1 ; \quad \theta = \pi/4$$

Then:

$$A_3 \cos\theta = -1/2 \; ; \; A_3 \cos\frac{\pi}{4} = -1/2$$

$$A_3 \cdot \frac{\sqrt{2}}{2} = -1/2 \; ; \; A_3 = -\frac{1}{\sqrt{2}}$$

Hence:

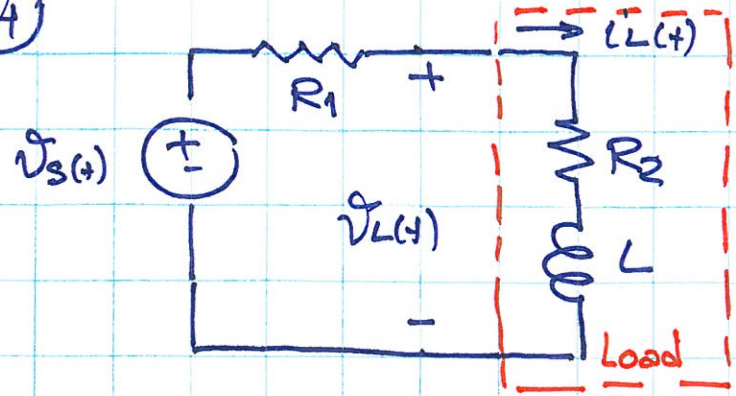
$$v_C(t) = e^{-\frac{t}{\sqrt{2}}} \times \left(-\frac{1}{\sqrt{2}}\right) \cos\left(\frac{t}{\sqrt{2}} - \pi/4\right) + 1/2$$

$$v_C(t) = \frac{1}{2} \cdot \left(1 - e^{-t/\sqrt{2}} \times \sqrt{2} \times \cos\left(\frac{t}{\sqrt{2}} - \pi/4\right)\right] u(t)$$

or, equivalently:

$$v_C(t) = \frac{1}{2}\left(1 - e^{-t/\sqrt{2}} \times \sqrt{2} \times \cos\left(\frac{\pi}{4} - t/\sqrt{2}\right)\right) u(t)$$

(4)

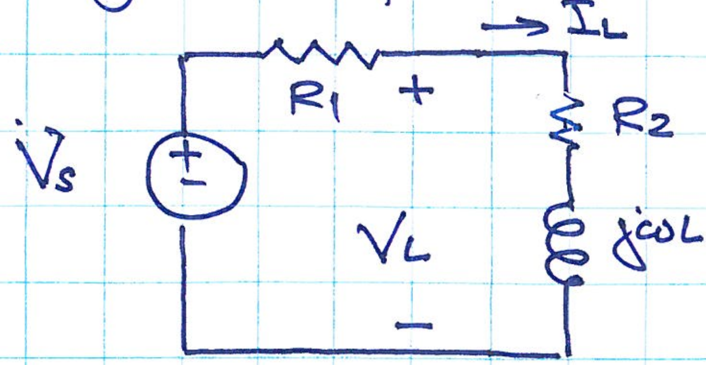



$R_1 = 50\,\Omega$

$R_2 = 350\,\Omega$

$L = 1\,H$

Steady-state: phasors




Sine $v_s(t) = \sqrt{2}\,100\cos(300t + 30^\circ)$

$$\underline{V}_s = 100\,e^{j\pi/6}$$

$\frac{\pi}{6} = 30^\circ$

$\omega = 300$

$V_{s\,eff} = 100$

$V_{s\,max} = \sqrt{2}\,100$

(a) $\underline{Z}_L = R_2 + j\omega L$

$$\underline{Z}_L = 350 + j\cdot 300 \times 1$$

$$\underline{Z}_L = 350 + j300$$

$$\underline{Z} = R_1 + \underline{Z}_L \;;\; \underline{Z} = 50 + 350 + j300$$

$$\underline{Z} = 400 + j300$$

$$\underline{I}_L = \frac{V_s}{\underline{Z}}$$

$$\underline{I}_L = \frac{100\, e^{j\pi/6}}{400 + j300}$$

Note that: $\underline{Z} = 400 + j300$

$$\underline{Z} = \sqrt{400^2 + 300^2} \cdot e^{j \arctan \frac{300}{400}}$$

$$\underline{Z} = 500\, e^{j \arctan 3/4}$$

$$\underline{Z} = 500\, e^{j 36.87^\circ}$$

Hence:

$$\underline{I}_L = \frac{100}{500} \frac{e^{j30^\circ}}{e^{j36.87^\circ}}$$

$$\underline{I}_L = \frac{1}{5} e^{j(-6.87^\circ)}$$

$$i_L(t) = \sqrt{2} \times \frac{1}{5} \cdot \cos(300t - 6.87^\circ)$$

(b): Complex power: $\underline{S}_L = V_L \cdot \underline{I}_L^*$ ← complex-conjugate

$$\underline{S}_L = Z_L \cdot \underline{I}_L \cdot \underline{I}_L^*$$

$$\underline{S}_L = (310 + j300) \cdot \frac{1}{5} e^{j(-6.87)} \cdot \frac{1}{5} e^{j(6.87)}$$

$$\underline{S}_L = (310 + j300) \times \frac{1}{25} \qquad \underline{S}_L = 14 + j\cdot 12 \ \text{(VA)}$$

Average power: $P_L = 14$ (W)