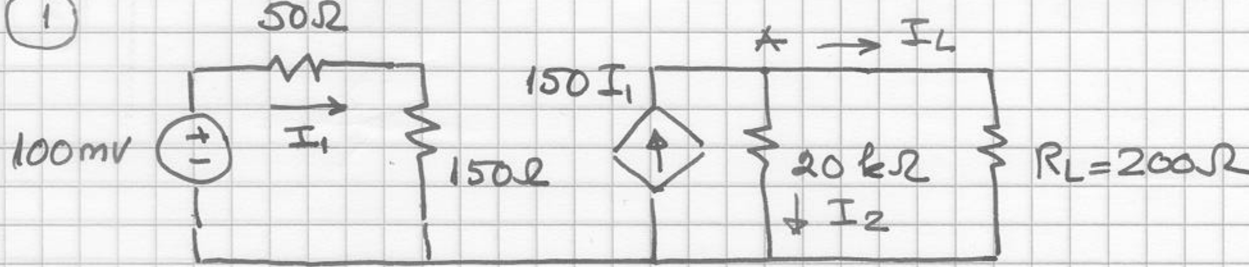
(1) (1)



$$100 \times 10^{-3} - 50 \times I_1 - 150 \cdot I_1 = 0$$

$$I_1 = \frac{100 \times 10^{-3}}{200}$$

$$I_1 = 0.5 \times 10^{-3}\ A, \quad I_1 = 0.5\,mA$$

KCL for node A: $150 I_1 = I_2 + I_L$

$$R_L \cdot I_L - 20 \times 10^{3} I_2 = 0$$

Hence,

$$I_2 + I_L = 150 \times 0.5 \times 10^{-3}$$

$$200 I_L = 20 \times 10^{3} I_2 \ ; \quad I_2 = 0.01 I_L$$

And $0.01 I_L + I_L = 150 \times 0.5 \times 10^{-3}$

$$I_L = \frac{75 \times 10^{-3}}{1.01} A \ ; \quad I_L = \frac{75}{1.01} \times 10^{-3} A$$

$$I_L = \frac{75}{1.01} mA$$

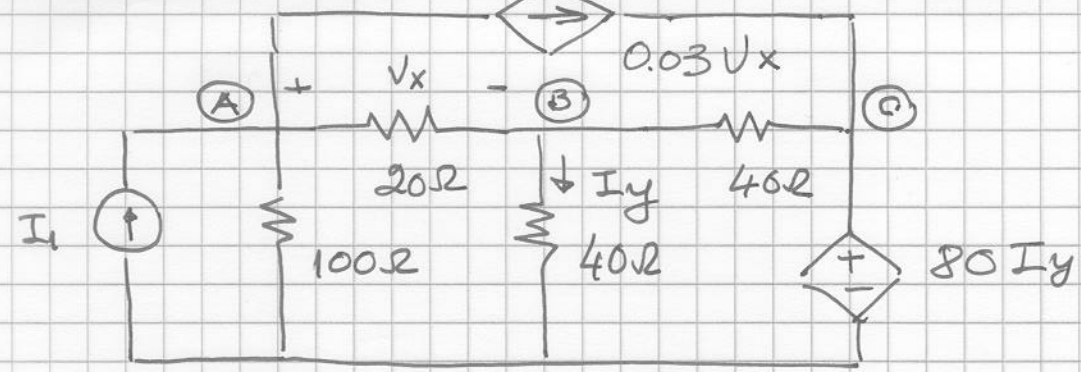
Power absorbed by the load is:

$$P_L = V_L \cdot I_L \ ; \quad P_L = R_L I_L^2$$

$$P_L = 200 \times \left(\frac{75}{1.01} \times 10^{-3} \right)^2 \ ; \quad P_L = \frac{2 \times 5625 \times 10^{-4}}{(1.01)^2}$$

$$P_L = \frac{11250}{(1.01)^2} \times 10^{-4} \ ; \quad P_L = \frac{1125}{(1.01)^2} \times 10^{-3} \ ; \quad P_L = \frac{1.125}{(1.01)^2} W$$

$P_L = 1.103\,W$



$I_1 = 0.4\,A$

Node equations:

$$\frac{V_A}{100} + \frac{V_A - V_B}{20} + 0.03\,V_x = I_1$$

$$\frac{V_B - V_A}{20} + \frac{V_B}{40} + \frac{V_B - V_C}{40} = 0$$

$$V_C = 80 \cdot I_y$$

Note that $V_x = V_A - V_B$

$$I_y = \frac{V_B}{40}$$

Hence:

$$V_A \cdot \left(\frac{1}{100} + \frac{1}{20}\right) - \frac{1}{20} \cdot V_B + 0.03 \cdot (V_A - V_B) = 0.4$$

$$V_B \cdot \left(\frac{1}{20} + \frac{1}{40} + \frac{1}{40}\right) - \frac{1}{20} V_A - \frac{1}{40} V_C = 0$$

$$V_C = 80 \times \frac{V_B}{40} \Rightarrow V_C = 2V_B$$

Hence:

$$V_A \left(\frac{1}{100} + \frac{1}{20} + \frac{3}{100}\right) - \left(\frac{1}{20} + \frac{3}{100}\right) V_B = 0.4$$

$$V_B \cdot \left(\frac{1}{20} + \frac{1}{40} + \frac{1}{40}\right) - \frac{1}{20} V_A - \frac{1}{40} \cdot 2V_B = 0$$

After arranging the terms:

$$V_A \cdot \frac{9}{100} - \frac{8}{100} \cdot V_B = 0.4$$

$$V_B \times \frac{1}{20} - V_A \cdot \frac{1}{20} = 0$$

From the second equation:

$$V_B = V_A$$

Substituting in the first equation gives:

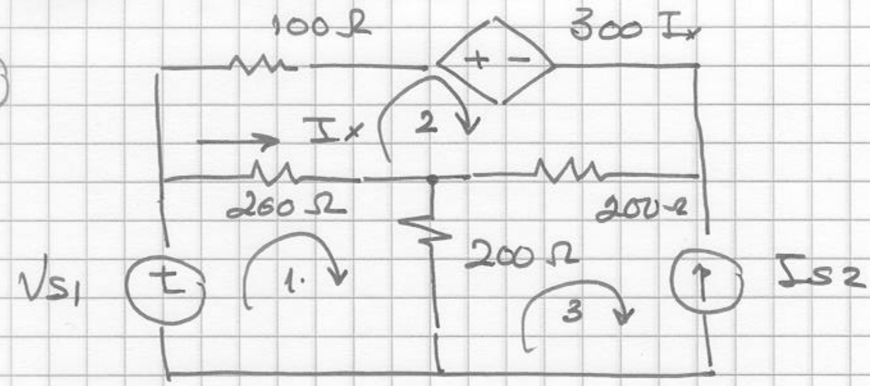
$$V_A \cdot \frac{9}{100} - V_A \cdot \frac{8}{100} = 0.4$$

$$\text{or } V_A = 40V$$

and $$V_B = 40V$$

Since $V_x = V_A - V_B \Rightarrow V_x = 0$

3



$V_{S1} = 250V$

$I_{S2} = 0.75A$

Let us denote mesh currents as I_1, I_2, and I_3.

Mesh equations are:

$$V_{S1} - 200(I_1 - I_2) - 200(I_1 - I_3) = 0$$

$$100 \cdot I_2 + 300 \cdot I_X + 200 \cdot (I_2 - I_3) + 200(I_2 - I_1) = 0$$

$$I_3 = -I_{S2}$$

Hence
$$400 I_1 - 200 I_2 - 200 I_3 = 250$$
$$500 I_2 - 200 I_1 - 200 I_3 + 300 I_X = 0$$
$$I_3 = -0.75$$

Note: $I_X = I_1 - I_2$

Hence,
$$400 I_1 - 200 I_2 - 200 I_3 = 250$$
$$500 I_2 - 200 I_1 - 200 I_3 + 300 \cdot (I_1 - I_2) = 0$$

Since $I_3 = -0.75$

$$400 \cdot I_1 - 200 I_2 - 200 \times (-0.75) = 250$$
$$500 I_2 - 200 I_1 - 200 \cdot (-0.75) + 300 I_1 - 300 I_2 = 0$$

$$400 I_1 - 200 I_2 = 100$$
$$200 I_2 + 100 I_1 = -150$$

After simplification:

$$4I_1 - 2I_2 = 1$$

$$10I_1 + 20I_2 = -15$$

$$\begin{pmatrix} 4 & -2 \\ 10 & 20 \end{pmatrix}\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} 1 \\ -15 \end{pmatrix}$$

$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} 4 & -2 \\ 10 & 20 \end{pmatrix}^{-1} \cdot \begin{pmatrix} 1 \\ -15 \end{pmatrix}$$

$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \frac{1}{100} \cdot \begin{pmatrix} 20 & 2 \\ -10 & 4 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -15 \end{pmatrix}$$

$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \frac{1}{100} \cdot \begin{pmatrix} -10 \\ -70 \end{pmatrix}$$

$I_1 = -1/10\ A$; $I_1 = -0.1A$

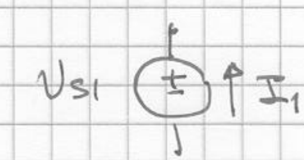
$I_2 = -7/10\ A$; $I_2 = -0.7A$

Since $Ix = I_1 - I_2$

$Ix = 0.6A$

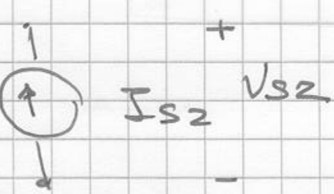
Power delivered by the voltage source $Vs1$ is:

$P_{s1} = Vs1 \cdot I_1$



$P_{s1} = 250 \times (-0.1)$; $P_{s1} = -25W$

$P_{s2} = Vs2 \cdot Is2$



$Vs2 = -200 \cdot (I_3 - I_1) - 200 \cdot (I_3 - I_2)$

$Vs2 = -400 \cdot I_3 + 200 I_1 + 200 I_2$

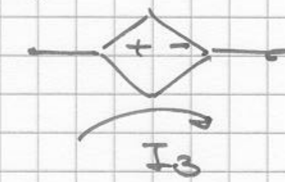
Note that $I_3 = -0.75 A$ Hence: $Vs2 = -400 \times (-0.75) +$

$I_1 = -0.1A$ $200 \times (-0.1) + 200 \times (-0.7)$

$I_2 = -0.7A$ $Vs2 = 140V$

$P_{s2} = 140 \times 0.75$; $P_{s2} = 105W$

Power delivered by the dependent source:



$$P_3 = (300 \times I_x) \times (-I_2)$$

$$P_3 = 300 \times 0.6 \times (+0.7)$$

$$P_3 = 126\,W$$

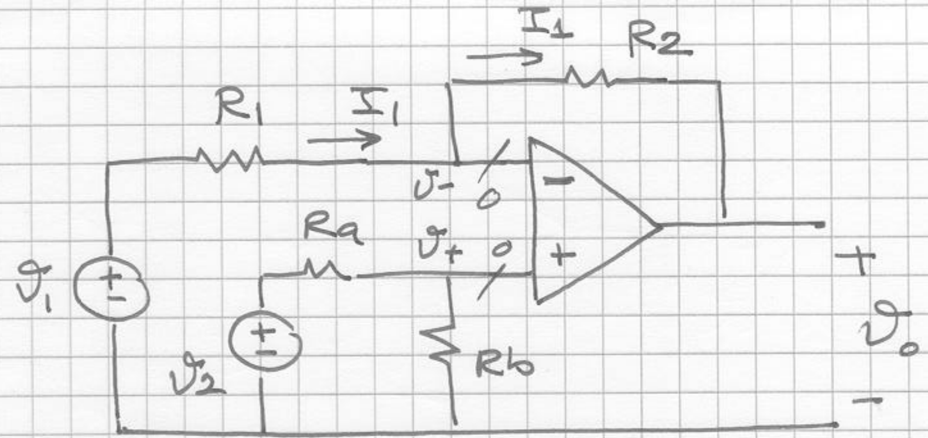
Total power delivered by all the sources is:

$$P = \sum P_i$$

$$P = -25 + 105 + 126$$

$$P = 206\,W$$

4



the opamp is ideal. Hence, the input current to the opamp is zero and:

$$v_+ = v_-$$

$$v_+ = \frac{R_b}{R_a + R_b} \cdot v_2$$

$$v_- = \frac{R_b}{R_a + R_b} \cdot v_2$$

$$I_1 = \frac{v_1 - v_-}{R_1} \quad ; \quad I_1 = \frac{v_1}{R_1} - \frac{R_b}{R_1(R_a + R_b)} v_2$$

$$v_0 = -R_2 \cdot I_1 + v_-$$

$$v_0 = -R_2 \cdot \left(\frac{v_1}{R_1} - \frac{R_b}{R_1(R_a+R_b)} v_2 \right) + \frac{R_b}{R_a+R_b} \cdot v_2$$

Hence,

$$v_0 = -\frac{R_2}{R_1} \cdot v_1 + \frac{R_2}{R_1} \cdot \frac{R_b}{R_a+R_b} v_2 + \frac{R_b}{R_a+R_b} \cdot v_2$$

or

$$v_0 = -\frac{R_2}{R_1} \cdot v_1 + \left(\frac{R_2}{R_1} + 1 \right) \cdot \frac{R_b}{R_a+R_b} \cdot v_2$$

Conditions: $\left| -\frac{R_2}{R_1} v_1 + \left(\frac{R_2}{R_1} + 1 \right) \cdot \frac{R_b}{R_a+R_b} \right| < V_{sat}$

If the circuit is to be used as a difference amplifier,

$$V_o = -\frac{R_2}{R_1}\cdot V_1 + \left(\frac{R_2}{R_1}+1\right)\frac{R_b}{R_a+R_b}\cdot V_2$$

$$V_o = -\frac{R_2}{R_1}\left(V_1 - V_2\right)$$

Then:

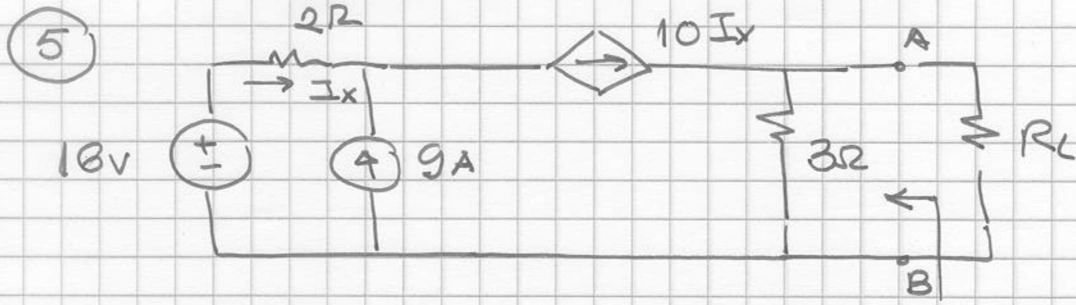
$$\left(\frac{R_2}{R_1}+1\right)\frac{R_b}{R_a+R_b} = \frac{R_2}{R_1}$$

$$\left(R_2+R_1\right)R_b = R_2\left(R_a+R_b\right)$$

$$\frac{R_1+R_2}{R_2} = \frac{R_a+R_b}{R_b}$$

$$\frac{R_1}{R_2} = \frac{R_a}{R_b}$$

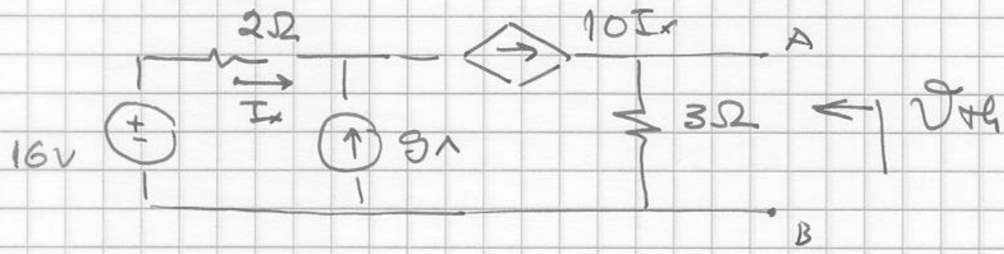
5



v_{th}
R_{th}

Let us find thevenin voltage:

$$I_x + 9 = 10 I_x \Rightarrow I_x = 1A$$



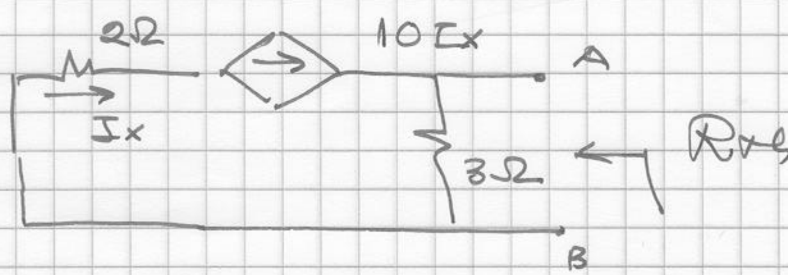
open-circuit

$$v_{th} = 10\, i_x \times 3$$

$$v_{th} = 30\, i_x$$

$$v_{th} = 30V$$

thevenin resistance:



Note that $i_x = 10 i_x \Rightarrow i_x = 0$

$$R_{th} = 3\Omega$$

$R_L = 3\Omega$: maximum power transfer

Proof of $R_L = R_{th}$ for maximum power transfer.

$$P_L = R_L \cdot \left(\frac{v_{th}}{R_L + R_{th}} \right)^2$$

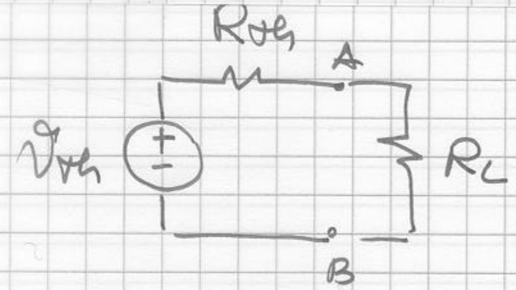
$$P_L = v_{th}^2 \cdot \frac{R_L}{(R_L + R_{th})^2};$$

$$\frac{\partial P_L}{\partial R_L} = \frac{(R_L + R_{th})^2 - R_L \cdot 2(R_L + R_{th})}{(R_L + R_{th})^2} \cdot v_{th}$$

$$\frac{\partial P_L}{\partial R_L} = 0 \text{ for } (R_L + R_{th})(R_{th} - R_L) = 0$$

i.e., $R_L = R_{th}$

Maximum power transferred.



$v_{th} = 30V$

$R_{th} = 3\Omega$

$$P_L = R_L \cdot I^2$$

$$= R_L \cdot \left(\frac{V_{th}}{R_{th} + R_L} \right)^2$$

if $R_L = R_{th}$

$$P_L = \frac{v_{th}^2}{4R_{th}}; \quad P_L = \frac{30^2}{4 \times 3}$$

$$= \frac{900}{12}$$

$$= 75W$$