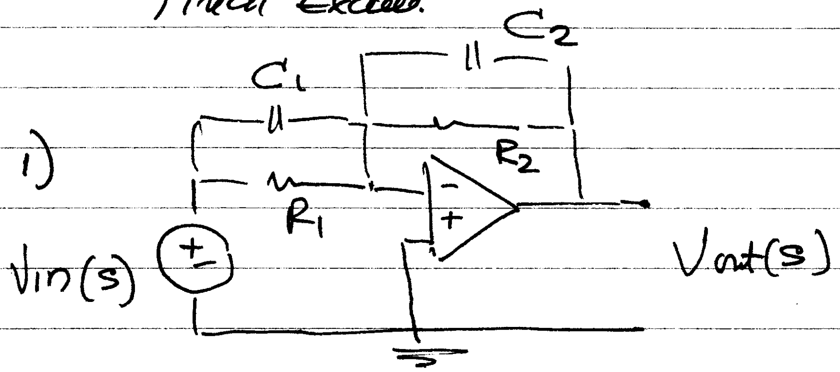
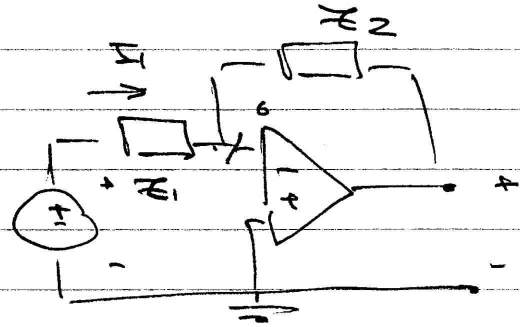
ENSC 320
Summer 2007

Final Exam:

1)



View the circuit as:



$$H(s) = \frac{V_{out}(s)}{V_{in}(s)}$$

$$V_{in} - (Z_1 + Z_2) I_1 - V_{out} = 0$$

$$I_1 = \frac{V_{in}}{Z_1}$$

$$V_{in} = (Z_1 + Z_2) \cdot \frac{V_{in}}{Z_1} + V_{out}$$

$$V_{in} \left(1 - \frac{Z_1 + Z_2}{Z_1}\right) = V_{out}$$

$$H = -\frac{Z_2}{Z_1}$$

$$Z_1 = \frac{R_1 \cdot \frac{1}{C_1 s}}{R_1 + \frac{1}{C_1 s}} \; ; \; Z_1 = \frac{R_1}{1 + R_1 C_1 s} \; ; \; Z_2 = \frac{R_2}{1 + R_2 C_2 s}$$

$$H(2) = -\frac{R_2}{1+R_2C_2S} \cdot \frac{1+R_1C_1S}{R_1}$$

$$H(s) = -\frac{R_2}{R_1} \cdot \frac{1+R_1C_1S}{1+R_2C_2S}$$

$$H(s) = -\frac{\cancel{R_2}}{\cancel{R_1}} \cdot \frac{\cancel{R_1}C_1}{R_2C_2} \cdot \frac{S+1/R_1C_1}{S+1/R_2C_2}$$

(a) $$H(s) = -\frac{C_1}{C_2} \cdot \frac{S+1/R_1C_1}{S+1/R_2C_2}$$

(b) · Impulse response.

$$h(t) = \mathcal{L}^{-1} H(s)$$

$$H(s) = -\frac{C_1}{C_2} \cdot \frac{s+a}{s+b} \qquad a = 1/R_1C_1 \quad b = 1/R_2C_2$$

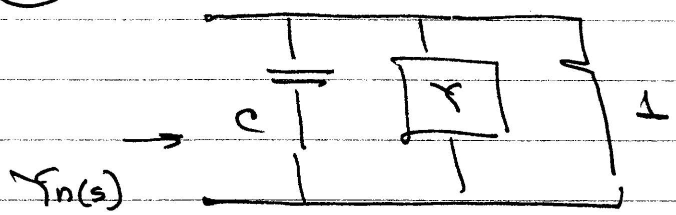
$$\frac{s+a}{s+b} = \frac{s+b+a-b}{s+b} = 1 + \frac{a-b}{s+b}$$

$$H(s) = -\frac{C_1}{C_2} \cdot \left(1 + \frac{a-b}{s+b}\right)$$

$$h(t) = -\frac{C_1}{C_2} \cdot \left(\delta(t) + (a-b) \cdot e^{-bt}\right) u(t)$$

$$h(t) = -\frac{C_1}{C_2} \cdot \left(\delta(t) + \left(\frac{1}{R_1C_1} - \frac{1}{R_2C_2}\right) e^{-\frac{t}{R_2C_2}}\right) u(t)$$

2



$$Y(s) = \frac{2s+1}{s^2+1}$$

$Y_{in}(s) = ?$

$Y_{in}(s) = Cs + Y(s) + 1$

$$Y_{in}(j\omega) = j\omega C + \frac{2j\omega + 1}{1-\omega^2} + 1$$

$Im\, Y_u(j1) = \phi$

$$\omega C + \frac{2\omega}{1-\omega^2} = \phi$$

$\omega = 0$

if $\omega = 1$ $\quad Y_u(j\omega) = \infty$

Stability: $Y(s) = Cs + \dfrac{2s+1}{s^2+1} + 1$

$$Y(s) = \frac{Cs(s^2+1) + 2s + 1 + s^2 + 1}{s^2+1}$$

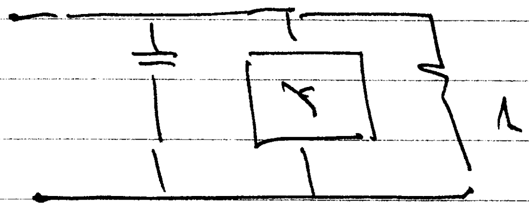
$$Y(s) = \frac{s^3 C + s^2 + (2+C)s + 1}{s^2+1}$$!

Look at: $Y(s) = \dfrac{2s+1}{s^2+1}$ $\quad$ zero: $s = -1/2$ LHP

pol: $s = \pm j1$ (ω-axis)

simple pole

(2) Correct.



at $W=3$

$$Y(s) = \frac{2s+1}{s^2+1}$$

$$Y_{in}(s) = Cs + \frac{2s+1}{s^2+1} + 1$$

$$Y_{in}(j\omega) = j\omega C + \frac{2j\omega+1}{1-\omega^2} + 1$$

$$\operatorname{Im} Y_{in}(j\omega) = \omega C + \frac{2\omega}{1-\omega^2}$$

$$\omega = 0$$

$$C + \frac{2}{1-\omega^2} = 0$$

$W=3$ $$C + \frac{2}{1-9} = 0$$

$$C = 1/4$$

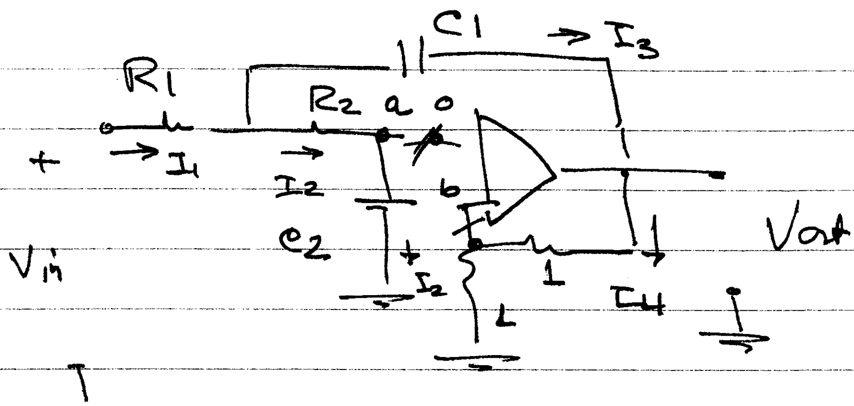
$$Y(j3) = \frac{1}{1-\omega^2} + 1$$

$$\omega = 3$$

$$Y(j3) = \frac{1}{1-9} + 1$$

$$= 1 - \frac{1}{8} = 0.75$$

(3)



Sallen key:

$R_1 = 1\Omega$

$R_2 = Q\Omega$

$C_1 = 1F$

$C_2 = 1/Q\ F$

$$V_{in} = R_1 \cdot I_1' + \left(R_2 + \frac{1}{C_2 s}\right) \cdot I_2 \qquad (1)$$

(2') $V_a = \frac{1}{C_2 s} \cdot I_2$

$V_b = V_a$

$$\left.\begin{array}{l} V_{out} = 2 \cdot I_4 \\ V_b = 1 \times I_4 \end{array}\right\} V_{out} = 2 \cdot V_b \Rightarrow V_b = \frac{V_{out}}{2}$$

$V_a = V_b$

$V_a = \frac{V_{out}}{2}$

$$\Rightarrow I_2 = \frac{C_2 s}{2} \cdot V_{out} \qquad (2)$$

(3) : $V_{in} = R_1 I_1 - \frac{1}{C_1 s} \cdot I_3 = V_{out}$

$V_{in} = R_1 I_1 + \frac{1}{C_1 s} \cdot I_3 + V_{out}$

(4) $I_3 = I_1 - I_2$

$I_3 = I_1 - \frac{C_2 s}{2} \cdot V_{out}$

Hence (from 3) : $V_{in} = R_1 \cdot I_1 + \frac{1}{C_1 s} \cdot \left(I_1 - \frac{C_2 s}{2} V_{out} \right) + V_{out}$

$V_{in} = \left(R_1 + \frac{1}{C_1 s} \right) I_1 - \frac{C_2}{2 C_1} V_{out} + V_{out}$

$V_{in} = \left(R_1 + \frac{1}{C_1 s} \right) I_1 + \left(1 - \frac{C_2}{2 C_1} \right) V_{out}$ (4)

(1) : $V_{in} = R_1 I_1 + \left(R_2 + \frac{1}{C_2 s} \right) \cdot I_2$

(4) $V_{in} = \left(R_1 + \frac{1}{C_1 s} \right) I_1 + \left(1 - \frac{C_2}{2 C_1} \right) V_{out}$

—

$V_{in} = R_1 I_1 + \left(R_2 + \frac{1}{C_2 s} \right) \cdot \frac{C_2 s}{2} V_{out}$

$V_{in} = \left(R_1 + \frac{1}{C_1 s} \right) I_1 + \left(1 - \frac{C_2}{2 C_1} \right) V_{out}$

$$V_{in} = \left(R_1 + \frac{1}{C_1 s}\right) \cdot \frac{V_{in} - \left(R_2 + \frac{1}{C_2 s}\right) \frac{C_2 s}{2} V_{out}}{R_1} + \left(1 - \frac{C_2}{2C_1}\right) V_{out}$$

$$V_{in} \left\{ 1 - \frac{R_1 + \frac{1}{C_1 s}}{R_1} \right\} = - \frac{\left(R_1 + \frac{1}{C_1 s}\right)\left(R_2 + \frac{1}{C_2 s}\right) \cdot \frac{C_2 s}{2}}{R_1} + \left(1 - \frac{C_2}{2C_1}\right) V_{out}.$$

$$H(s) = \frac{V_{out}}{V_{in}}$$

$$H(s) = \frac{-\frac{1}{C_1 s}}{-\left(R_1 + \frac{1}{C_1 s}\right)\left(R_2 + \frac{1}{C_2 s}\right)\frac{C_2 s}{2} + \left(1 - \frac{C_2}{2C_1}\right) R_1}$$

$$H(s) = \frac{1}{\left(R_1 C_1 s + 1\right)\left(R_2 C_2 s + 1\right) \cdot \frac{1}{2} - \left(1 - \frac{C_2}{2C_1}\right) \cdot R_1 C_1 s}$$

$$H(s) = \frac{1 \times 2}{\left(1 + R_1 C_1 s\right)\left(1 + R_2 C_2 s\right) - \left(2C_1 - C_2\right) R_1 s}$$

let $R_1 = 1 \quad C_1 = 1$ $\qquad R_1 C_1 = 1$

$R_2 = Q \quad C_2 = 1/Q$ $\qquad R_2 C_2 = 1$

$$H(s) = \frac{2}{(1+s)(1+s) - \left(2 - \frac{1}{Q}\right)s}$$

$$H(s) = \frac{2}{1 + s^2 + 2s - 2s + \frac{s}{Q}}$$

$$H(s) = \frac{2}{1 + s^2 + s/Q}$$

R ok ✓

$$H(s) = \frac{2}{s^2 + \frac{1}{Q}s + 1}$$

$$H(s) = \frac{2}{s^2 + \frac{1}{Q}s + 1}$$

Zeros: 2 at infin

Pole: $s^2 + \frac{1}{Q}s + 1 = 0 \quad s_{1/2} = -\frac{1}{2Q} \pm \sqrt{\left(\frac{1}{2Q}\right)^2 - 1}$

$s^2 + as + b = 0$

$s_{1/2} = -\frac{a}{2} \pm \sqrt{\left(\frac{a}{2}\right)^2 - 1}$

$$s_{1/2} = -\frac{1}{2Q} \pm j\sqrt{1 - \left(\frac{1}{2Q}\right)^2}$$

$$H(jw) = \frac{2}{1-w^2 + \frac{1}{Q} jw}$$

$$|H(jw)| = \frac{2}{\sqrt{(1-w^2)^2 + \left(\frac{w}{Q}\right)^2}}$$

$$\measuredangle H(jw) = -\operatorname{arctg} \frac{w/Q}{1-w^2}$$

Type of the circuit $\quad w=0 \quad H(0)=2$

$\quad w=\infty \quad H(\infty)=0$

$1/2$ Power: $\quad (1-w^2)^2 = \left(\frac{w}{Q}\right)^2$

$$1-w^2 = \pm \frac{w}{Q}$$

$$w^2 \pm \frac{w}{Q} - 1 = 0 \qquad w_{1/2} = -\frac{w}{2Q} \pm \sqrt{\left(\frac{w}{2Q}\right)^2 + 1}$$

$$w_{3/4} = \frac{w}{2Q} \pm \sqrt{\left(\frac{w}{2Q}\right)^2 + 1}$$

Bandw. $\quad w_3 - w_1 = \frac{w}{Q}$

Peak: $\quad y = (1-w^2)^2 + \left(\frac{w}{Q}\right)^2$

$$\partial y / \partial w = 2(1-w^2) \cdot 2w + 2 \cdot \frac{w}{Q} \cdot \frac{1}{Q}$$

$$w=0 \; ; \quad 4(1-w^2) + \frac{2}{Q^2} = 0 \qquad 4 + \frac{2}{Q^2} = 4w^2$$

$$w^2 = 1 + \frac{1}{2Q^2}$$

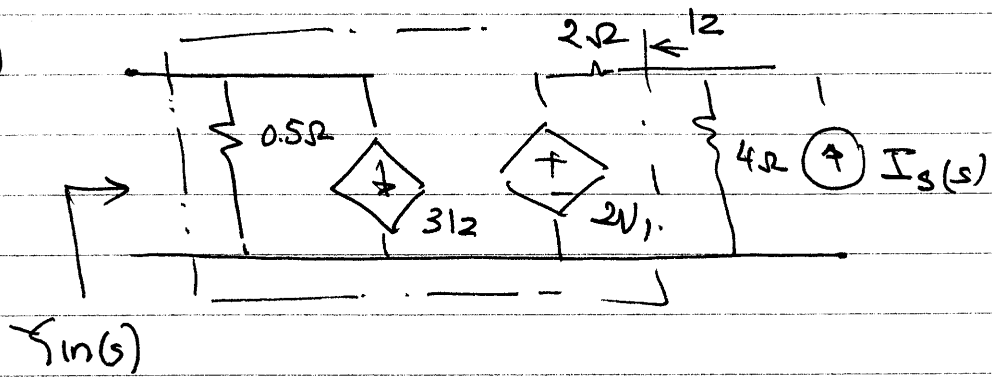
bandpass filter. if proper ch Q:

Low pass filter. ✓ in general

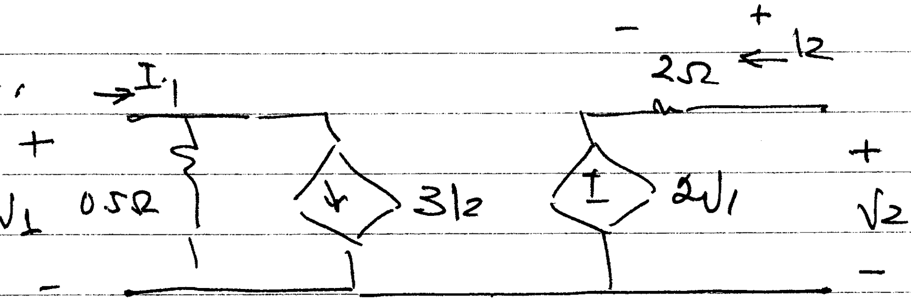
$k=2$ at $w=0$

Butterworth Designs.

4)



Two-port:



$$I_1 = 2V_1 + 3I_2$$

$$V_2 = 2I_2 + 2V_1 \Rightarrow I_2 = \frac{V_2 - 2V_1}{2}$$

$$I_1 = 2V_1 + 3 \cdot \frac{V_2 - 2V_1}{2}$$

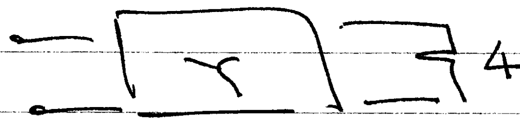
$$I_1 = -V_1 + \frac{3}{2} V_2$$

$$I_2 = \frac{V_2 - 2V_1}{2}$$

$$Y = \begin{bmatrix} -1 & 3/2 \\ -1 & 1/2 \end{bmatrix}$$

Input impedance:

$$I_1 = -V_1 + 1.5V_2$$
$$I_2 = -V_1 + 0.5V_2$$



open circuited

$$Y = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$$

$$V_2 = -4 \cdot I_2$$

$$Y_{in} = \frac{V_1}{I_1}$$

$$\begin{cases} I_1 = y_{11}V_1 + y_{12}V_2 \\ I_2 = y_{21}V_1 + y_{22}V_2 \end{cases}$$

$$I_1 = -V_1 + 1.5 \cdot (-4I_2)$$

$$I_2 = -V_1 + 0.5 \cdot (-4I_2)$$

$$I_2 = -V_1 - 2I_2$$

$$3I_2 = -V_1$$

$$I_2 = -\frac{1}{3}V_1$$

$$I_1 = -V_1 + 1.5(-4 \cdot (-1/3)\, V_1)$$

$$I_1 = -V_1 + 2V_1 ; \quad I_1 = V_1 \quad Y_{in} = 1\ \underline{S}$$