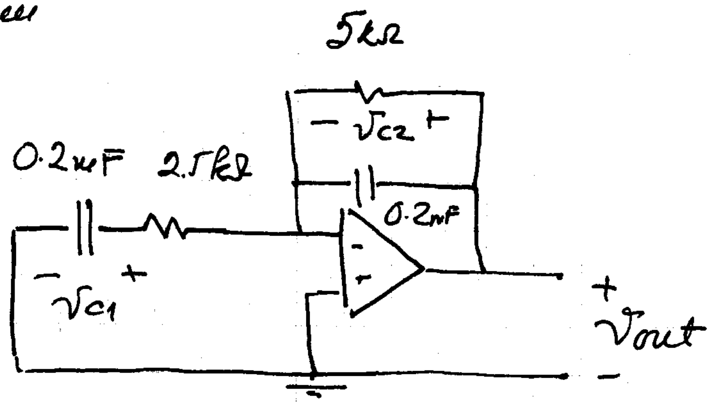
Final Exam

Spring 2008

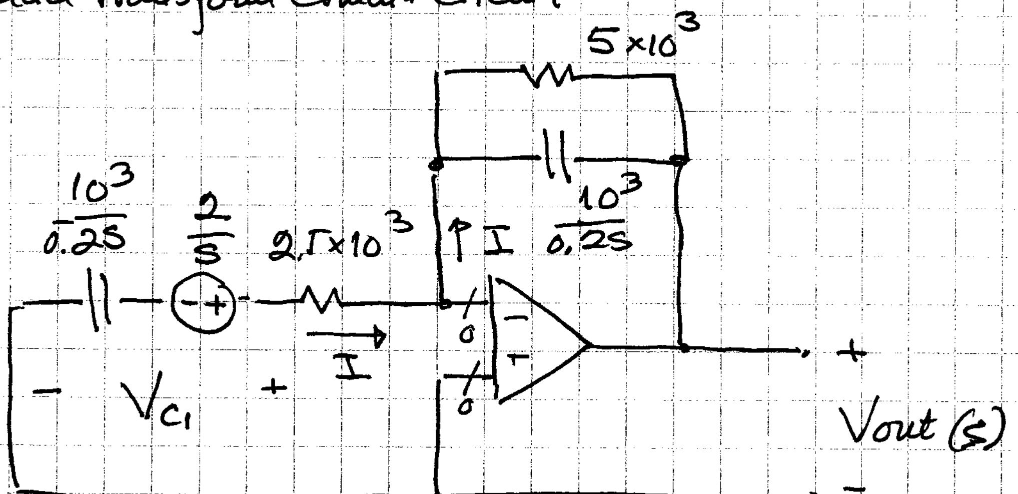
# ①




$V_{C1}(0-) = 2V$ $\quad R_1 = 2.5k\Omega$ $\quad C_1 = 0.2uF$

$V_{C2}(0-) = 0V$ $\quad R_2 = 5k\Omega$ $\quad C_2 = 0.2uF$

Laplace transform domain circuit:




Node equations:

$V_{C1}(0-)$

$$I(s) = \frac{-\frac{2}{s}}{2.5\times10^3 + \frac{10^3}{0.2s}}$$

$$= \frac{-2\times10^{-3}}{2.5s + 0.5\times10}$$

$$= \frac{-8\times10^{-4}}{s+2}$$

$$V_{out}(s) = I(s) \times \frac{5\times10^3 \times \frac{10^3}{0.2s}}{5\times10^3 + \frac{10^3}{0.2s}}$$

$$V_{out}(s) = \frac{-8\times10^{-4}}{s+2} \times \frac{5\times\frac{10^4}{2s}}{5+\frac{10^4}{2s}}$$

$$= -8\times\frac{\cancel{10^{-4}}}{s+2} \times \frac{5\times\cancel{10^4}}{10s+10}$$

$$= -8 \cdot \frac{5}{10\cdot(s+1)(s+2)}$$

$$= \frac{-4}{(s+1)(s+2)}$$

Partial fract expansn gn:

$$-\frac{4}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$$

$$A = \lim_{s\to-1} (s+1)\times\frac{-4}{(s+1)(s+2)}$$

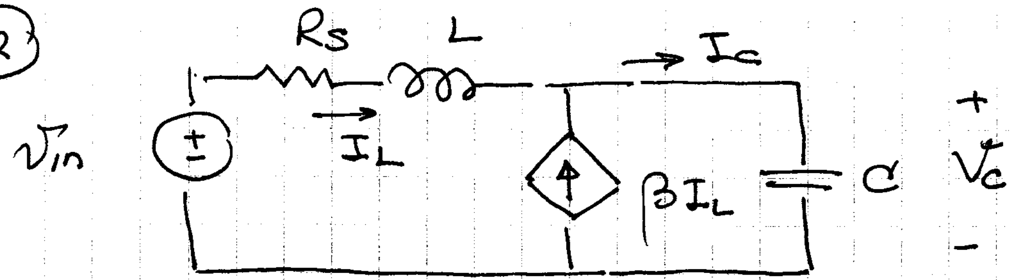
$$= \frac{-4}{-1+2} \; ; \; A = -4$$

$$B = \lim_{s\to-2} (s+2)\times\frac{-4}{(s+1)(s+2)}$$

$$= \frac{-4}{-2+1} \; ; \; B = 4$$

Hence: $V_{out}(s) = \frac{-4}{s+1} + \frac{4}{s+2}$ ; $V_{out}(t) = 4(e^{-2t} - e^{-t})u(t)\,V$

# 2




In "s"-domain:

$$V_{in}(s) = (R_s + SL) \cdot I_L + V_c \qquad (1)$$

$$I_c = Cs \cdot V_c$$

$$I_L = -\beta \cdot I_L + I_c \implies (1+\beta) I_L = I_c$$

$$V_c = \frac{1}{Cs} \cdot I_c$$

$$V_c = \frac{1}{Cs} \cdot (1+\beta) \cdot I_L$$

then (1) become:

$$V_{in}(s) = (R_s + SL) \cdot I_L + \frac{1}{Cs}(1+\beta) \cdot I_L$$

$$H(s) = \frac{I_L(s)}{V_{in}(s)}$$

Hence:

$$H(s) = \frac{1}{R_s + SL + \frac{1}{Cs}(1+\beta)}$$

$$H(s) = \frac{Cs}{LCs^2 + R_s Cs + (1+\beta)}$$

Frequency response:

$$H(j\omega) = \frac{j\omega C}{1+\beta - LC\omega^2 + j\omega R_s C}$$

Amplitude.

$$|H(j\omega)|^2 = \frac{\omega^2 C^2}{(1+\beta - LC\omega^2)^2 + (\omega R_s C)^2}$$

Finding $\omega_m$ :

$$1+\beta - LC\omega^2 = 0$$

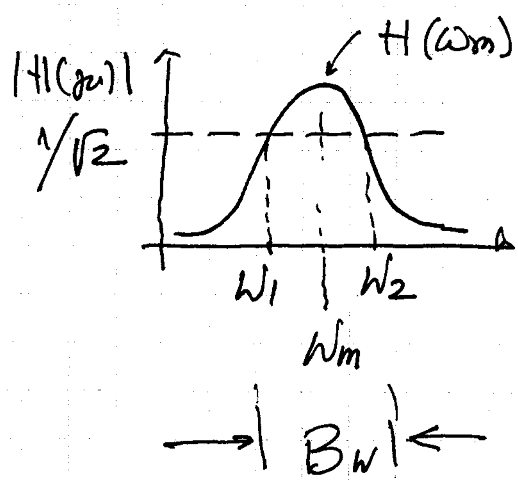
$$\omega_m = \sqrt{\frac{1+\beta}{LC}}$$

Then.

$$|H_m|^2 = |H(j\omega)|^2 \Big|_{\omega=\omega_m}$$

$$|H_m|^2 = \frac{\omega^2 C^2}{(\omega R_s C)^2} \Big|_{\omega=\omega_m}$$

$$H_m = \frac{1}{R_s}$$




Bandwidth:

$$\frac{|H(j\omega)|^2}{H_m^2} = 1/2$$

$$(1+\beta - LC\omega^2)^2 = (\omega R_s C)^2$$

$$1+\beta - LC\omega^2 = \pm \omega R_s C$$

$$LC\omega^2 \pm \omega R_s C - (1+\beta) = 0$$

$$\omega^2 \pm \frac{\omega R_s}{L} - \frac{1+\beta}{LC} = 0$$

$$\omega_{1/2} = -\frac{R_s}{2L} \pm \sqrt{\left(\frac{R_s}{2L}\right)^2 + \frac{1+\beta}{LC}}$$

$$\omega_{3/4} = \frac{R_s}{2L} \pm \sqrt{\left(\frac{R_s}{2L}\right)^2 + \frac{1+\beta}{LC}}$$

positive solutions for $\omega$ only. (choose +)

$$B\omega = \omega_3 - \omega_4 = \frac{R_s}{L}$$

Bandwidth

Hence:

$$I = I_1 + I_2$$

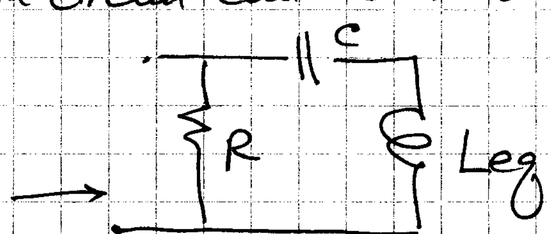
$$I = \frac{L_1 + L_2 - 2M}{s(L_1 L_2 - M^2)} \cdot V$$

OR, equivalently: $$V = s \cdot \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} \cdot I$$

$$L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$$

$$L_{eq} = \frac{2 \times 10 - 2^2}{2 + 10 - 2 \times 2} \; ; \; L_{eq} = \frac{16}{8} \; ; \; L_{eq} = 2H$$

The circuit can be viewed as:




Zin(s)

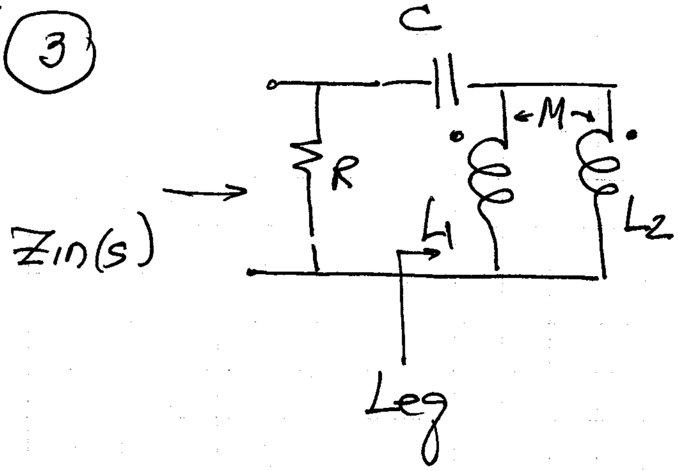
$$Z_{in}(s) = \frac{R \cdot \left(s \cdot L_{eq} + \frac{1}{Cs}\right)}{R + s L_{eq} + \frac{1}{Cs}}$$

$$Z_{in}(s) = \frac{R \cdot (1 + C L_{eq} \cdot s^2)}{1 + RCs + C L_{eq} \cdot s^2}$$

$$Z_{in}(s) = \frac{1 \times (1 + 0.8 \times 10^{-3} \times 2 s^2)}{1 + 1 \times 0.8 \times 10^{-3} s + 0.8 \times 10^{-3} \times 2 s^2}$$

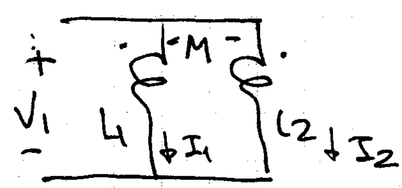
$$Z_{in}(s) = \frac{1 + 1.6 \times 10^{-3} s^2}{1 + 0.8 \times 10^{-3} s + 1.6 \times 10^{-3} s^2}$$

# 3




$R = 1\Omega$
$L_1 = 2H$
$L_2 = 10H$
$M = 2H$
$C = 0.8mF$

Find Leq




$$I = I_1 + I_2$$

$$V = L_1 s \cdot I_1 + M s \cdot I_2$$

$$V = L_2 s \cdot I_2 + M \cdot s \cdot I_1$$

$$L_1 I_1 + M I_2 = L_2 I_2 + M \cdot I_1$$

$$(L_1 - M) I_1 = (L_2 - M) I_2 \ ; \ I_2 = \frac{L_1 - M}{L_2 - M} \cdot I_1$$

Hence,
$$V = L_1 \cdot s \cdot I_1 + M s \cdot \frac{L_1 - M}{L_2 - M} \cdot I_1$$

$$I_1 = \frac{V}{L_1 s + M s \cdot \frac{L_1 - M}{L_2 - M}} \ ; \ I_1 = \frac{(L_2 - M) \cdot V}{s \cdot (L_1 L_2 - \cancel{L_1 M} + \cancel{L_1 M} - M^2)}$$

$$I_1 = \frac{L_2 - M}{s \cdot (L_1 L_2 - M^2)} \cdot V$$

$$I_2 = \frac{L_1 - M}{s (L_1 L_2 - M^2)} \cdot V$$

$$Z_{in}(s) = \frac{s^2 + \frac{1}{1.6\times10^{-3}}}{s^2 + \frac{0.8\times10^{-3}}{1.6\times10^{-3}}s + \frac{1}{1.6\times10^{-3}}}$$

$$Z_{in}(s) = \frac{s^2+625}{s^2+0.5s+625}$$

By definition, resonance occurs at:

$$Im\{Z_{in}(j\omega)\} = 0$$

$$Z(j\omega) = \frac{625-w^2}{(625-w^2)+0.5j\omega}$$

$$= (625-w^2)\times\frac{625-w^2-0.5j\omega}{(625-\omega^2)^2+(0.5\omega)^2}$$

$Im\{Z(j\omega)\}=0$ at $(625-w^2)\times(-0.5w)=0$

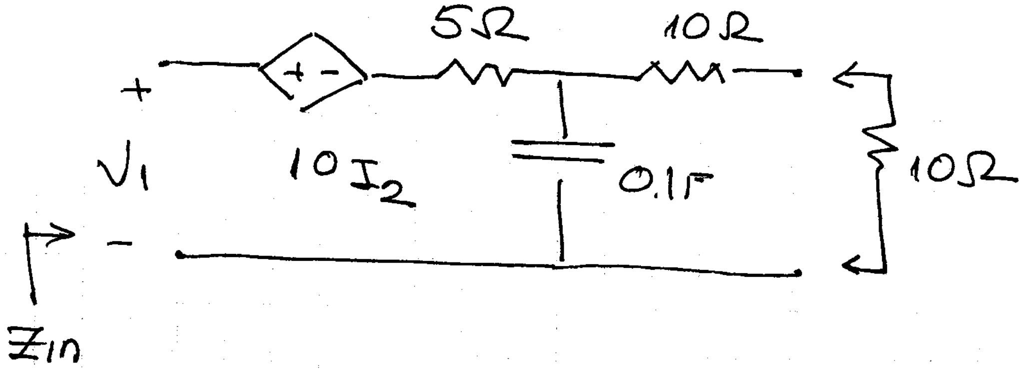
Trivial solution: $w=0$

Non-trivial solution: $625-w^2=0$

$w=25$ rad/s

at resonance: $Z(j\omega)\Big|_{w=25} = 0$

#4




Consider the two-port without the resistor load.
KVL and KCL equations:

$$V_1 - 10 \times I_2 - 5 \times I_1 - \frac{1}{0.1s} \cdot (I_1 + I_2) = 0$$

$$V_2 - 10 \times I_2 - \frac{1}{0.1s} \cdot (I_1 + I_2) = 0$$

OR

$$V_1 = 10 \times I_2 + 5 \times I_1 + \frac{10}{s} \cdot (I_1 + I_2)$$

$$V_2 = 10 \times I_2 + \frac{10}{s} \cdot (I_1 + I_2)$$

Hence,

$$V_1 = \left(5 + \frac{10}{s}\right) I_1 + 10\left(1 + \frac{1}{s}\right) \cdot I_2$$

$$V_2 = \frac{10}{s} \times I_1 + 10 \cdot \left(1 + \frac{1}{s}\right) I_2$$

$$Z = \begin{pmatrix} 5 + 10/s & 10(1 + 1/s) \\ 10/s & 10(1 + 1/s) \end{pmatrix}$$

$Z_{in} = ?$

The two port is now terminated by the resistive load:

$$V_2 = -10 \times I_2$$

From:
$$\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix}$$

$$V_2 = z_{21} I_1 + z_{22} I_2$$

$$V_2 = -10 I_2$$

Then:
$$-10 \times I_2 = \frac{10}{s} \times I_1 + 10(1 + 1/s) \cdot I_2$$

$$(-20 - 10/s) I_2 = 10/s \cdot I_1$$

Hence,
$$I_2 = \frac{-10}{20s + 10} \times I_1$$

$$I_2 = -\frac{1}{1 + 2s} \cdot I_1$$

From:
$$V_1 = z_{11} \cdot I_1 + z_{12} \cdot I_2$$

$$V_1 = (5 + 10/s) I_1 + 10 \cdot (1 + 1/s) \cdot \frac{-1}{1 + 2s} \cdot I_1$$

$$Z_{in} = V/I_1$$

And
$$Z_{in} = 5 + \frac{10}{s} - \frac{10(1+s)}{s(1+2s)}$$

$$= 5 + \frac{2s + 1 - (1+s)}{s(1+2s)} \times 10$$

$$Z_{in} = 5 + \frac{s}{s(1+2s)} \times 10$$

$$= 5 + \frac{1}{1+2s} \times 10$$

$$Z_{in} = \frac{10s+15}{1+2s} \; ; \; Z_{in} = \frac{5s+7.5}{s+0.5}$$

17 $v_1(t) = 10u(t)$; $V_1(s) = 10/s$

The easiest way to find $i_1(t)$ is from:

$$I_1 = V_1 / Z_{in}$$

$$I_1 = \frac{10}{s} \times \frac{s+0.5}{5s+7.5}$$

$$I_1 = \frac{10s+5}{s(5s+7.5)} \; ; \; I_1 = \frac{2s+1}{s(s+1.5)}$$

Partial fraction expansion leads to:

$$\frac{2s+1}{s(s+1.5)} = \frac{A}{s} + \frac{B}{s+1.5} \qquad A = \lim_{s\to 0} s \times \frac{2s+1}{s(s+1.5)}$$

$$A = \frac{1}{1.5}$$

$$A = 2/3$$

$$B = \lim_{s\to -1.5} (s+1.5) \times \frac{2s+1}{s(s+1.5)}$$

$$= \frac{2\times(-1.5)+1}{-1.5}$$

$$= +2/1.5 \; ; \quad B = +4/3$$

Hence, Laplace Transform of current $i_1(t)$ is: 

$$I_1 = \frac{2}{3s} + \frac{4}{3}\cdot\frac{1}{s+1.5}$$

Hence;

$$i_1(t) = \frac{2}{3}\cdot u(t) + \frac{4}{3}\cdot e^{-1.5t}\, u(t)$$

or $$i_1(t) = \frac{2}{3}\left(1+2e^{-1.5t}\right)\cdot u(t)$$