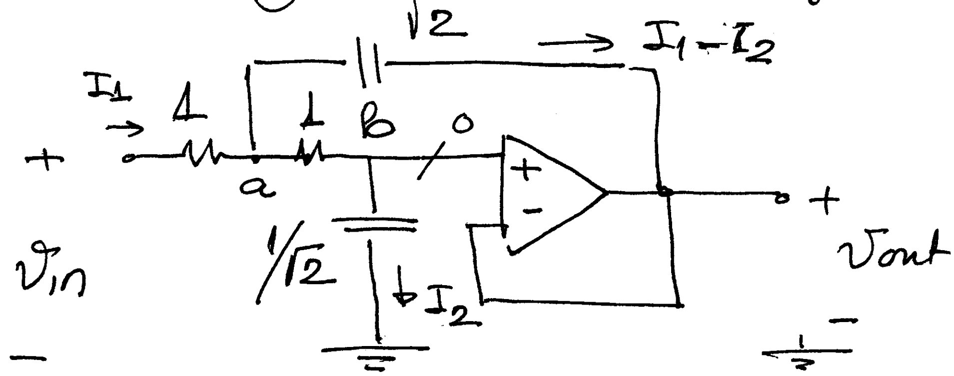
Midterm #2 Summer 2006

Sallen-key normalized low-pass filter



$R_1 = R_2 = 1$

$C_1 = \sqrt{2}$

$C_2 = 1/\sqrt{2}$

Ideal op-amp

$V_{OUT} = V_b$

$$I_2 = \frac{1}{\sqrt{2}} s \cdot V_{out} \qquad (1)$$

$$V_{in} - 1 \times I_1 - \left(1 + \frac{\sqrt{2}}{s}\right) I_2 = 0$$

KVL:

$$V_{in} = I_1 + \left(1 + \frac{\sqrt{2}}{s}\right) \cdot I_2 \qquad (2)$$

$$V_{in} = I_1 + \left(1 + \frac{\sqrt{2}}{s}\right) \cdot \frac{1}{\sqrt{2}} s \cdot V_{out}$$

$$V_{in} = I_1 + \frac{s + \sqrt{2}}{\sqrt{2}} \cdot V_{out}$$

$$I_1 = V_{in} - \frac{s + \sqrt{2}}{\sqrt{2}} \cdot V_{out}$$

KVL:

$$V_{in} - 1 \times I_1 - \frac{1}{\sqrt{2}s}(I_1 - I_2) - V_{out} = 0$$

$$V_{in} = \left(1 + \frac{1}{\sqrt{2}s}\right) I_1 - \frac{1}{\sqrt{2}s} \cdot I_2 + V_{out}$$

Hence,

$$V_{in} = \left(1 + \frac{1}{\sqrt{2}s}\right) \cdot \left(V_{in} - \frac{s+\sqrt{2}}{\sqrt{2}} V_{out}\right) - \frac{1}{\sqrt{2}s} \cdot \frac{1}{\sqrt{2}} s V_{out} + V_{out}$$

$$V_{in}\left(1 - 1 - \frac{1}{\sqrt{2}s}\right) = -\left(1 + \frac{1}{\sqrt{2}s}\right) \cdot \frac{s+\sqrt{2}}{\sqrt{2}} V_{out} + \frac{1}{2} V_{out}$$

$$-\frac{1}{\sqrt{2}s} \cdot V_{in} = -\frac{(\sqrt{2}s+1)(s+\sqrt{2})}{2s} V_{out} + \frac{1}{2} V_{out}$$

$$V_{in} = \left(\frac{(\sqrt{2}s^2 + s + 2s + \sqrt{2})}{\sqrt{2}} - \frac{s}{\sqrt{2}}\right) V_{out}$$

$$V_{in} = \frac{\sqrt{2}s^2 + 2s + \sqrt{2}}{\sqrt{2}} V_{out}$$

$$\frac{V_{out}}{V_{in}} = \frac{1}{s^2 + \sqrt{2}s + 1}$$

Zeros: 2 at infinity

Poles: 2 finite poles at $s^2 + \sqrt{2}s + 1 = 0$

$$s_{1/2} = -\frac{\sqrt{2}}{2} \pm \sqrt{\left(\frac{\sqrt{2}}{2}\right)^2 - 1}$$

$$= -\frac{\sqrt{2}}{2} \pm \sqrt{-1/2}$$

$$= \frac{-\sqrt{2}}{2} \pm j\frac{\sqrt{2}}{2}$$

$$H(j\omega) = \frac{1}{(1-\omega^2) + \sqrt{2} \cdot j\omega}$$

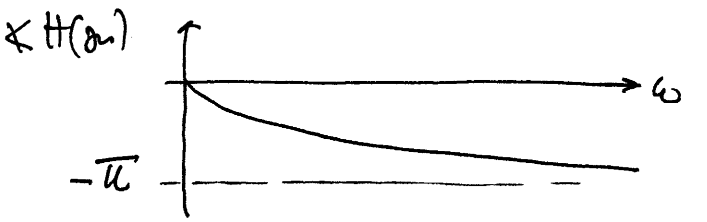
$$|H(j\omega)|^2 = \frac{1}{(1-\omega^2)^2 + 2\omega^2} \quad \text{or} \quad |H(j\omega)| = \frac{1}{\sqrt{(1-\omega^2)^2 + 2\omega^2}}$$

$$|H(j\omega)| = \frac{1}{\sqrt{1+\omega^4}}$$

$$\sphericalangle H(j\omega) = -\mathrm{arctg}\, \frac{\sqrt{2}\omega}{1-\omega^2}$$

Amplitude frequency respon:

Low pass filter
(Butterworth design)



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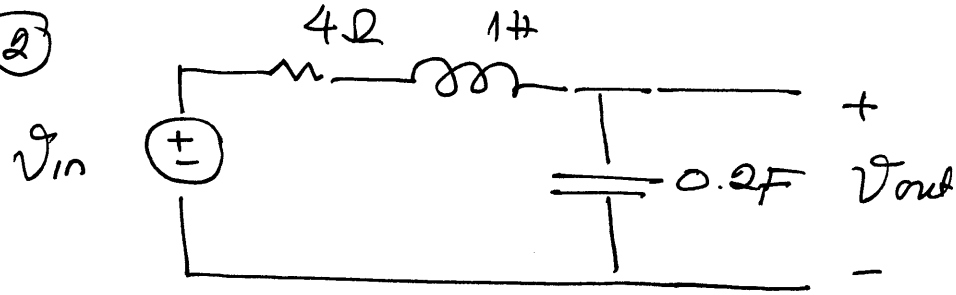

$$V_{out}(s) = \frac{1/Cs}{R + Ls + \frac{1}{Cs}}$$

$$H(s) = \frac{1}{LC} \cdot \frac{1}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

$$H(s) = \frac{1}{0.2} \cdot \frac{1}{s^2 + 4s + 5}$$

$$H(s) = \frac{5}{s^2+4s+5} \quad ; \quad H(s) = \frac{5}{(s+2)^2+1}$$

Impulse response:

$$h(t) = \mathcal{L}^{-1}\{H(s)\}$$

$$h(t) = \mathcal{L}^{-1}\left\{\frac{5}{(s+2)^2+1}\right\}$$

$$h(t) = 5e^{-2t} \cdot \sin t \; u(t)$$

complete response
particular solution
natural response
transient response

Step response:

$$V_{out}(s) = \frac{5}{(s+2)^2+1} \cdot V_{in}(s) \quad ; \quad V_{in}(s) = 1/s$$

(step function)

$$V_{out}(s) = \frac{5}{(s+2)^2+1} \times \frac{1}{s}$$

$$V_{out}(s) = \frac{A}{s} + \frac{Bs+C}{(s+2)^2+1}$$

$$A = \lim_{s \to 0} s \cdot V_{out}(s)$$

$$= \lim_{s \to 0} \frac{5}{(s+2)^2+1}$$

$$A = 1$$

$$V_{out}(s) = \frac{1}{s} + \frac{Bs+C}{(s+2)^2+1}$$

$$\frac{Bs+C}{(s+2)^2+1} = -\frac{1}{s} + \frac{5}{s[(s+2)^2+1]}$$

$$\frac{Bs+C}{(s+2)^2+1} = \frac{-(s+2)^2-1+5}{s[(s+2)^2+1]}$$

$$= \frac{-s^2-4s\cancel{-4-1+5}}{s[(s+2)^2+1]}$$

$$Bs+C = -s-4$$

$$B=-1; \quad C=-4$$

Hence: $V_{out}(s) = \frac{1}{s} - \frac{s+4}{(s+2)^2+1}$

$$V_{out}(s) = \frac{1}{s} - \frac{s+2}{(s+2)^2+1} - \frac{2}{(s+2)^2+1}$$

$$V_{out}(t) = (1 - e^{-2t}\cos t - 2e^{-2t}\sin t)\,u(t)$$

forced steady-state

transient

Complete response

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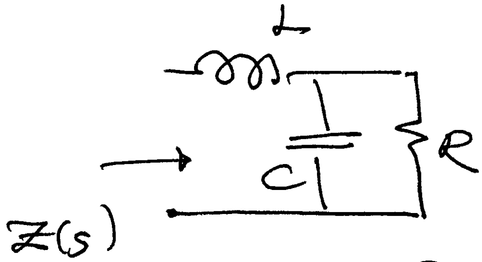

$$Z(s) = Ls + \frac{R \cdot \frac{1}{Cs}}{R + \frac{1}{Cs}}$$

$$Z(s) = Ls + \frac{R}{1+RCs}$$

$$Z(j\omega) = j\omega L + \frac{R}{1+j\omega RC}$$

$$Z(j\omega) = j\omega L + \frac{R(1-j\omega RC)}{(1+j\omega RC)(1-j\omega RC)}$$

$$Z(j\omega) = \frac{R}{1+(\omega RC)^2} + j\omega\left(L - \frac{R^2C}{1+(\omega RC)^2}\right)$$

Resonance:

$$Z(j\omega) = \text{Real}$$

$$X(\omega) = 0$$

$$\omega\left(L - \frac{R^2C}{1+(\omega RC)^2}\right) = 0$$

$\omega = 0$: Trivial solution

$$L - \frac{R^2C}{1+(\omega RC)^2} = 0 \; ; \quad \omega_r = \frac{1}{RC}\sqrt{\frac{R^2C}{L} - 1}$$

$$\omega_r = \sqrt{\frac{1}{LC} - \frac{1}{R^2C^2}}$$

$$\omega_r = \omega_0\sqrt{1 - \frac{L}{CR^2}} \; ; \quad \omega_0 = \frac{1}{\sqrt{LC}}$$