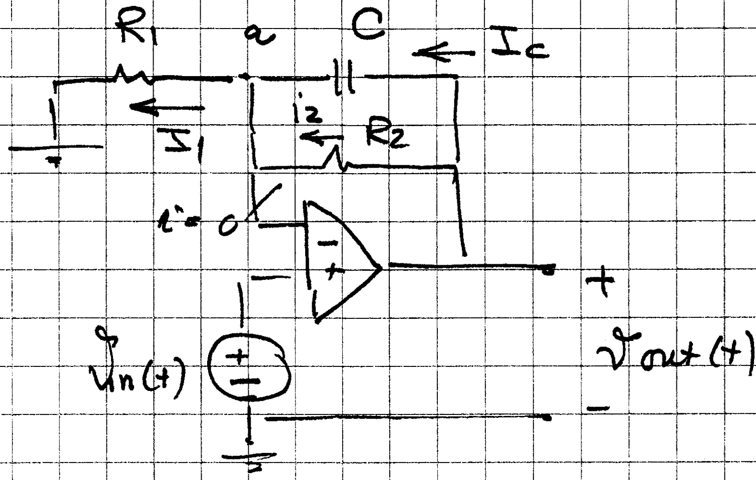
Midterm #2: ENSC 320

1)



$C = 1\mu F$

R_1, R_2 ?

$V_a = V_{in}$; input current $= 0$ Ideal op amp

in s-domain.

$$I_1 = \frac{V_{in}}{R_1}$$

$$V_a = V_{in}$$

$$I_c + I_2 = I_1$$

$$\frac{1}{Cs} \cdot I_c = R_2 \cdot I_2$$

$$V_{out} - R_2 \cdot I_2 - V_{in} = 0$$

Hence:

$$I_c + I_2 = \frac{V_{in}}{R_1}$$

$$I_C = R_2 C s \cdot I_2$$

or:

$$I_2 = \frac{1}{1 + R_2 C s} \cdot \frac{V_{in}}{R_1}$$

Hence, $V_{out} = R_2 \cdot \dfrac{1}{1+R_2Cs} \cdot \dfrac{V_{in}}{R_1} + V_{in}$

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)}$$

$$H(s) = \frac{R_2}{R_1} \cdot \frac{1}{1+R_2Cs} + 1$$

$$H(s) = \frac{\frac{R_2}{R_1} + 1 + R_2Cs}{1+R_2Cs}$$

$$H(s) = \frac{\frac{R_2+R_1}{R_1R_2} + Cs}{\frac{1}{R_2} + Cs}$$

$$H(s) = \frac{s + \frac{1}{C}\cdot\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}{s + \frac{1}{R_2C}}$$

$C = 1\mu F$

$$H(s) = \frac{s+4}{s+2}$$

Hence: $\dfrac{1}{R_2C} = 2$ $\qquad R_2C = 1/2$

$\dfrac{1}{C}\cdot\left(\dfrac{1}{R_1} + \dfrac{1}{R_2}\right) = 4$ $\qquad C = 1\mu F$

$R_2 = 0.5 \times 10^6$

$R_2 = 500\ k\Omega$

$\dfrac{1}{R_1} + \dfrac{1}{R_2} = 4\times10^{-6}$

$\dfrac{1}{R_1} = 4\times10^{-6} - 2\times10^{-6}$; $R_1 = 500\ k\Omega$

• Step respn: $H(s) = \frac{s+4}{s+2}$

$$V_{out}(s) = \frac{1}{s} \times H(s)$$

$$= \frac{s+4}{s(s+2)}$$

$$\frac{s+4}{s(s+2)} = \frac{A}{s} + \frac{B}{s+2}$$

$$A + B = 1$$

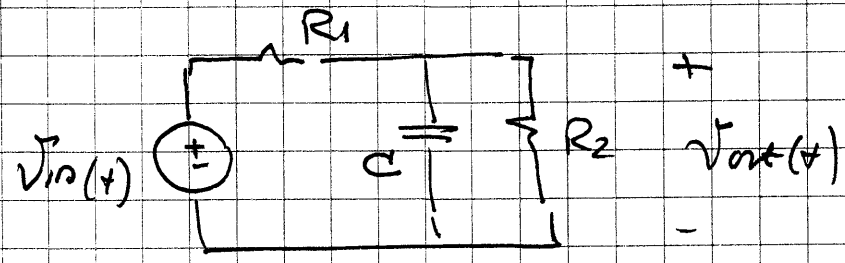
$$2A = 4 \quad ; \quad A = 2, \; B = -1$$

$$V_{out}(s) = \frac{2}{s} - \frac{1}{s+2}$$

$$v_{out}(t) = 2u(t) - e^{-2t}u(t)$$

4

2)



$R_1 = 50\Omega$

$R_2 = 200\Omega$

$C = 25 mF$

$V_{in}(t) = 10e^{-2t}\, u(t)$

$V_c(0^-) = 4V$

$$H(s) = \frac{R_2 \cdot \frac{1}{Cs}}{R_2 + \frac{1}{Cs}} \cdot \frac{1}{R_1 + \frac{R_2 \cdot 1/Cs}{R_2 + 1/Cs}}$$

$$H(s) = \frac{R_2}{1 + R_2 Cs} \cdot \frac{1}{R_1 + \frac{R_2}{1 + R_2 Cs}}$$

$$H(s) = \frac{R_2}{R_1 + R_2 + R_1 R_2 \cdot Cs}$$

$$H(s) = \frac{\frac{1}{CR_1}}{\frac{R_1 + R_2}{R_1 R_2 C} + S} \qquad R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

$$H(s) = \frac{\frac{1}{CR_1}}{S + \frac{1}{R_{eq} \cdot C}}$$

$$H(s) = \frac{\frac{1}{2.5 \times 10^{-3}} \cdot \frac{1}{50}}{S + \frac{1}{40 \times 2.5 \times 10^{-3}}} \qquad R_{eq} = \frac{50 \times 200}{250}$$

$R_{eq} = 40\Omega$

$$H(s) = \frac{\frac{100\cancel{0}}{2.5\times 5\cancel{0}}}{s + \frac{1000}{40\times 2.5}}$$

$$H(s) = \frac{8}{s+10}$$

- Zero state respons:

$$v_{in}(t) = 10e^{-2t}u(t)$$

$$V_{in}(s) = \frac{10}{s+2}$$

$$V_{out}(s) = \frac{8}{s+10}\cdot\frac{10}{s+2}$$

$$= \frac{A}{s+10} + \frac{B}{s+2}$$

$$A+B=0$$

$$2A+10B=80$$

$$2A-10A=80$$

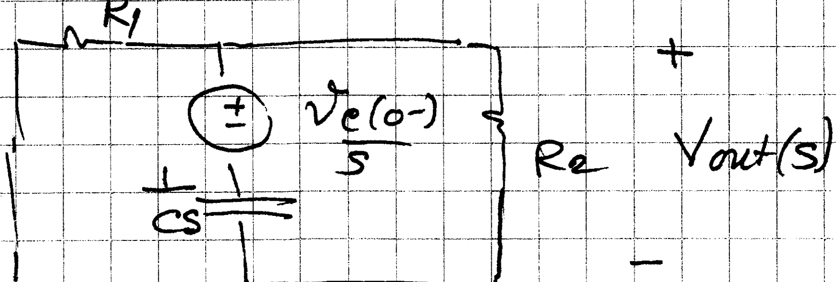
$$A=-10$$

$$B=10$$

$$V_{out}(s) = -\frac{10}{s+10} + \frac{10}{s+2}$$

$$v_{zero\text{-}sta}(t) = \left(-10e^{-10t} + 10e^{-2t}\right)u(t)$$

- Zero-input respon:



$$\bar{V}_{out}(s) = \frac{\frac{R_1 \cdot R_2}{R_1 + R_2}}{\frac{R_1 R_2}{R_1 + R_2} + \frac{1}{Cs}} \times \frac{v_c(0-)}{s}$$

$$\bar{V}_{out} = \frac{R_{eq}}{R_{eq} + \frac{1}{Cs}} \times \frac{v_c(0-)}{s}$$

$$= \frac{R_{eq} \cdot C}{R_{eq} \cdot C s + 1} \cdot v_c(0-)$$

$$= \frac{1}{s + \frac{1}{R_{eq} \cdot C}} \cdot v_c(0-)$$

$$\bar{V}_{out} = \frac{1}{s + \frac{1}{40 \times 2.5 \times 10^{-3}}} \cdot 4$$

$$= \frac{4}{s + 10}$$

$$v_{out_{zero-inp}} = 4e^{-10t} u(t)$$

– complete respons:

$$v_{out}(t) = \left(-10e^{-10t} + 10e^{-2t} + 4e^{-10t}\right) u(t)$$

$$v_{out}(t) = \left(-6e^{-10t} + 10e^{-2t}\right) u(t)$$

Forced response: $10e^{-2t}u(t)$

Natural resp.: $-6e^{-10t}u(t)$

- Steady-state: ϕ

transient: $\left(-6e^{-10t}+10e^{-2t}\right)u(t)$

Let $v_{in}(t)=10e^{-10}u(t)$; $V_c(0-)=0$

Then $V_{out}(s)=\frac{8}{s+10}\times V_{in}(s)$

$$=\frac{8}{s+10}\times\frac{10}{s+10}$$

$$=\frac{80}{(s+10)^2}$$

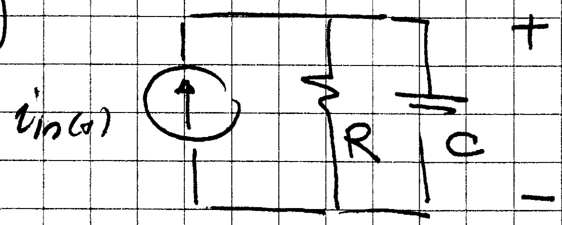
$$v_{out}(t)=80te^{-10t}u(t)$$

! not well defin.

Note that $v_{in}(t)=10e^{-10}u(t)$

-10 is the natural frequency of the circuit.

3)



$R = 0.25\Omega$
$C = 1F$

$$H(s) = \frac{\frac{1}{Cs} \times R}{R + \frac{1}{Cs}} \quad ; \quad H(s) = \frac{R}{1 + RCs}$$

$$H(s) = \frac{1/C}{s + 1/RC}$$

$$H(s) = \frac{1}{s + \frac{1}{0.25}} \quad ; \quad H(s) = \frac{1}{s+4}$$

$$h(t) = e^{-4t} u(t)$$

$$i_{in}(t) = 3e^{-t} u(t)$$

$$V_{out}(t) = h(t) * i_{in}(t)$$

$$V_{out}(t) = \int_{-\infty}^{+\infty} e^{-4\tau} u(\tau) \cdot 3e^{-(t-\tau)} u(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{+\infty} 3e^{-t} \cdot e^{-3\tau} u(\tau) u(t-\tau)\, d\tau$$

$t < 0 \quad V_{out}(t) = 0$

$t \geq 0 \quad V_{out}(t) = \int_0^t 3e^{-t} \cdot e^{-3\tau}\, d\tau$

$$= 3e^{-t} \cdot \int_0^t e^{-3\tau}\, d\tau = 3e^{-t} \cdot \frac{e^{-3\tau}}{-3} \Big|_0^t$$

$$V_{out}(t) = -e^{-t}\left(e^{-3t} - 1\right) u(t) \; ; \; V_{out}(t) = \left(e^{-t} - e^{-4t}\right) u(t)$$