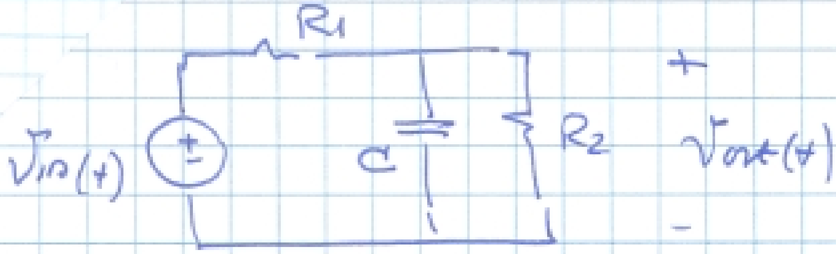
1



$R_1 = 50\Omega$

$R_2 = 200\Omega$

$C = 2.5\,mF$

$v_{in}(t) = 10e^{-2t}u(t)$

$v_C(0^-) = 4V$

$$H(s) = \frac{R_2 \cdot \frac{1}{Cs}}{R_2 + \frac{1}{Cs}} \cdot \frac{1}{R_1 + \frac{R_2 \cdot 1/Cs}{R_2 + 1/Cs}}$$

$$H(s) = \frac{R_2}{1 + R_2 Cs} \cdot \frac{1}{R_1 + \frac{R_2}{1 + R_2 Cs}}$$

$$H(s) = \frac{R_2}{R_1 + R_2 + R_1 R_2 \cdot Cs}$$

$$H(s) = \frac{\frac{1}{CR_1}}{\frac{R_1 + R_2}{R_1 R_2 C} + S}$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

$$H(s) = \frac{\frac{1}{CR_1}}{S + \frac{1}{R_{eq} \cdot C}}$$

$$H(s) = \frac{\frac{1}{2.5 \times 10^{-3}} \cdot \frac{1}{50}}{S + \frac{1}{40 \times 2.5 \times 10^{-3}}}$$

$$R_{eq} = \frac{50 \times 200}{250}$$

$$R_{eq} = 40\Omega$$

$$H(s) = \frac{\dfrac{1000}{2.5 \times 50}}{s + \dfrac{1000}{40 \times 2.5}}$$

$$H(s) = \frac{8}{s+10}$$

– Zero state respons:

$$v_{in}(t) = 10e^{-2t}u(t)$$

$$V_{in}(s) = \frac{10}{s+2}$$

$$V_{out}(s) = \frac{8}{s+10} \cdot \frac{10}{s+2}$$

$$= \frac{A}{s+10} + \frac{B}{s+2}$$

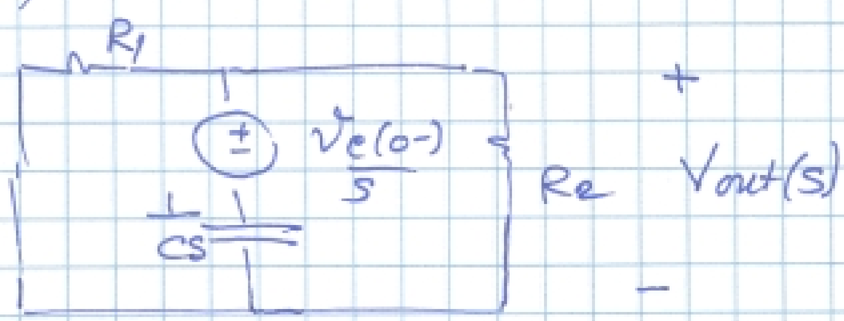
$$A + B = 0$$
$$2A + 10B = 80$$

$$2A - 10A = 80$$
$$A = -10$$
$$B = 10$$

$$V_{out}(s) = -\frac{10}{s+10} + \frac{10}{s+2}$$

$$v_{zero\text{-}sta}(t) = \left(-10e^{-10t} + 10e^{-2t}\right)u(t)$$

– zero-input respon:



③

$$V_{out}(s) = \frac{\frac{R_1 \cdot R_2}{R_1 + R_2}}{\frac{R_1 R_2}{R_1 + R_2} + \frac{1}{Cs}} \times \frac{v_c(0-)}{s}$$

$$V_{out} = \frac{R_{eq}}{R_{eq} + \frac{1}{Cs}} \times \frac{v_c(0-)}{s}$$

$$= \frac{R_{eq} \cdot C}{R_{eq} \cdot C s + 1} \cdot v_c(0-)$$

$$= \frac{1}{S + \frac{1}{R_{eq} \cdot C}} \cdot v_c(0-)$$

$$V_{out} = \frac{1}{S + \frac{1}{40 \times 2.5 \times 10^{-3}}} \cdot 4$$

$$= \frac{4}{s + 10}$$

$$v_{out_{zero-inp}} = 4e^{-10t} u(t)$$

– complete respons:

$$v_{out}(t) = (-10e^{-10t} + 10e^{-2t} + 4e^{-10t}) u(t)$$

$$v_{out}(t) = (-6e^{-10t} + 10e^{-2t}) u(t)$$

Forced respons: $10e^{-2t}u(t)$

Natural resp: $-6e^{-10t}u(t)$

- Steady-state: ϕ

transient: $(-6e^{-10t}+10e^{-2t})u(t)$

Let $v_{in}(t)=10e^{-10}u(t)$; $V_c(0-)=0$

then $V_{out}(s)=\frac{8}{s+10}\times V_{in}(s)$

$$=\frac{8}{s+10}\times\frac{10}{s+10}$$

$$=\frac{80}{(s+10)^2}$$

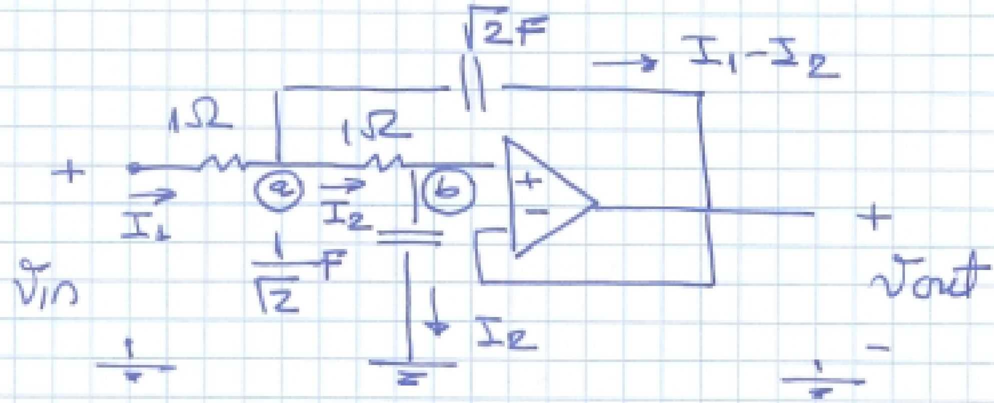
$$v_{out}(t)=80te^{-10t}u(t)$$

! not well defn.

note that $v_{in}(t)=10e^{-10}u(t)$

-10 is the natural frequency of the circuit.

2



Sallen-key filter:

$$V_{out} = V_b \quad (1)$$

$$I_2 = \frac{1}{\sqrt{2}} s \cdot V_{out} \quad (2)$$

KVL: $$V_a = \left(1 + \frac{\sqrt{2}}{s}\right) I_2 \quad (3)$$

using (2): $$= \left(1 + \frac{\sqrt{2}}{s}\right) \times \frac{s}{\sqrt{2}} \cdot V_{out}$$

$$= \left(\frac{s}{\sqrt{2}} + 1\right) \cdot V_{out} \quad (4)$$

KVL: $$V_{in} - V_a = 1 \times I_1$$

$$V_{in} = V_a + I_1$$

Hence: $$I_1 = V_{in} - V_a$$

From (4): $$I_1 = V_{in} - \left(\frac{s}{\sqrt{2}} + 1\right) \cdot V_{out} \quad (5)$$

Voltage across capacitor of $\sqrt{2}$F is:

$$V_a - V_{out} = \frac{1}{\sqrt{2}s} \cdot (I_1 - I_2) \qquad (6)$$

Substituting (2), (4), and (5) into (6) gives:

$$\left(\frac{s}{\sqrt{2}} + 1\right) \cdot V_{out} - V_{out} = \frac{1}{\sqrt{2}s} \times \left(V_{in} - \left(\frac{s}{\sqrt{2}} + 1\right) V_{out} - \frac{1}{\sqrt{2}} s V_{out}\right)$$

Hence,

$$\frac{s}{\sqrt{2}} \cdot V_{out} = \frac{1}{\sqrt{2}s} \cdot V_{in} - \frac{1}{\sqrt{2}s} \cdot \left(\frac{s}{\sqrt{2}} + 1 + \frac{s}{\sqrt{2}}\right) V_{out}$$

$$\frac{s}{\sqrt{2}} \cdot V_{out} = \frac{1}{\sqrt{2}s} \cdot V_{in} - \frac{1}{\sqrt{2}s} \cdot \left(s\sqrt{2} + 1\right) \cdot V_{out} \quad / \cdot \sqrt{2}s$$

$$s^2 V_{out} = V_{in} - \left(s\sqrt{2} + 1\right) V_{out}$$

or:

$$\frac{V_{out}}{V_{in}} = \frac{1}{s^2 + \sqrt{2}s + 1}$$

Zeros: Two at infinity.

Poles: Two finite poles: $s^2 + \sqrt{2}s + 1 = 0$

$$s_{1/2} = -\frac{\sqrt{2}}{2} \pm \sqrt{\left(\frac{\sqrt{2}}{2}\right)^2 - 1}$$

$$s_{1/2} = -\frac{\sqrt{2}}{2} \pm j\frac{\sqrt{2}}{2}$$

$$s_{1/2} = \frac{1}{\sqrt{2}}(-1 \pm j) \quad \text{(on a circle)}$$

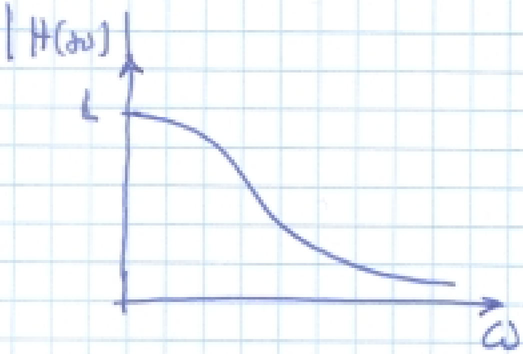
Frequency respons:

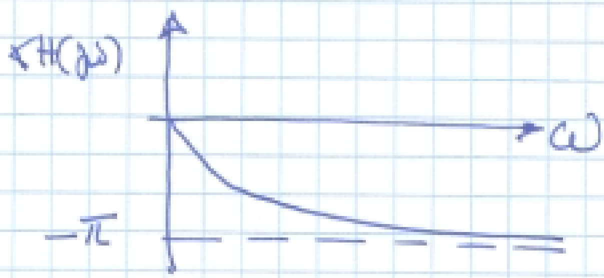
$$H(s) = \frac{1}{s^2 + \sqrt{2}s + 1}$$

$$H(j\omega) = \frac{1}{1 - \omega^2 + j\sqrt{2}\omega}$$

$$|H(j\omega)| = \frac{1}{\sqrt{(1-\omega^2)^2 + 2\omega^2}}$$

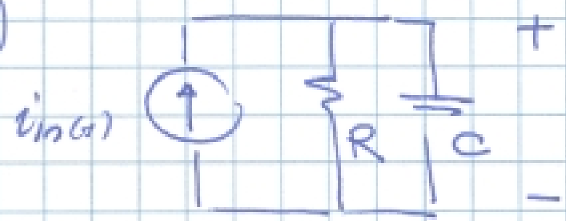
$$\measuredangle H(j\omega) = -\operatorname{arctg} \frac{\sqrt{2}\omega}{1-\omega^2}$$





this is a low-pass filter.

3)



$R = 0.25\Omega$
$C = 1F$

$$H(s) = \frac{\frac{1}{Cs} \times R}{R + \frac{1}{Cs}} \quad ; \quad H(s) = \frac{R}{1+RCs}$$

$$H(s) = \frac{1/C}{s + 1/RC}$$

$$H(s) = \frac{1}{s + \frac{1}{0.25}} \quad ; \quad H(s) = \frac{1}{s+4}$$

$$h(t) = e^{-4t} u(t)$$

$$i_{in}(t) = 3e^{-t} u(t)$$

$$V_{out}(t) = h(t) * i_{in}(t)$$

$$v_{out}(t) = \int_{-\infty}^{\infty} e^{-4\tau} u(\tau) \cdot 3e^{-(t-\tau)} u(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} 3e^{-t} \cdot e^{-3\tau} u(\tau) u(t-\tau)\, d\tau$$

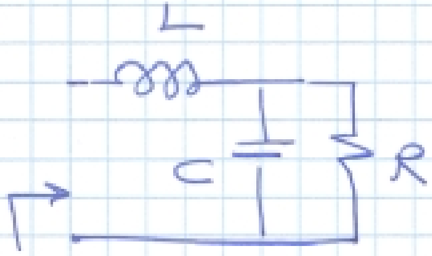
$t < 0 \quad v_{out}(t) = 0$

$t \geq 0 \quad v_{out}(t) = \int_0^t 3e^{-t} \cdot e^{-3\tau}\, d\tau$

$$= 3e^{-t} \cdot \int_0^t e^{-3\tau}\, d\tau = 3e^{-t} \cdot \frac{e^{-3\tau}}{-3} \Big|_0^t$$

$$v_{out}(t) = -e^{-t}\left(e^{-3t} - 1\right) u(t) \; ; \; v_{out}(t) = \left(e^{-t} - e^{-4t}\right) u(t)$$

4



$Z(s)$

$$Z(s) = Ls + \frac{R \cdot \frac{1}{Cs}}{R + \frac{1}{Cs}}$$

$$Z(s) = Ls + \frac{R}{1+RCs}$$

$$Z(j\omega) = j\omega L + \frac{R}{1+j\omega RC}$$

$$Z(j\omega) = j\omega L + \frac{R(1-j\omega RC)}{1+(\omega RC)^2}$$

$$Z(j\omega) = \frac{R}{1+(\omega RC)^2} + j\left(\omega L - \frac{\omega R^2 C}{1+(\omega RC)^2}\right)$$

Resonance condition:

$$\mathrm{Im}\left(Z(j\omega)\right) \equiv 0$$

$$\omega L - \frac{\omega R^2 C}{1+(\omega RC)^2} = 0$$

$w=0$: Trivial solution

$$L - \frac{R^2 C}{1+(\omega RC)^2} = 0$$

$$w_r = \sqrt{\frac{1}{LC} - \frac{1}{(RC)^2}}$$

$$w = \frac{1}{RC} \cdot \sqrt{\frac{R^2 C}{L} - 1}$$

Let $\omega_0 = \frac{1}{\sqrt{LC}}$

Then

$$\omega_r = \frac{1}{\sqrt{LC}} \cdot \sqrt{1 - \frac{L}{R^2C}}$$

$$\omega_r = \omega_0 \sqrt{1 - \frac{L}{R^2C}}$$