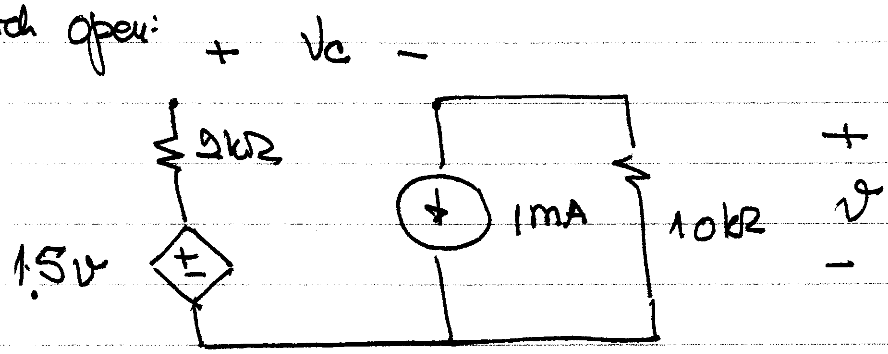
i) Switch open:



$$v = -10\times10^{3}\times10^{-3}$$

$$v = -10$$

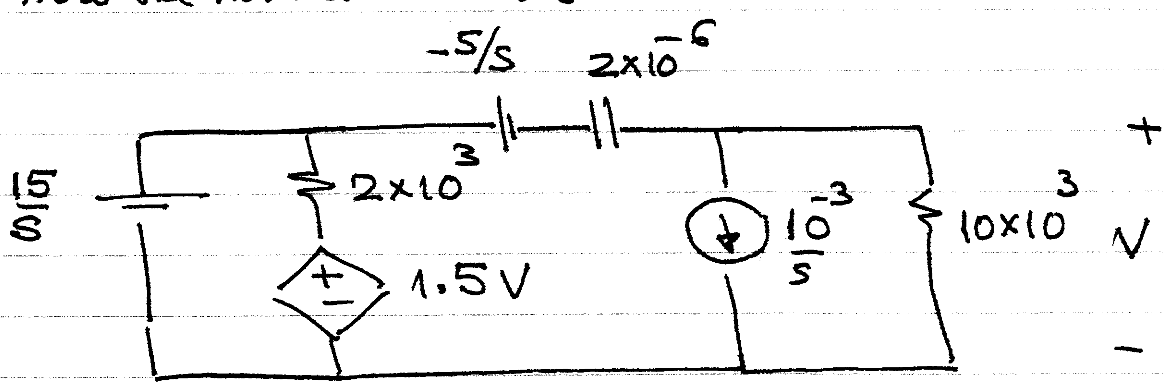
$$V_c - 1.5v + v = 0$$

$$V_c = 0.5v$$

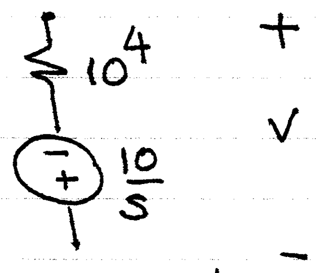
$$V_c = -5V$$

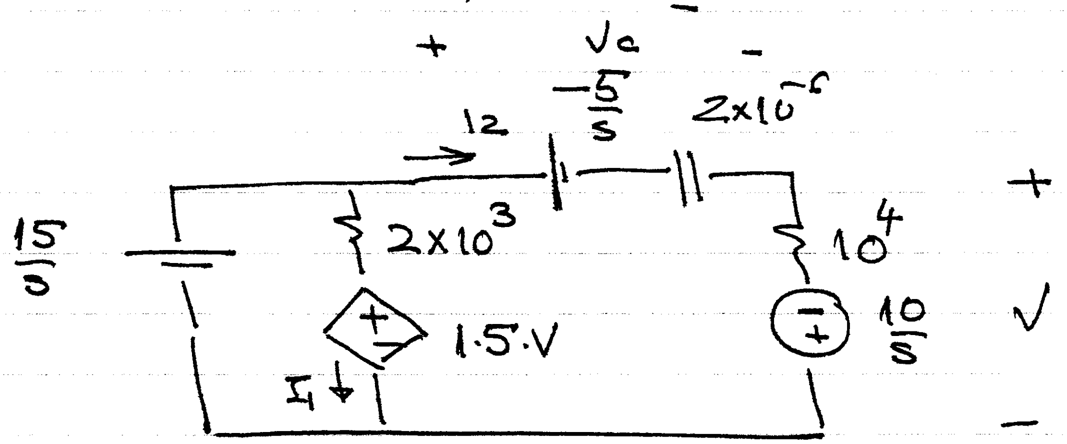
Switch closed

Note the initial conditions



thevenen's equivalent





$$\frac{15}{s} - 2\times10^3 I_1 - 1.5V = 0 \qquad (1)$$

$$\frac{15}{s} - \left(-\frac{5}{s}\right) - \frac{1}{2\times10^{-6}s}\cdot I_2 - V = 0 \qquad (2)$$

$$V - 10^4\cdot I_2 + \frac{10}{s} = 0 \qquad (3)$$

Hence

$$\frac{15}{s} = 2\times10^3 I_1 + 1.5V \qquad (1)$$

$$\frac{20}{s} = \frac{1}{2\times10^{-6}s}\cdot I_2 + V \qquad (2)$$

$$V = 10^4 I_2 - \frac{10}{s} \qquad (3)$$

$$\frac{15}{s} = 2\times 10^{3} I_1 + 1.5\cdot\left(10^4 I_2 - \frac{10}{s}\right) \qquad (1)$$

$$\frac{20}{s} = \frac{1}{2\times 10^{-6} s}\cdot I_2 + 10^4 I_2 - \frac{10}{s} \qquad (2)$$

From (2): $\dfrac{30}{s} = \left(10^4 + \dfrac{1}{2\times 10^{-6} s}\right) I_2$

$$I_2 = \frac{30}{s}\cdot\frac{1}{10^4 + \frac{10^6}{2}\frac{1}{s}}$$

$$I_2 = \frac{30}{s}\cdot\frac{1}{10^4\left(1 + \frac{50}{s}\right)}$$

$$I_2 = 10^{-3}\cdot\frac{3}{s+50}$$

Hence: $V = 10^4\cdot I_2 - \dfrac{10}{s}$

$$V = 10^4\cdot\left(10^{-3}\cdot\frac{3}{s+50}\right) - \frac{10}{s}$$

$$V = \frac{30}{s+50} - \frac{10}{s}$$

$$V = \frac{3s - s + 50}{s(s+50)}\, 10$$

$$V = 10\cdot\frac{2s - 50}{s(s+50)}$$

$$V = 10 \cdot \frac{s+s+50}{s(s+50)}$$

$$\frac{A}{s} + \frac{B}{s+50}$$

$$A+B=2$$

$$50A=-50$$

or/ $$v = 10 \cdot \frac{3}{s+50} - 10 \cdot \frac{1}{s}$$

$$A=-1$$

$$B=3$$

$$v(t) = 10e^{-50t}\,u(t) - 10\,u(t)$$

Find $V_c(t)$:

$$V_c = -\frac{5}{s} + \frac{1}{2\times10^{-6}s} \cdot I_2$$

$$V_c = -\frac{5}{s} + \frac{10^6}{2s} \cdot 10^{-3} \cdot \frac{3}{s+50}$$

$$V_c = -\frac{5}{s} + \frac{10^{+3}}{2s}\,\frac{3}{s+50}$$

$$V_c = -\frac{5}{s} + \frac{500\times3}{s(s+50)} \;;\; V_c = \frac{-5(s+50)+1500}{s(s+50)}$$

$$V_c = \frac{1250-5s}{s(s+50)}$$

Since: $$\frac{1250-5s}{s(s+50)} = \frac{A}{s} + \frac{B}{s+50}$$

$$1250 = 50A \qquad A=25$$

$$A+B=-5 \qquad B=-30$$

$$V_C = \frac{25}{s} - \frac{30}{s+50}$$

$$v_C(t) = 25\,u(t) - 30\,e^{-50t}\,u(t)$$

Check: $V_C(t) = 25\,u(t) - 30\,e^{-50t}\,u(t)$

$$v_C(t) = 5\left(5 - 6e^{-50t}\right)u(t)$$

$$v_C(0) = -5 \quad ✓$$

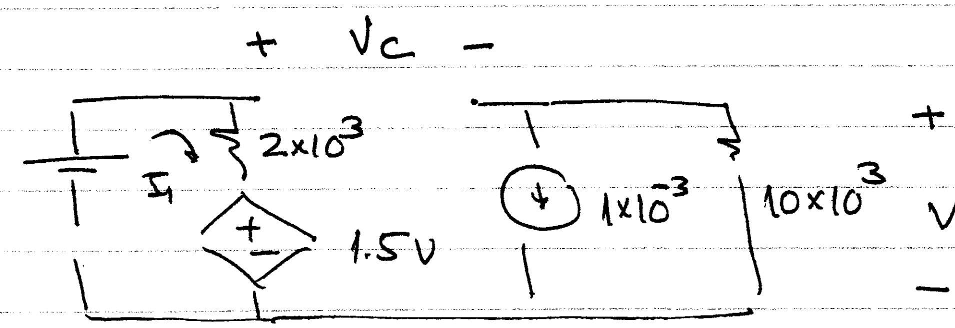
$$V_C(\infty) = 25$$

$$V_C(s) = \frac{1250 - 5s}{s(s+50)}$$

$\lim\limits_{s\to\infty} s\cdot V_C(s) = -5$ ✓ $\qquad \lim\limits_{s\to\infty} s\cdot V_C(s) = v_C(0)$

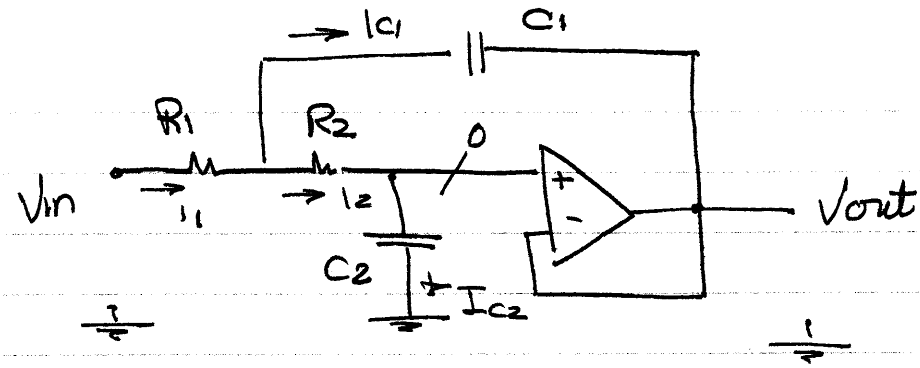
$\lim\limits_{s\to 0} s\cdot V_C(s) = 25 \qquad \lim\limits_{s\to 0} s\cdot V_C(s) = v_C(\infty)$

Check:



$V = -10^{4} \times 10^{-3}$; $V = -10\,V$; $V_C = V + 15$; $V_C = 25\,V$ ✓

2)



$C_1 = 2Q$

$C_2 = \frac{1}{2Q}$

Ideal op-amp:

KVL + KCL:

$$V_{in} = R_1 I_1 + R_2 I_2 + V_{out} \qquad (1)$$

$$V_{in} = R_1 I_1 + \frac{1}{C_1 s} \cdot I_{C1} + V_{out} \qquad (2)$$

KCL :

$$I_1 = I_2 + I_{C1} \qquad (3)$$

$$I_{C2} = C_2 s V_{out}$$

$$I_2 = I_{C2}$$

From (1) and (2): $R_2 I_2 = \frac{1}{C_1 s} I_{C1} \Rightarrow I_{C1} = R_2 C_1 s I_2$

Then:

$$I_1 = I_2 + I_{C1}$$

$$I_1 = I_2 + R_2 C_1 s I_2 \quad ; \quad I_1 = (1 + R_2 C_1 s) I_2$$

From (1) $V_{in} = R_1 \cdot (1 + R_2 C_1 s) I_2 + R_2 I_2 + V_{out}$

$$I_2 = C_2 s V_{out}$$

$$V_{in} = \left(R_1 (1 + R_2 C_1 s) + R_2 \right) C_2 s V_{out} + V_{out}$$

$$\frac{V_{out}}{V_{in}} = \frac{1}{1 + C_2 s\,(R_1 + R_1 R_2 C_1 s + R_2)}$$

$$\frac{V_{out}}{V_{in}} = \frac{1}{1 + (C_2 R_1 + C_2 R_2)s + R_1 R_2 C_1 C_2 s^2}$$

$R_1 = 1$

$R_2 = 1$

$C_1 = 2Q$

$C_2 = \frac{1}{2Q}$

$$\frac{V_{out}}{V_{in}} = \frac{1}{1 + \frac{1}{Q}s + s^2}$$

zeros: 2 at infinity

poles: $s^2 + \frac{1}{Q}s + 1 = 0 \quad s_{1/2} = -\frac{1}{2Q} \pm \sqrt{\left(\frac{1}{2Q}\right)^2 - 1}$

let $Q = 1/\sqrt{2}$

$$H(s) = \frac{1}{1 + \sqrt{2}s + s^2} \quad ; \quad s_{1/2} = -\frac{1}{\sqrt{2}} \pm \sqrt{\frac{1}{2} - 1}$$

$$s_{1/2} = -\frac{1}{\sqrt{2}} \pm j\frac{1}{\sqrt{2}}$$

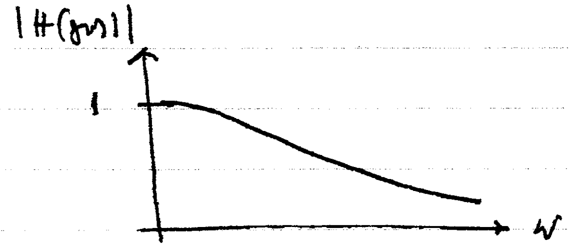
$$s_{1/2} = \frac{-1 \pm j}{\sqrt{2}}$$

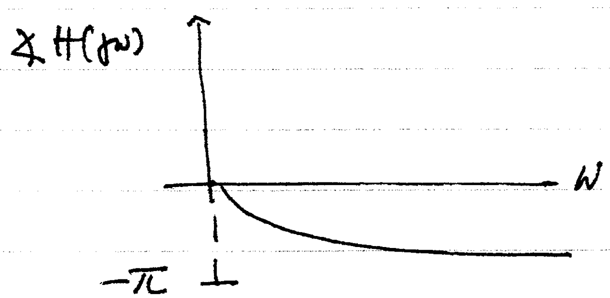
$$H(j\omega) = \frac{1}{1+\sqrt{2}j\omega-\omega^2}$$

$$|H(j\omega)|^2 = \frac{1}{(1-\omega^2)^2+2\omega^2}$$

$$\measuredangle H(j\omega) = -\arctan\frac{\sqrt{2}\omega}{1-\omega^2}$$

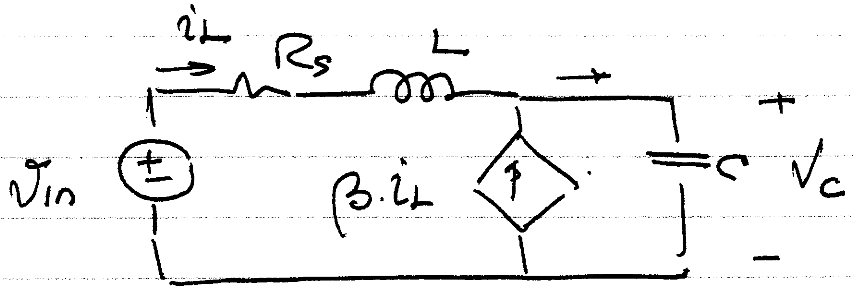
Plot:





Low pass Sallen an key filter

3.



$$V_{in} = (R_S + SL)\cdot I_L + V_C$$

$$I_C = CS \cdot V_C$$

$$I_L = -\beta I_L + I_C \Rightarrow (1+\beta) I_L = I_C$$

$$V_C = \frac{1}{CS}\cdot I_C$$

$$V_C = \frac{1}{CS}\cdot (1+\beta) I_L$$

$$V_{in} = (R_S + SL)\cdot I_L + \frac{1}{CS}\cdot(1+\beta)\cdot I_L$$

$$H(s) = \frac{I_L}{V_{in}}$$

$$H(s) = \frac{1}{R_S + SL + \frac{1}{CS}\cdot(1+\beta)}$$

$$H(s) = \frac{CS}{LCS^2 + R_S CS + (1+\beta)}$$

$$H(j\omega) = \frac{j\omega C}{1+\beta - LC\omega^2 + j\omega R_S C}$$

$$|H(j\omega)|^2 = \frac{\omega^2 C^2}{(1+\beta - LC\omega^2)^2 + (\omega R_s C)^2}$$

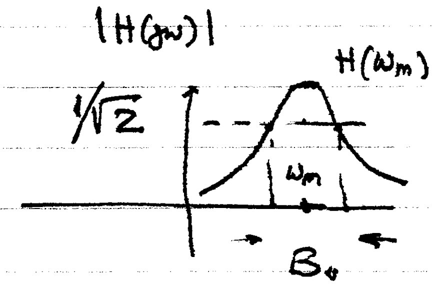
ω_m:

$$1+\beta - LC\omega^2 = 0$$

$$\omega = \sqrt{\frac{1+\beta}{LC}}$$

$$|H_m|^2 = \frac{\omega^2 C^2}{(\omega R_s C)^2}$$

$$H_m = \frac{1}{R_s}$$



Bandwidth:

$$\frac{|H(j\omega)|^2}{H_m^2} = \frac{1}{2}$$

$$(1+\beta - LC\omega^2)^2 = (\omega R_s C)^2$$

$$1+\beta - LC\omega^2 = \pm \omega R_s C$$

$$LC\omega^2 \pm \omega R_s C - (1+\beta) = 0$$

$$\omega^2 \pm \frac{\omega R_s}{L} - \frac{1+\beta}{LC} = 0$$

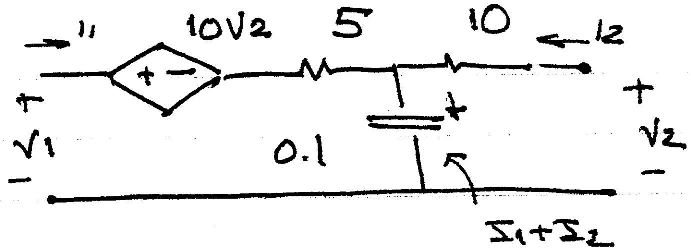
$$\omega_{1/2} = -\frac{R_s}{2L} \pm \sqrt{\left(\frac{R_s}{2L}\right)^2 + \frac{1+\beta}{LC}}$$

$\oplus$

$$\omega_{3/4} = \frac{R_s}{2L} \pm \sqrt{\left(\frac{R_s}{2L}\right)^2 + \frac{1+\beta}{LC}}$$

$$\omega_3 - \omega_1 = \frac{R_s}{L} : BW \ \oplus$$

4:



$$v_1 - 10V_2 = 5\cdot I_1 + \frac{1}{0.1s}(I_1+I_2)$$

$$V_2 = 10\cdot I_2 + \frac{1}{0.1s}(I_1+I_2)$$

$$v_1 - 10V_2 = \left(5+\frac{10}{s}\right)I_1 + \frac{10}{s}I_2$$

$$V_2 = \frac{10}{s}I_1 + \left(10+\frac{10}{s}\right)I_2$$

$$v_1 = 10\cdot\left\{\frac{10}{s}I_1 + \left(10+\frac{10}{s}\right)I_2\right\} + \left(5+\frac{10}{s}\right)I_1 + \frac{10}{s}I_2$$

$$V_2 = \frac{10}{s}I_1 + \left(10+\frac{10}{s}\right)I_2$$

$$v_1 = \left(\frac{100}{s}+5+\frac{10}{s}\right)I_1 + \left(100\cdot\frac{1+s}{s}+\frac{10}{s}\right)I_2$$

$$V_2 = \frac{10}{s}I_1 + 10\cdot\frac{1+s}{s}I_2$$

$$v_1 = \frac{110+5s}{s}I_1 + \frac{110+\overset{100}{s}}{s}I_2$$

$$V_2 = \frac{10}{s}I_1 + 10\cdot\frac{1+s}{s}\cdot I_2$$

$$Z = \begin{pmatrix} \frac{110+5s}{s} & \frac{110+100s}{s} \\ \frac{10}{s} & 10\,\frac{1+s}{s} \end{pmatrix}$$

$$V_1 = \frac{110+5s}{s} I_1 + \frac{110+100s}{s} I_2$$

$$V_2 = \frac{10}{s} I_1 + 10 \cdot \frac{1+s}{s} I_2$$

$Z_{in} = ?$

$$V_2 = -10 \cdot I_2$$

$$\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix}$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$$V_2 = -10 I_2$$

$$-10 I_2 = \frac{10}{s} I_1 + 10 \cdot \frac{1+s}{s} I_2$$

$$\left(-10 - 10 \cdot \frac{1+s}{s}\right) I_2 = \frac{10}{s} I_1$$

$$\frac{-20s-10}{s} I_2 = \frac{10}{s} \cdot I_1 \qquad I_2 = -\frac{10}{20s+10} \cdot I_1$$

$$I_2 = -\frac{1}{2s+1} \cdot I_1$$

$$V_1 = \frac{110+5s}{s} I_1 + \frac{110+100s}{s} \cdot I_2$$

$$V_1 = \frac{110+5s}{s} I_1 + \frac{110+100s}{s} \cdot \left(\frac{-1}{2s+1}\right) I_1$$

$$V_1 = \left(\frac{110+5s}{s} - \frac{110+100s}{s(2s+1)} \right) I_1$$

$$Z_{in} = \frac{(110+5s)(2s+1) - (110+100s)}{s(2s+1)}$$

$$Z_{in} = \frac{220s + 10s^2 + \cancel{110} + 5s - \cancel{110} - 100s}{s(2s+1)}$$

$$Z_{in} = \frac{125s + 10s^2}{s(2s+1)}$$

$$Z_{in} = 5\,\frac{25+2s}{2s+1} \quad ; \quad Z_{in} = 5 \cdot \frac{25+2s}{1+2s}$$

$$v_1(t) = 10\,u(t)$$

$$V_1(s) = \frac{10}{s}$$

$$I_1 = \frac{V_1}{Z_{in}}$$

$$I_1(s) = \frac{10}{s} \cdot \frac{1+2s}{5(25+2s)}$$

$$I_1 = \frac{2}{s} \cdot \frac{1+2s}{25+2s} \quad ; \quad I_1 = \frac{A}{s} + \frac{B}{25+2s}$$

$$I_1 = \frac{A}{s} + \frac{B}{25+2s} \;; \quad A = \lim_{s \to 0} s I_1 = \lim_{s \to 0} 2 \cdot \frac{1+2s}{25+2s} = 2/25$$

$$B = \lim \left(25+2s\right) \cdot \frac{2}{s} \cdot \frac{1+2s}{25+2s}$$

$$s \to -\frac{25}{2}$$

$$B = \frac{2}{-25} \cdot 2 \cdot \left(1 + 2 \times \frac{-25}{2}\right)$$

$$B = -\frac{4}{25} \cdot \left(1 - 25\right)$$

$$B = \frac{4 \times 24}{25}$$

$$I_1(s) = \frac{2}{25} \cdot \frac{1}{s} + \frac{96}{25} \cdot \frac{1}{25+2s}$$

$$i_1(t) = \frac{2}{25} \cdot u(t) + \frac{48}{25} \cdot e^{-\frac{25}{2}t}$$