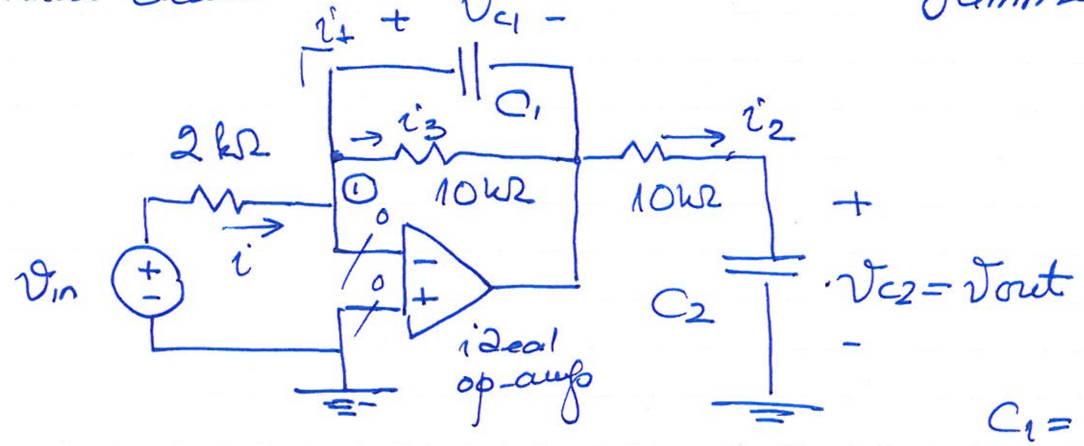
Final exam Summer 2011

(1)



$C_1 = 0.2 mF$
$C_2 = 0.5 mF$
$v_{C1}(0-) = 80 mV$
$v_{C2}(0-) = 0 V$
$V_{in} = 400u(t)\ mV$

· Zero-state response:
Assume all initial conditions are 0.

Apply Laplace transform.

(1) $V_{in} - 2\times10^3 \cdot I = 0$ Here $I = \dfrac{400 \times \frac{1}{s} \times 10^{-3}}{2\times10^3}$

$I = \dfrac{200}{s} \times 10^{-6}$

(2) $I_3 \times 10 \times 10^3 = \dfrac{1}{C_1 s} \times I_1$

$I_3 \times 10^4 = \dfrac{1}{0.2\times10^{-3} \cdot s} \cdot I_1$

$I_3 = \dfrac{1}{2s} \cdot I_1$

KCL for node (1) gives:

(3) $I = I_3 + I_1$

$I = \frac{1}{2s} I_1 + I_1$; $I_1 = \dfrac{2s}{1+2s} \cdot I$

$I_1 = \dfrac{400}{1+2s} \times 10^{-6}$

Then:

$$I_3 = \frac{1}{2s} \times \frac{400}{1+2s} \times 10^{-6}$$

$$I_3 = \frac{200}{s(1+2s)} \times 10^{-6}$$ (2)

KVL around the outer loop and the input to the op-amp:

(4) $10\times10^{3}\cdot I_3 + 10\times10^{3}\cdot I_2 + V_{C2} = 0$

(5) $I_2 = C_2 s \cdot V_{C2}$; $I_2 = 0.5\times10^{-3}\cdot s\, V_{C2}$

Then:

$$10^4\cdot I_3 + 10^4\cdot 0.5\times10^{-3}\cdot s\cdot V_{C2} + V_{C2} = 0$$

$$V_{C2} = -\frac{10^4}{5s+1}\times I_3$$

$$V_{C2} = -\frac{10^4}{1+5s}\times\frac{200\times10^{-6}}{s(1+2s)}$$

$$V_{C2} = -\frac{2}{s(1+2s)(1+5s)}$$

$$V_{C2} = -\frac{0.2}{s(s+0.2)(s+0.5)}$$

$V_{out} = V_{C2}$

Hence: $$V_{out} = -\frac{0.2}{s(s+0.2)(s+0.5)}$$

Inverse Laplace Transform:

$$V_{out} = \frac{K_0}{s} + \frac{K_1}{s+0.2} + \frac{K_2}{s+0.5}$$

$$K_0 = \lim_{s\to 0} s \times V_{out} \; ; \; K_0 = \lim_{s\to 0} s \times \left(- \frac{0.2}{s(s+0.2)(s+0.5)} \right)$$

$$K_0 = \frac{-0.2}{0.2 \times 0.5} \; ; \; K_0 = -2$$

$$K_1 = \lim_{s\to -0.2} (s+0.2) \cdot V_{out} \; ; \; K_1 = \lim_{s\to -0.2} \frac{-0.2}{s \cdot (s+0.5)}$$

$$K_1 = \frac{-0.2}{(-0.2)(-0.2+0.5)}$$

$$K_1 = 10/3$$

$$K_2 = \lim_{s\to -0.5} (s+0.5) \cdot V_{out} \; ; \; K_2 = \lim_{s\to -0.5} \frac{-0.2}{s(s+0.2)}$$

$$K_2 = \frac{-0.2}{-0.5 \cdot (-0.5+0.2)}$$

$$K_2 = -4/3$$

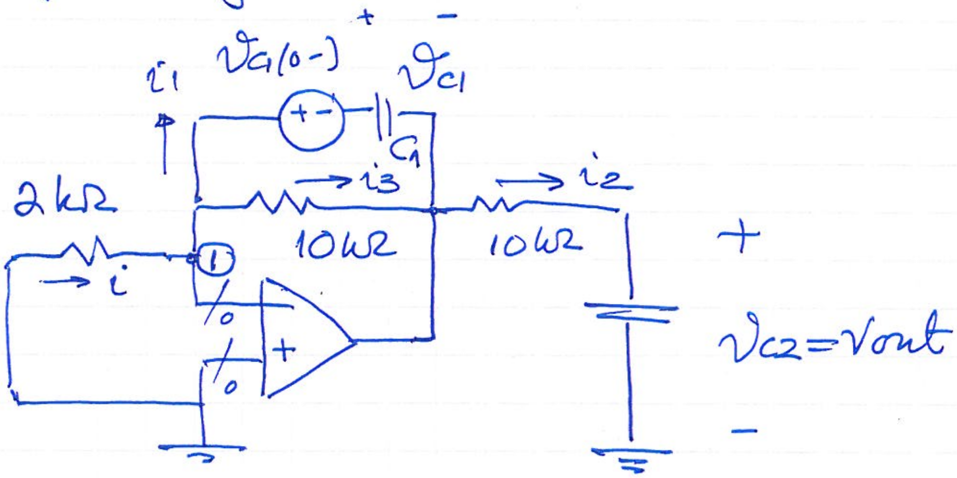
Hence, $V_{out} = \frac{-2}{s} + \frac{10/3}{s+0.2} - \frac{4/3}{s+0.5}$

$$v_{out_{ZS}}(t) = -2 \cdot u(t) + \frac{10}{3} \cdot e^{-0.2t} u(t) - \frac{4}{3} e^{-0.5t} u(t)$$

Zero-state response

$$v_{out_{ZS}}(t) = -\left(2 - 3.33 e^{-0.2t} + 1.33 e^{-0.5t}\right) u(t)$$

- Zero-input response



After applying Laplace Transform:

(6) $$I \times 2 \times 10^3 = 0 \Rightarrow I = 0$$

(7) $$\frac{v_{C1}(0-)}{s} + \frac{1}{C_1 s} \times I_1 - 10 \times 10^3 \cdot I_3 = 0$$

$$\frac{80 \times 10^{-3}}{s} + \frac{1}{0.2 \times 10^{-3} s} \cdot I_1 = 10^4 \cdot I_3$$

(8) $$I_1 + I_3 = 0$$

$$\frac{80 \times 10^{-3}}{s} = \left(\frac{10^3}{0.2\,s} + 10^4 \right) \cdot I_3$$

$$I_3 = \frac{80 \times 10^{-3}}{s} \cdot \frac{0.2 s}{10^3 + 2 \cdot 10^3 s}$$

$$I_3 = \frac{16 \times 10^{-6}}{1 + 2s}; \quad I_3 = \frac{8 \times 10^{-6}}{s + 0.5}$$

$$I_1 = -\frac{8 \times 10^{-6}}{s + 0.5}$$

KVL around the outer loop gives:

$$(9) \quad 10\times10^{3}\cdot I_3 + 10\times10^{3}\cdot I_2 + V_{C2} = 0$$

$$(10) \quad V_{C2} = \frac{1}{C_2 s}\cdot I_2$$

$$10^4\cdot\frac{8\times10^{-6}}{s+0.5} + 10^4\times C_2 s\cdot V_{C2} + V_{C2} = 0$$

$$V_{C2} = -\frac{8\times10^{-2}}{s+0.5}\cdot\frac{1}{10^4\times0.5\times10^{-3}s+1}$$

$$V_{C2} = -\frac{8\times10^{-2}}{s+0.5}\times\frac{1}{5s+1}$$

$$V_{C2} = -\frac{1.6\times10^{-2}}{(s+0.2)(s+0.5)}$$

Inverse Laplace Transform:

$$V_{C2} = \frac{K_1}{s+0.2} + \frac{K_2}{s+0.5} \quad ; \quad V_{out} = \frac{K_1}{s+0.2} + \frac{K_2}{s+0.5}$$

$$K_1 = \lim_{s\to-0.2}(s+0.2)\cdot\frac{-1.6\times10^{-2}}{(s+0.2)(s+0.5)} \quad ; \quad K_1 = \frac{-1.6\times10^{-2}}{-0.2+0.5}$$

$$K_1 = -\frac{16}{3}\times10^{-2}$$

$$K_1 = -0.0533$$

$$K_2 = \lim_{s\to-0.5}(s+0.5)\cdot\frac{-1.6\times10^{-2}}{(s+0.2)(s+0.5)} \quad ; \quad K_2 = \frac{-1.6\times10^{-2}}{-0.5+0.2} \quad ; \quad K_2 = \frac{16}{3}\times10^{-2}$$

$$K_2 = 0.0533$$

Zero-input response.

$$V_{out_{zi}}(t) = \left(-0.0533\, e^{-0.2t} + 0.0533\, e^{-0.5t}\right) u(t)$$

- Complete response

$$V_{out}(t) = -\left(2 - 3.33e^{-0.2t} + 1.33e^{-0.5t}\right) u(t)$$
$$+ \left(-0.0533\, e^{-0.2t} + 0.0533\, e^{-0.5t}\right) u(t)$$

$$V_{out}(t) = -\left(2 - 3.28e^{-0.2t} + 1.28e^{-0.5t}\right) u(t)$$

- General form of the natural response:

$$V_{nat}(t) = \left(K_1 e^{-0.2t} + K_2 e^{-0.5t}\right) u(t)$$

due to natural frequencies only.

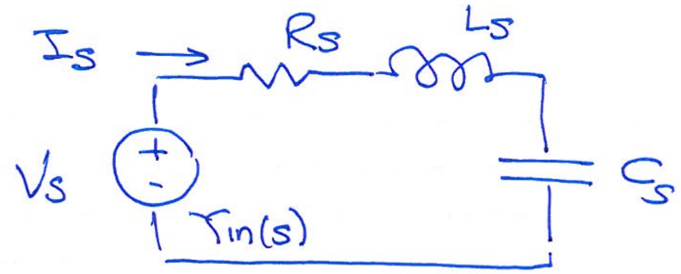
- Transient response.

$$V_{out_{Tran}}(t) = \left(3.28\, e^{-0.2t} - 1.28\, e^{-0.5t}\right) u(t)$$

- Steady state response.

$$V_{out_{ss}}(t) = -2u(t)$$

2



$$H(s) = \frac{I_s}{V_s}$$

$$V_s = \left(R_s + s \cdot L_s + \frac{1}{sC_s}\right) \cdot I_s$$

$$H(s) = Y_{in}(s)$$

$$Y_{in}(s) = \frac{1}{R_s + sL_s + \frac{1}{sC_s}}$$

- $$Y_{in}(s) = \frac{sC_s}{1 + sR_sC_s + s^2 L_s C_s} \; ; \; Y_{in}(s) = \frac{\frac{1}{L_s} \cdot s}{s^2 + \frac{R_s}{L_s} s + \frac{1}{L_sC_s}}$$

- peak frequ.:

$$Y_{in}(j\omega) = \frac{1}{L_s} \cdot \frac{j\omega}{\left(\frac{1}{L_sC_s} - \omega^2\right) + j \cdot \frac{R_s}{L_s}\omega}$$

$$|Y_{in}(j\omega)| = \frac{\omega}{L_s} \cdot \frac{1}{\sqrt{\left(\frac{R_s}{L_s}\omega\right)^2 + \left(\frac{1}{L_sC_s} - \omega^2\right)^2}}$$

Peak frequency: $\omega_m = \frac{1}{\sqrt{L_sC_s}}$

- Half-power frequen W_1 and W_2:

$$\left(\frac{R_s}{L_s} \cdot W\right)^2 = \left(\frac{1}{L_sC_s} - W^2\right)^2$$

$$\pm \frac{R_s}{L_s} \cdot W = \frac{1}{L_sC_s} - W^2$$

$$W^2 \pm \frac{R_s}{L_s} W - \frac{1}{L_sC_s} = 0$$

$$W_{1/2} = -\frac{R_s}{2L_s} \pm \sqrt{\left(\frac{R_s}{2L_s}\right)^2 + \frac{1}{L_sC_s}}$$

$$W_{3/4} = \frac{R_s}{2L_s} \pm \sqrt{\left(\frac{R_s}{2L_s}\right)^2 + \frac{1}{L_sC_s}}$$

Positive ω only: $W_1 = -\frac{R}{2L_s} + \sqrt{\left(\frac{R_s}{2L_s}\right)^2 + \frac{1}{L_sC_s}}$

$$W_3 = \frac{R}{2L_s} + \sqrt{\left(\frac{R_s}{2L_s}\right)^2 + \frac{1}{L_sC_s}}$$

- Bandwidth. $B_W = W_3 - W_1$

$$B_W = \frac{R_s}{L_s}$$

- $Q_{circ} = \frac{W_m}{B_W}$; $Q_{circ} = \frac{1}{\sqrt{L_sC_s}} \cdot \frac{L_s}{R_s}$

$$Q_{circ} = \frac{1}{R_s} \sqrt{\frac{L_s}{C_s}}$$

· Frequen: $\measuredangle H(jw) = 0$

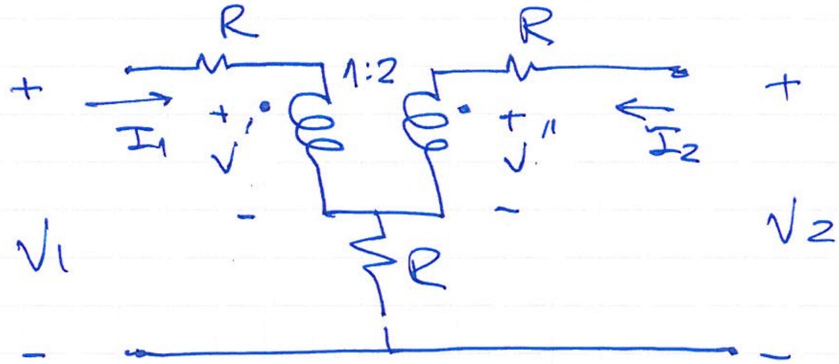
i.e., $H(jw) = \text{Real}$

$$\frac{1}{L_s C_s} - w^2 = 0 \; ; \; w = \frac{1}{\sqrt{L_s C_s}}$$

$$Y_{in}(jw) = \frac{1}{L_s} \cdot \frac{jw}{j \cdot \frac{R_s}{L_s} \cdot w}$$

$$Y_{in}(jw) = \frac{1}{R_s} \quad \text{(resonant frequency)}$$

3



z
y parameters
t

$$\frac{V'}{V''} = 1/2$$

ideal transformer

$$\frac{I_1}{I_2} = -2$$

$$\begin{cases} V_1 - R \cdot I_1 - V' - R \cdot (I_1 + I_2) = 0 \\ V_2 - R \cdot I_2 - V'' - R(I_1 + I_2) = 0 \end{cases}$$

$$V_1 - 2R \cdot I_1 - R \cdot I_2 - V' = 0$$

$$V_2 - R \cdot I_1 - 2R\, I_2 - V'' = 0$$

$$V'' = 2 \cdot V'$$

$$I_1 = -2 I_2$$

$$V_1 - 2R I_1 - R \cdot I_2 - V' = 0 \quad /(-2) \text{ multiply wise } (-2)$$

$$V_2 - R \cdot I_1 - 2R \cdot I_2 - 2V' = 0$$

Add:

$$-2V_1 + 4R \cdot I_1 + 2R I_2 + V_2 - R I_1 - 2R I_2 = 0$$

$$-2V_1 + V_2 + 3R \cdot I_1 = 0 \; ; \; I_1 = \frac{2V_1 - V_2}{3R}$$

$$I_2 = -\frac{1}{2}\cdot I_1$$

$$I_2 = \frac{V_2 - 2V_1}{6R}$$

Hence: $2V_1 - V_2 = 3R\cdot I_1$

$-2V_1 + V_2 = 6R\cdot I_2$

$$\begin{pmatrix} 2 & -1 \\ -2 & 1 \end{pmatrix}\cdot\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} 3R & 0 \\ 0 & 6R \end{pmatrix}\begin{pmatrix} I_1 \\ I_2 \end{pmatrix}$$

det = 2 − 2 = 0

- Z-parameters do not exist

- $I_1 = \frac{2}{3R}\cdot V_1 - \frac{1}{3R}\cdot V_2$

$I_2 = -\frac{1}{3R}\cdot V_1 + \frac{1}{6R}\cdot V_2$

$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} 2/3R & -1/3R \\ -1/3R & 1/6R \end{pmatrix}\cdot\begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$

Y-parameters: $\frac{1}{R}\cdot\begin{pmatrix} 2/3 & -1/3 \\ -1/3 & 1/6 \end{pmatrix}$

or $\frac{1}{6R}\cdot\begin{pmatrix} 4 & -2 \\ -2 & 1 \end{pmatrix}$

t-parameters:

$$2V_1 - V_2 = 3R \cdot I_1$$

$$-2V_1 + V_2 = 6R \cdot I_2 \quad (\text{or: } I_1 = -2I_2)$$

Here: $2V_1 = V_2 + 3R \cdot (-2I_2)$

$$I_1 = -2I_2$$

$$V_1 = \frac{1}{2}V_2 - 3R \cdot I_2$$

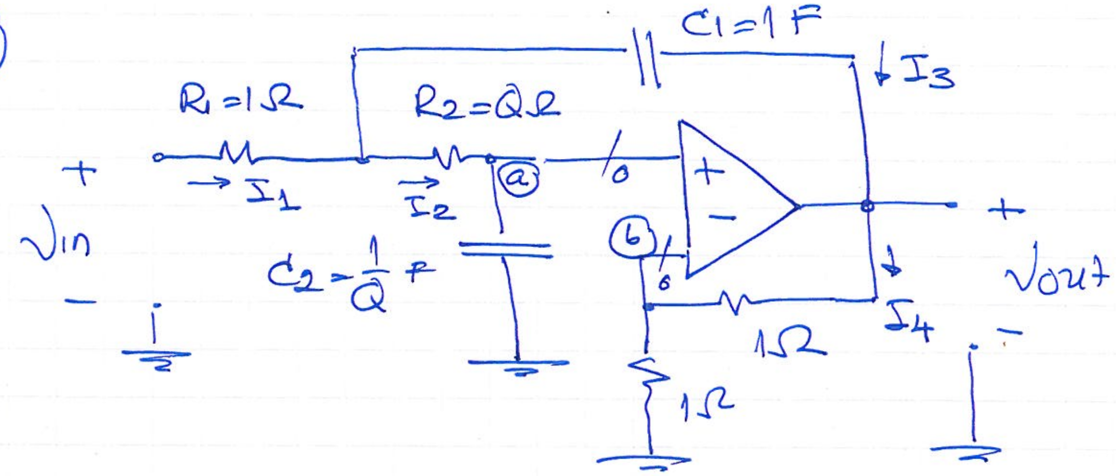
$$I_1 = -2I_2$$

$$\begin{pmatrix} V_1 \\ I_1 \end{pmatrix} = \begin{pmatrix} 1/2 & -3R \\ 0 & -2 \end{pmatrix} \cdot \begin{pmatrix} V_2 \\ I_2 \end{pmatrix}$$

$$\begin{pmatrix} V_1 \\ I_1 \end{pmatrix} = \begin{pmatrix} 1/2 & 3R \\ 0 & 2 \end{pmatrix} \cdot \begin{pmatrix} V_2 \\ -I_2 \end{pmatrix}$$

t-parameters: $\begin{pmatrix} 1/2 & 3R \\ 0 & 2 \end{pmatrix}$

4



$R_1 = 1\Omega$
$R_2 = Q\Omega$
$C_1 = 1F$
$C_2 = 1/Q\ F$

KVL: $$V_{in} = R_1 \cdot I_1 + \left(R_2 + \frac{1}{C_2 s}\right) \cdot I_2 \qquad (1)$$

$$V_a = \frac{1}{C_2 s} \cdot I_2 \qquad (2)$$

$$V_b = V_a$$

$$V_{out} = 1 \times I_4 + 1 \times I_4 \ ; \ V_{out} = 2 \times I_4$$

$$V_b = 1 \times I_4$$

Hence $$V_{out} = 2 \times V_b \ ; \ V_b = \frac{V_{out}}{2}$$

Since $V_a = V_b \Rightarrow V_a = \frac{V_{out}}{2}$

From (2): $I_2 = C_2 s \cdot V_a$; $I_2 = \frac{C_2 s}{2} \times V_{out}$

KVL: $$V_{in} - R_1 \cdot I_1 - \frac{1}{C_1 s} \cdot I_3 - V_{out} = 0 \qquad (3)$$

KCL: $$I_3 - I_1 + I_2 = 0 \qquad (4)$$

From (3):

$$V_{in} = R_1 \cdot I_1 + \frac{1}{C_1 s} \cdot I_3 + V_{out}$$

Substituting (4) into (3) gives: (3): $I_3 = I_1 - I_2$

$$V_{in} = R_1 \cdot I_1 + (I_1 - I_2) \cdot \frac{1}{C_1 s} + V_{out}$$

Since: $I_2 = \frac{C_2 s}{2} V_{out}$

We have:

$$V_{in} = R_1 \cdot I_1 + \frac{1}{C_1 s} \cdot \left(I_1 - \frac{C_2 s}{2} \cdot V_{out} \right) + V_{out}$$

$$V_{in} = \left(R_1 + \frac{1}{C_1 s} \right) I_1 + \left(1 - \frac{C_2}{2 C_1} \right) \cdot V_{out} \quad (4')$$

Hence:

$$(1) \quad V_{in} = R_1 \cdot I_1 + \left(R_2 + \frac{1}{C_2 s} \right) I_2$$

$$(4') \quad V_{in} = \left(R_1 + \frac{1}{C_1 s} \right) I_1 + \left(1 - \frac{C_2}{2 C_1} \right) V_{out}$$

Since: $I_2 = \frac{C_2 s}{2} V_{out}$, (1) becomes

$$(1) \quad V_{in} = R_1 \cdot I_1 + \left(R_2 + \frac{1}{C_2 s} \right) \cdot \frac{C_2 s}{2} \cdot V_{out}$$

$$(4') \quad V_{in} = \left(R_1 + \frac{1}{C_1 s} \right) I_1 + \left(1 - \frac{C_2}{2 C_1} \right) V_{out}$$

Eliminate I_1 by substn (1) into (4')

$$(4') \quad V_{in} = \left(R_1 + \frac{1}{C_1 s} \right) \cdot \frac{V_{in} - \left(R_2 + \frac{1}{C_2 s} \right) \cdot \frac{C_2 s}{2} \cdot V_{out}}{R_1} +$$

$$+ \left(1 - \frac{C_2}{2 C_1} \right) V_{out}$$

Hence,

$$V_{in}\left(1-\frac{R_1+\frac{1}{C_1 s}}{R_1}\right)=-\left(R_1+\frac{1}{C_1 s}\right)\cdot\frac{\left(R_2+\frac{1}{C_2 s}\right)\cdot\frac{C_2 s}{2}}{R_1}V_{out}$$

$$+\left(1-\frac{C_2}{2C_1}\right)\cdot V_{out}$$

$$H(s)=\frac{V_{out}}{V_{in}}$$

$$H(s)=\frac{1-\frac{R_1+\frac{1}{C_1 s}}{R_1}}{-\left(R_1+\frac{1}{C_1 s}\right)\left(R_2+\frac{1}{C_2 s}\right)\cdot\frac{C_2 s}{2R_1}+\left(1-\frac{C_2}{2C_1}\right)}$$

$$H(s)=\frac{-1/C_1 s}{-\left(R_1+\frac{1}{C_1 s}\right)\cdot\left(R_2+\frac{1}{C_2 s}\right)\cdot\frac{C_2 s}{2}+\left(1-\frac{C_2}{2C_1}\right)R_1}$$

$$H(s)=\frac{1}{(1+R_1C_1s)\cdot(1+R_2C_2s)\cdot\frac{1}{2}-\left(1-\frac{C_2}{2C_1}\right)\cdot R_1\cdot C_1 s}$$

$$H(s)=\frac{1\times 2}{(1+R_1C_1s)(1+R_2C_2s)-(2C_1-C_2)\cdot R_1 s}$$

Let $R_1=1$
$R_2=Q$
$C_1=1$
$C_2=1/Q$
Then
$$H(s)=\frac{2}{(1+s)(1+s)-\left(2-\frac{1}{Q}\right)\cdot s}$$

$$H(s)=\frac{2}{s^2+2s+1-2s+s/Q}$$

$$H(s) = \frac{2}{s^2 + \frac{1}{Q}s + 1}$$

- Zeros: Two zeros at infinity

Poles: $s^2 + \frac{1}{Q}s + 1 = 0$

$$s_{1/2} = -\frac{1}{2Q} \pm \sqrt{\left(\frac{1}{2Q}\right)^2 - 1}$$

$$s_{1/2} = -\frac{1}{2Q} \pm j\sqrt{1 - \left(\frac{1}{2Q}\right)^2}$$

- $$H(j\omega) = \frac{2}{1 - \omega^2 + \frac{1}{Q} \cdot j\omega}$$

$$|H(j\omega)| = \frac{2}{\sqrt{(1-\omega^2)^2 + \left(\frac{\omega}{Q}\right)^2}}$$

$$\measuredangle H(j\omega) = -\operatorname{arctg} \frac{\omega/Q}{1-\omega^2}$$

- Type of the circuit: $\omega = 0$: $H(0) = 2$

$\omega \to \infty$: $H(\infty) = 0$

1/2 power: $(1-\omega^2)^2 = \left(\frac{\omega}{Q}\right)^2$; $1 - \omega^2 = \pm \omega/Q$

$$\omega^2 \pm \frac{\omega}{Q} - 1 = 0$$

$$\omega_{1/2} = -\frac{\omega}{2Q} \pm \sqrt{\left(\frac{\omega}{2Q}\right)^2 + 1} \; ; \; \omega_{3/4} = \frac{\omega}{2Q} \pm \sqrt{\left(\frac{\omega}{2Q}\right)^2 + 1}$$

Bandwidth: $w_3 - w_1 = \frac{\omega}{Q}$

Peak: $y = (1-\omega^2)^2 + \left(\frac{\omega}{Q}\right)^2$

Minimum of y occurs at:

$dy/dw = 0$; $dy/dw = 2(1-\omega^2) \times 2w + 2\frac{\omega}{Q} \cdot \frac{1}{Q}$

$dy/dw = 0$ for: $\omega = 0$

$$4(1-\omega^2) + 2 \cdot \frac{1}{Q^2} = 0$$

$$4\omega^2 = 4 + 2/Q^2$$

$$\omega^2 = 1 + \frac{1}{2Q^2}$$

in general, this is a low passfilter since $H(0)=2$

Butterworth design

this could be a bandpass filter if Q is chosen properly.