Algebraic methods and optimization for signal processing and source separation of multiway signals and data sets

Seminar Vancouver

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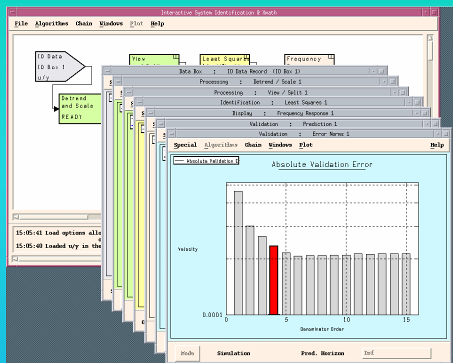
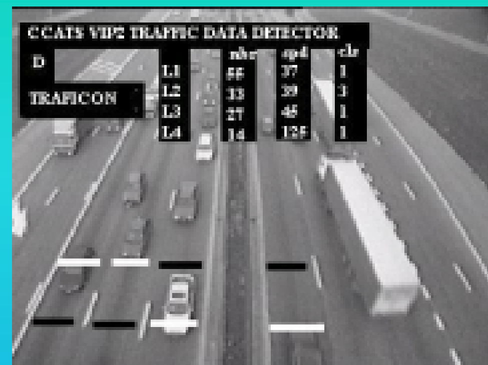
Vectors, Matrices, and Tensors for multiway data and signals

System Identification and Control

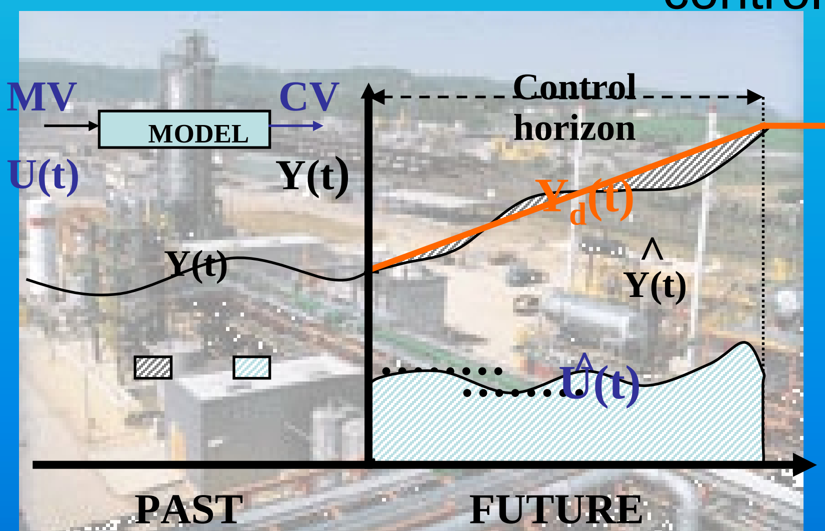


Subspace identification



Traffic modelling and control

Satellite control



Model Predictive Control

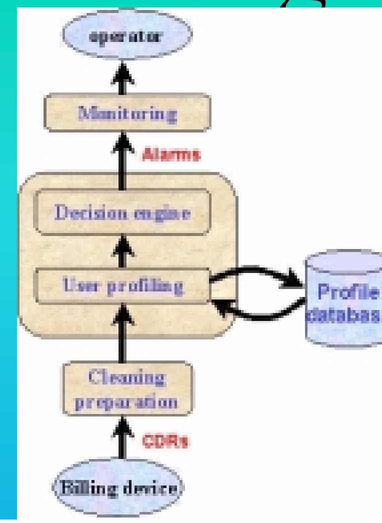
Identification and prediction of time series (stock exchange, physical phenomena, …)

Data Processing and Data Mining

- Fraud Detection

Credit cards

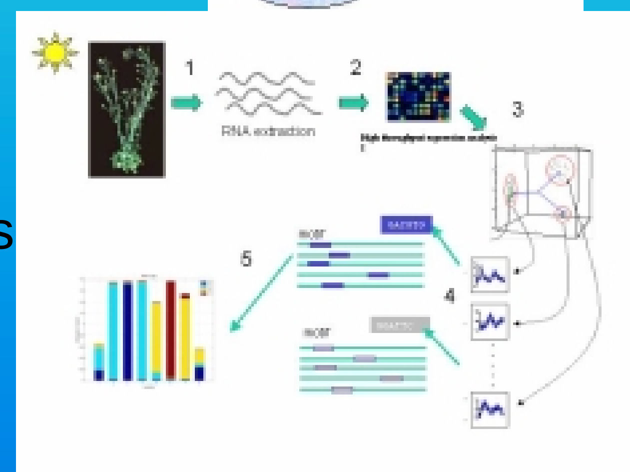
Mobile telephony



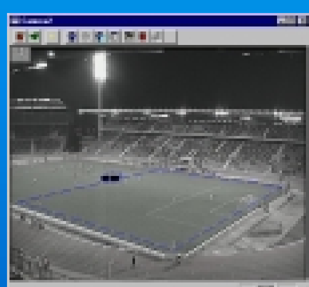
- Bio Informatics

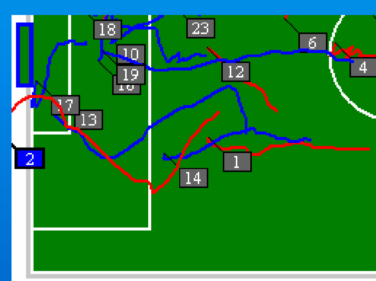
Genetic Sequence Modelling

Micro Arrays



- Sports and Technology





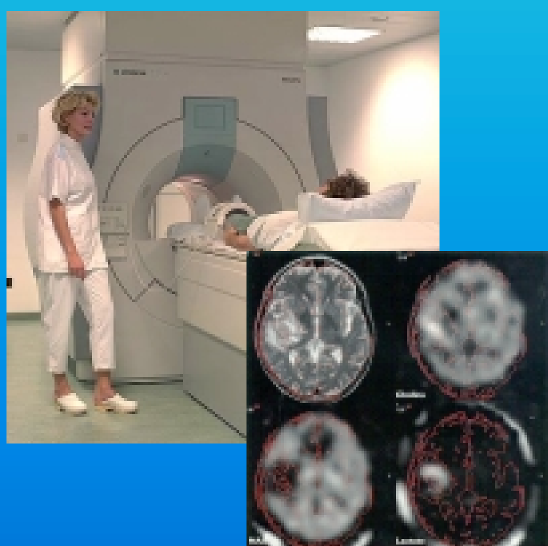
Biomedical Signal Processing

- improved algorithms for medical diagnostics (accuracy, efficiency, automation)

Measurement ⇨ Signal Enhancement ⇨ Parameter Estimation ⇨ Improved Diagnosis

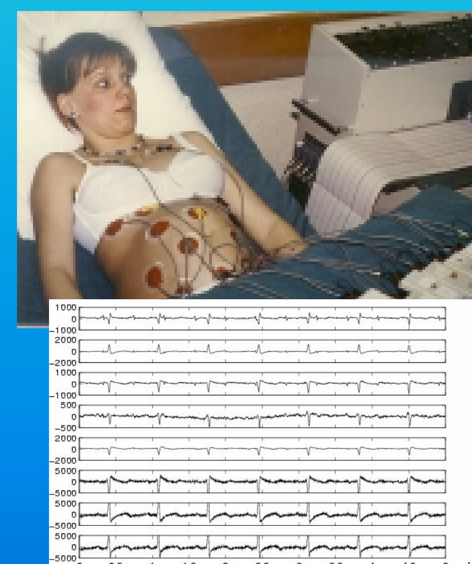
(heart beat, magnetisation) (FFT, filtering, SVD) (reliable algorithms)

- Applications:



Nuclear Magnetic Resonance

Near Infra-Red Spectroscopy

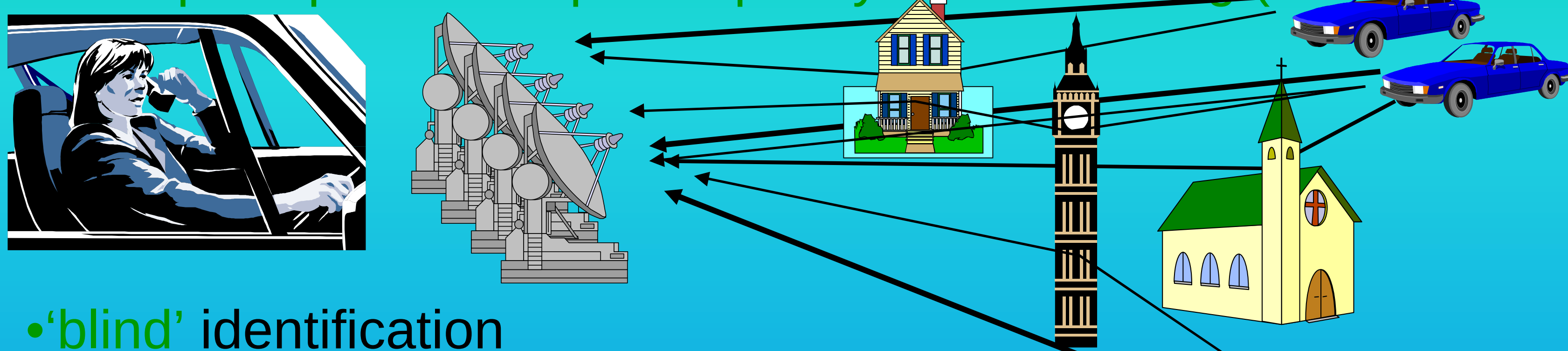


Foetal ECG

ESAT-SISTA
Info: +32-(0)16-321709 or
http://www.esat.kuleuven.ac.be/sista

Signal processing, signal separation & filtering

- Multiple input-& multiple output system modeling (identification)



- ‘blind’ identification
- exploit a priori information & signal structure
    - ‘finite alphabet’ (communications signals)
    - ON/OFF (speech signals)
    - known signal components (biomedical signals)
- performance versus implementation complexity trade-off

Aim: Improved high-performance (next generation) signal separation and filtering techniques

What do we know in the matrix case ?

- Concept : Oriented signal to signal ratios (see also papers around 1990)
- Computation : Generalized singular value decomposition
- Application : Fetal ECG signal intervals with relevant contributions FECG and with contributions that should be rejected MECG

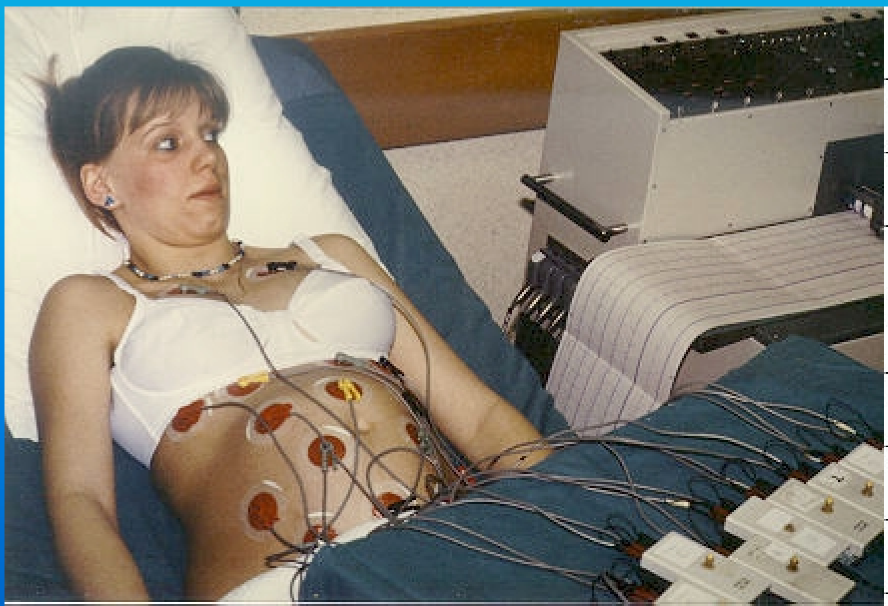
Application :
Multiple signal sources

extract the fetal electrocardiogram (FECG) from multilead potential recordings on the mother's skin

mechanical activity initiated by electrical activity

Visualization as ECG or FECG

Abdominal and thoracic recordings



Multiple signals in matrix form

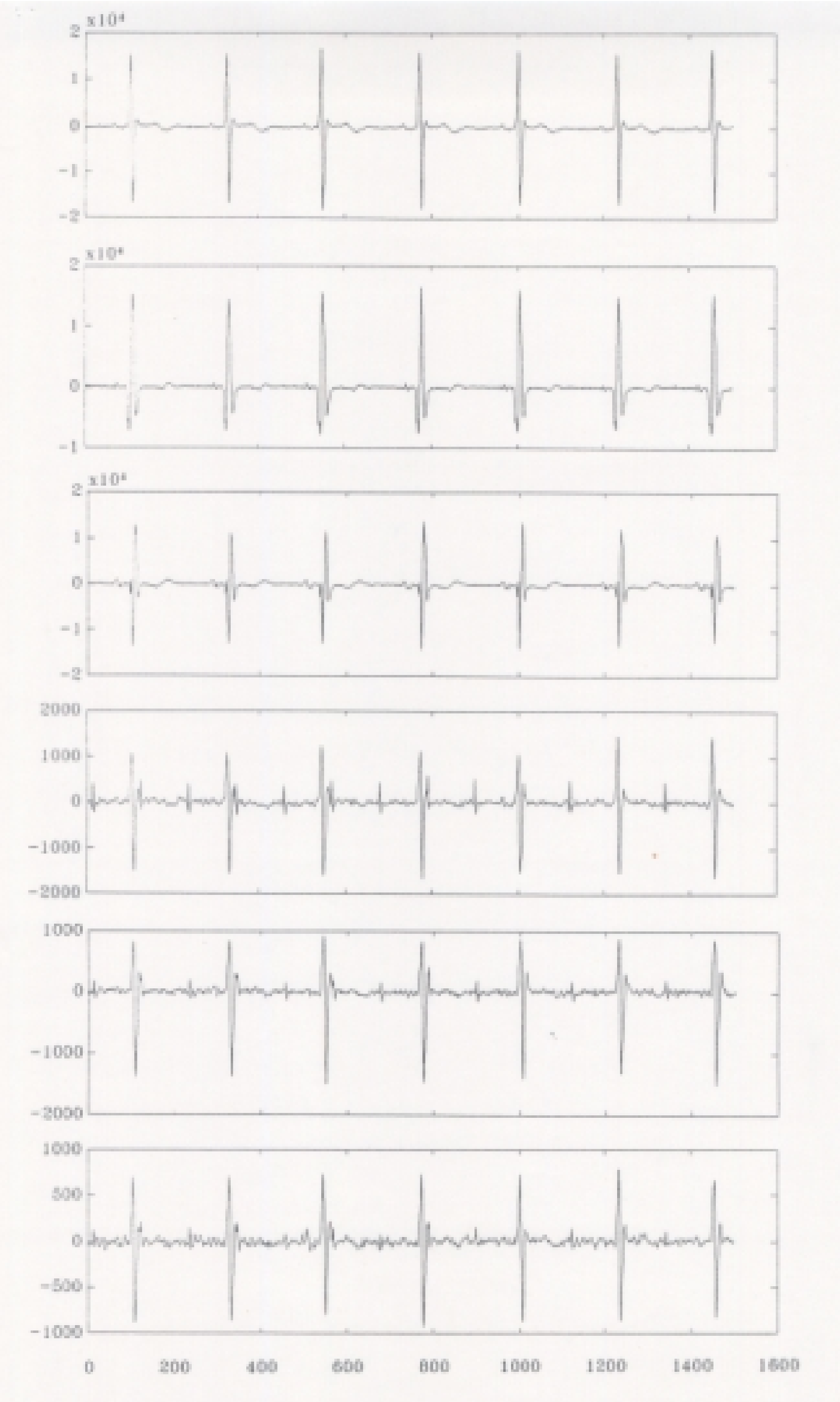
Cutaneous electrode measurements more safe for the patient

But Mixtures of sources measured

Electrode 1

Electrode 2

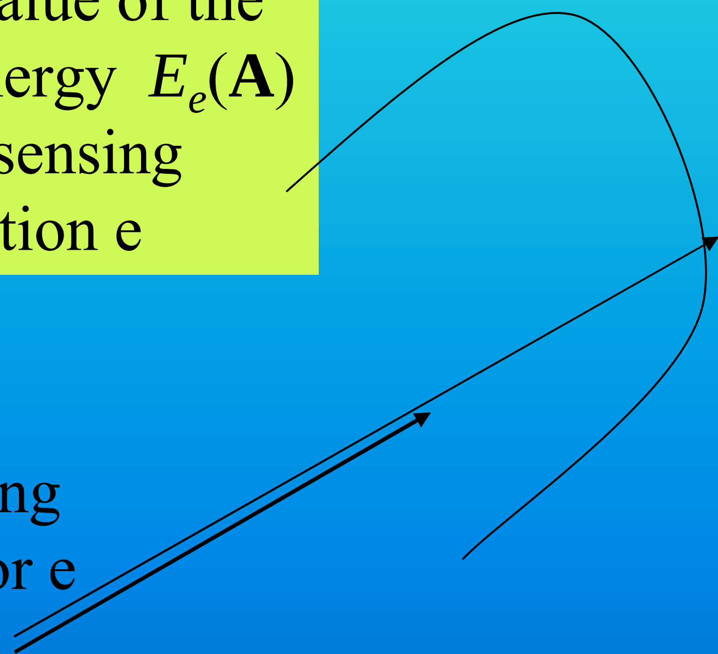
Electrode 6



Oriented energy : make linear combination e of signals A

$$E_e(\mathbf{A}) = \sum_{i=1}^{n} (e^T a_i)^2 = \left\| e^T \mathbf{A} \right\|^2$$

Plot the value of the oriented energy $E_e(\mathbf{A})$ in the sensing direction e



$E_e(\mathbf{A})$

Unit Sensing Vector e

We are interested in the extremal values (maxima, minima, saddle points) of the orientede energy and subspaces of extremal oriented energy

Optimal sensing direction(s) **e**

Spaces of **optimal oriented energy** can be computed with the singular value decomposition SVD $\mathbf{A} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^T$

Left columns of U largest contributions : most valuable linear combination -->signal component

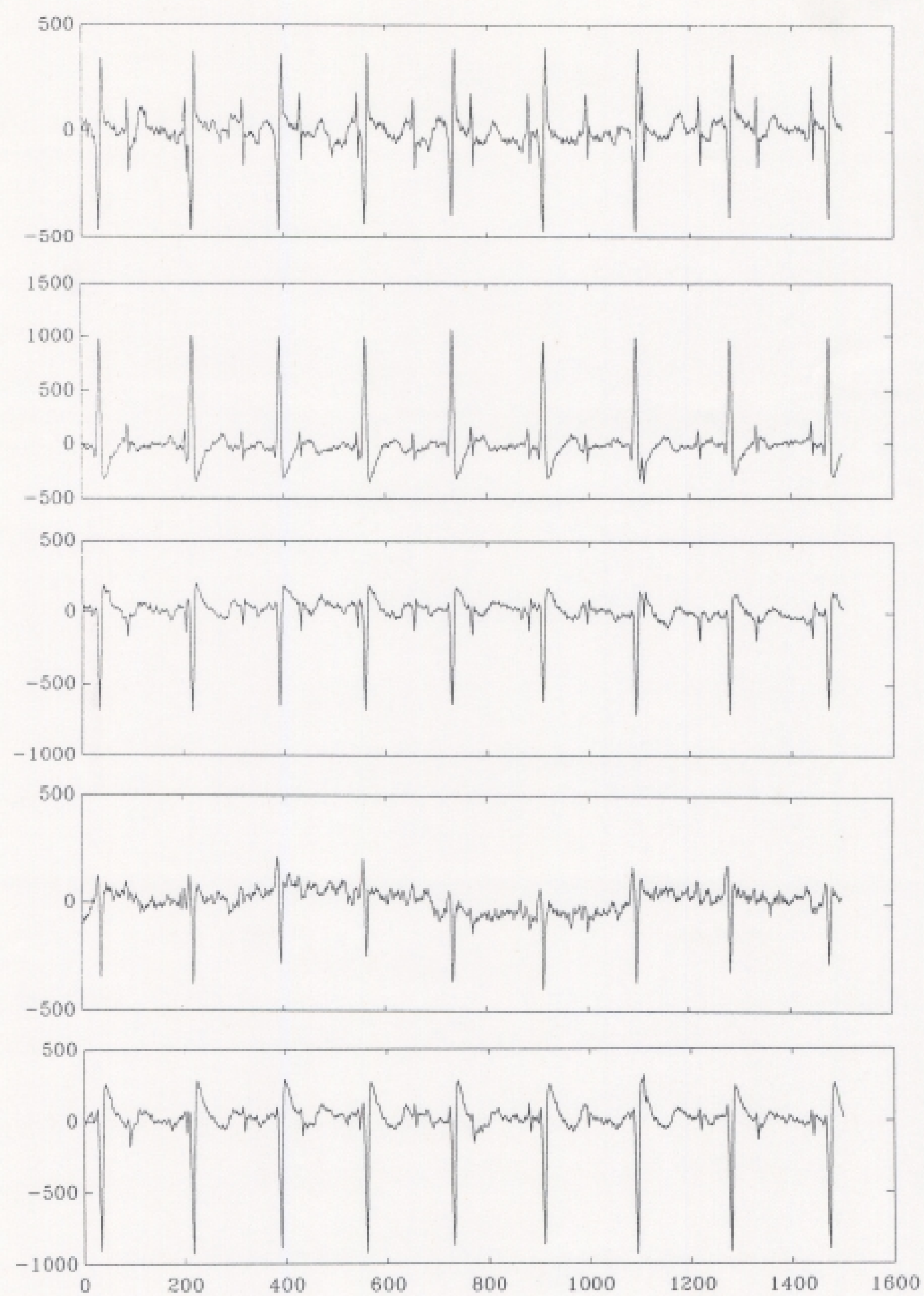
k-dimensional dominant subspace is spanned by the k left columns of **U** ---> orthogonal

Singular Value Decomposition SVD of an mxn measurement Matrix M

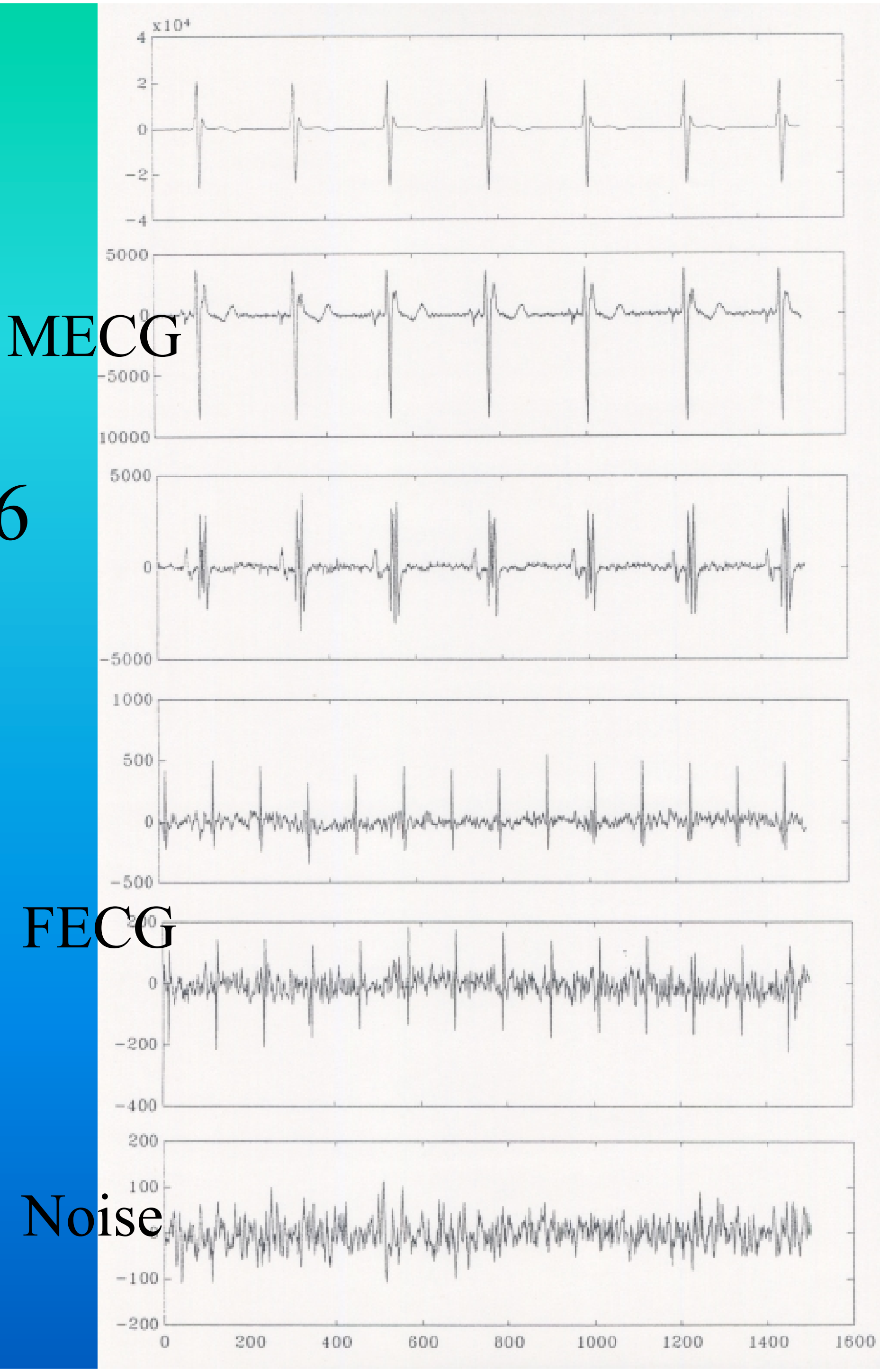
Σ is diagonal and U and V orthogonal

$$M = U \; \Sigma \; V^T$$

Further electrode measurements with mixture of MECG and FECG not suitable for gynecologist



Processed measurements:
using U of SVD of data matrix.
strongest directions in 6 dimensional
space are detected
Interpretation :
3 dominant signals =Mother ECG
2 next =Fetal ECG
last=noise

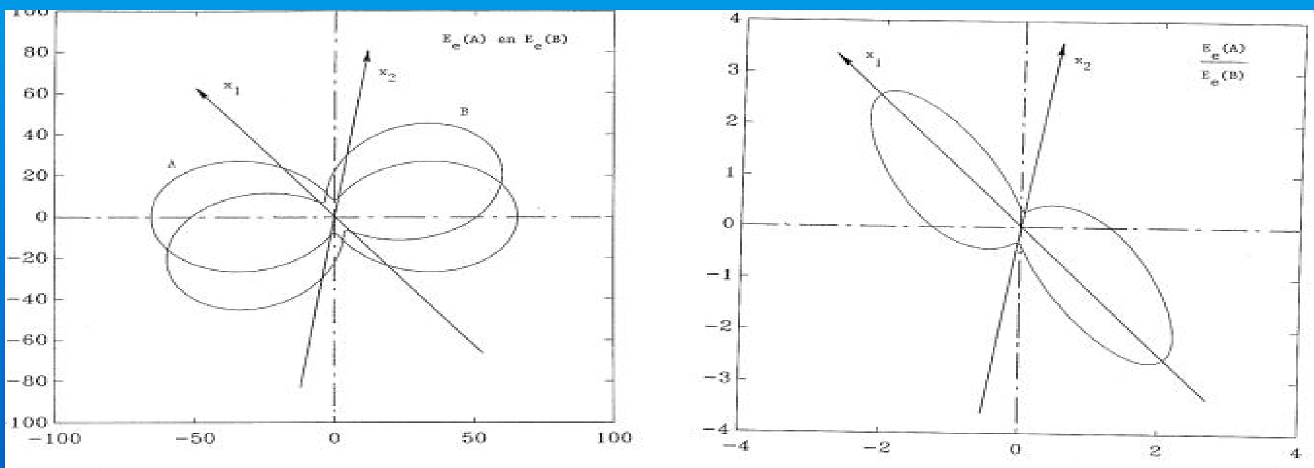


Oriented energy : make linear combination e of signals A

$$E_e(\mathbf{A}) = \sum_{i=1}^{n} (e^T a_i)^2 = \left\| e^T \mathbf{A} \right\|^2$$

Oriented signal to signal ratio : make linear combination e of "good" signals A versus that of "bad" signals B

$$E_e(\mathbf{A}) / E_e(\mathbf{B}) = \sum_{i=1}^{n} (e^T a_i)^2 / \sum_{i=1}^{n} (e^T b_i)^2$$



Plot the value in the sensing direction e

Application of GSVD :

Measured mxn matrix of data : row i electrode signal i
Column j time instant j

mxn’matrix **A** submatrix of columns corresponding to intervals where the desired signal is present

mxn’’ matrix **B** submatrix of columns corresponding to intervals where the desired signal is present

Perform GSVD (Generalized Singular Value Decomposition) of pair **A B**

First three columns x_1 x_2 x_3 of **Q** consistute a basis for the space of fetal heart--> project onto these columns

Optimal sensing direction(s) **e**

Spaces of **optimal oriented signal to signal ratio** can be computed with the **generalized svd GSVD** of the mxn and mxl matrix pair **A, B**

$\mathbf{A} = \mathbf{Q}^{-1}\, \boldsymbol{\Sigma}' \mathbf{V}^T$ with **Q** square nonsingular

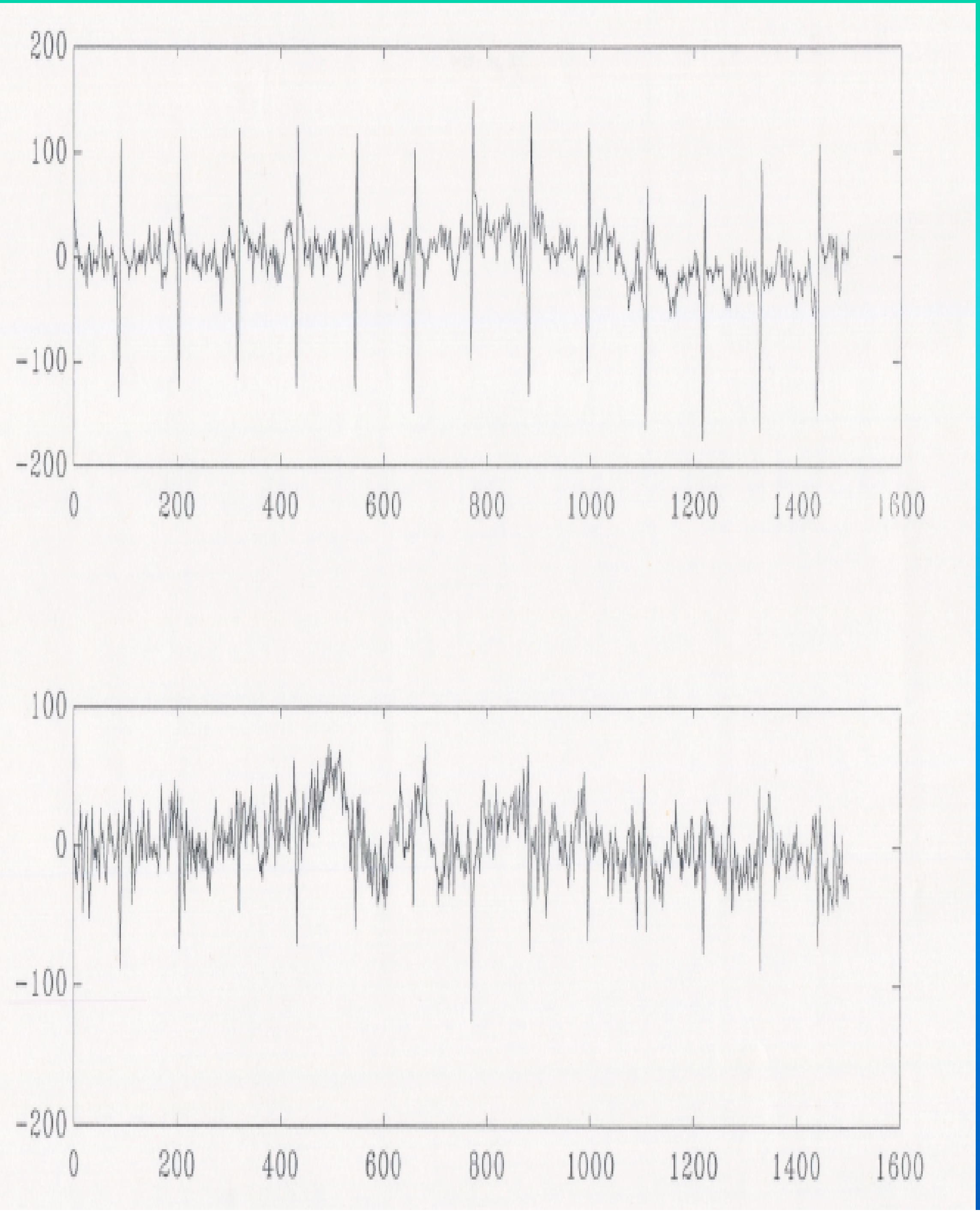
$\mathbf{B} = \mathbf{Q}^{-1}\, \boldsymbol{\Sigma}'' \mathbf{U}^T$ with **U** and **V** orthogonal

With $\boldsymbol{\Sigma}' = \mathrm{diag}(\sigma_1', \sigma_2', 0\ 0..)$ $\boldsymbol{\Sigma}'' = \mathrm{diag}(\sigma_1'', \sigma_2'', ,0\ 0..)$

And $(\sigma_1'/\sigma_1'') \geq (\sigma_2'/\sigma_2'') \geq \ldots > 0$

k-dimensional dominant subspace is spanned by the k left columns of **Q** ---> not orthogonal

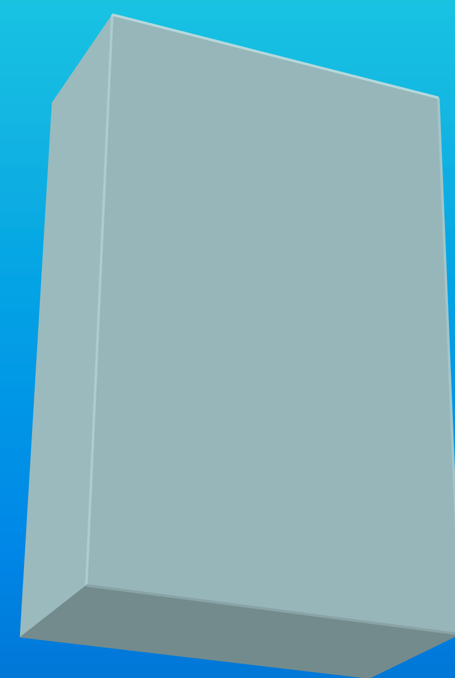
Processed signals with GSVD by projecting the measured data signals onto Directions x_1 and x_2 of maximal oriented FECG signal versus MECG signal
--> reveals the relevant information for the gynecologist heart rate and shape of FECG



What are multiway data and signals ?

Many signals and data sets have several types of variables involved space, time, frequency,

Time intervals, space intervals or frequency intervals of interest



- Black and white image sequences
- RGB color images
- RGB color image sequences
- Microarray image sequences
- NMR image spectra

Tensors

A , B , C , D …

3 or more variables : x,y, z space coordinates, time, color…
Tensor algebra, multilinear algebra, …

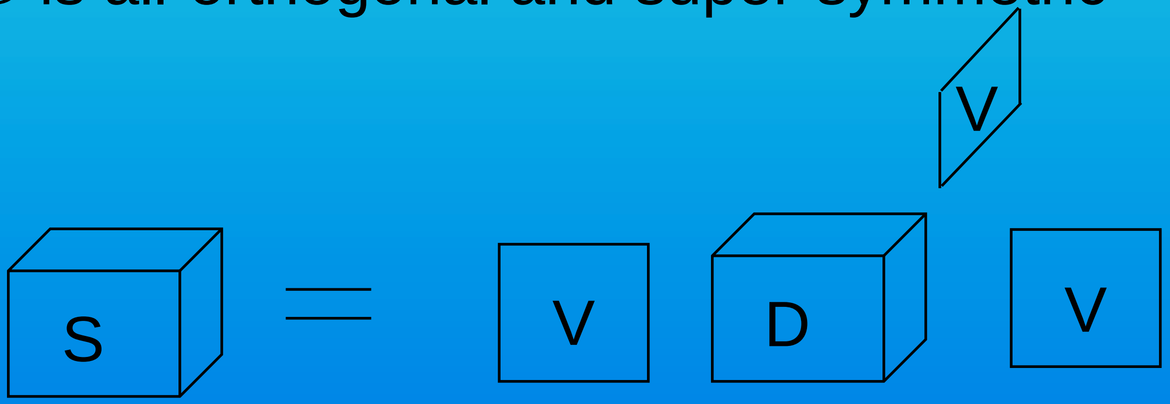
Tensor A of “good” signals and tensor B of “bad” signals

Higher Order EVD

2nd order : D is diagonal

3rd order :

D is all-orthogonal and super-symmetric



V is calculated as singular vector matrix of matrix unfolding of 4th order tensor C_4 : improve V by diagonalising D

Can the **notions** be generalized ?
How can the optimal directions be **computed** ?

- Need generalization of concepts
- Need generalization of the computations
- **Then test out on applications**

Tensor product

$$A \times_1 \times_2 .. \times_l B = \sum_{i_l=1}^{I_l} \dots \sum_{i_1=1}^{I_1} (a_{i_1 i_2 \dots i_l i_{l+1} \dots i_m} b_{i_1 i_2 \dots i_l i_{l+1} \dots i_n})$$

Can **ONLY** be performed if the size I_j in the direction j of the two tensors is the same for j=1…l

Notation at the Level of entries and summations

Frobenius norm of an m-th order tensor A

$$\|A\|_F^2 = \sum_{i_m=1}^{I_m} \dots \sum_{i_1=1}^{I_1} \left(a_{i_1 i_2 .. i_m}\right)^2$$

Oriented energy in sensing direction e

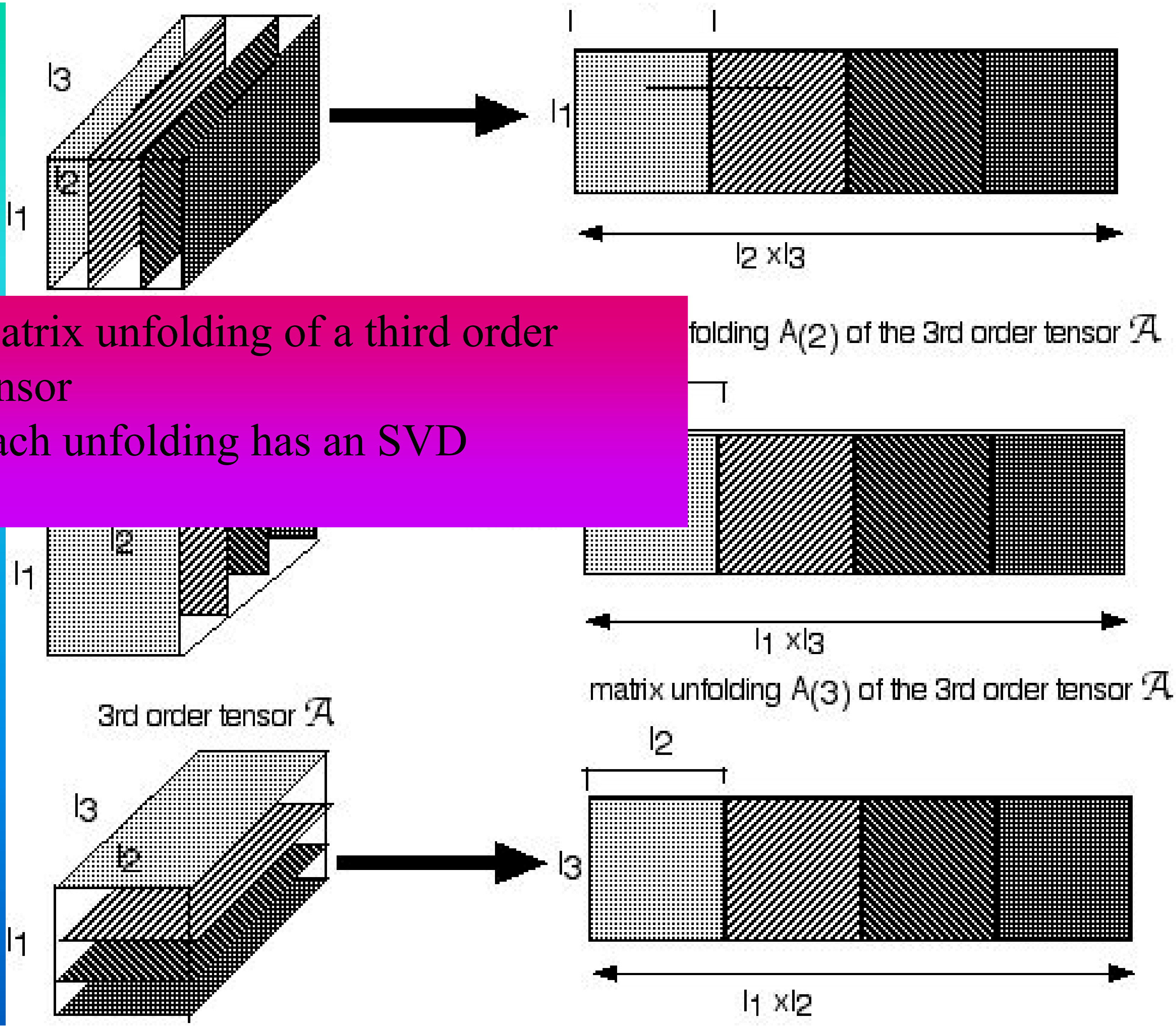
$$E_e(\mathcal{A}) = \sum_{i_2=1}^{I_2} .. \sum_{i_n=1}^{I_n} \left(\sum_{i_1=1}^{I_1} (e_{i_1} a_{i_1 i_2 .. i_n}) \right)^2 = \left\| \mathcal{A} \times_1 e^T \right\|_F^2$$

Oriented signal to signal ratio in sensing direction e

$$\frac{E_e(\mathcal{A})}{E_e(\mathcal{B})} = \frac{\sum_{i_2=1}^{I_2} .. \sum_{i_n=1}^{I_n} \left(\sum_{i_1=1}^{I_1} (e_{i_1} a_{i_1 i_2 \ldots i_n}) \right)^2}{\sum_{i_2=1}^{I_2} .. \sum_{i_m=1}^{I_m} \left(\sum_{i_1=1}^{I_1} (e_{i_1} b_{i_1 i_2 \ldots i_m}) \right)^2} = \frac{\left\| \mathcal{A} \times_1 e^T \right\|_F^2}{\left\| \mathcal{B} \times_1 e^T \right\|_F^2} \quad (12)$$

Matrix unfolding of a third order tensor

Each unfolding has an SVD



Computation of the oriented energy of an nth order tensor A in an l-th order sensing tensor E

- Unfold the tensor in the directions i_1, i_2,..i_l to the left and in the directions i_l, i_{l+1},..i_n to the right. The result is a matrix **A**
- Compute the SVD of $\mathbf{A}=\mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^T$
- The space of the left k columns of **U** has the dominant oriented energy.
- Refold the k columns as **k l-th order dominant tensors E_1.. E_k**

Computation of the oriented signal to signal ratio of a pair of tensors : A of order n and B of order m **in an l-th order sensing tensor E**

- Unfold the tensors **A** and **B** in the directions $i_1, i_2,..i_l$ to the left and in the other directions to the right. The result is a matrix **A** and a matrix **B**
- Compute the GSVD of the matrix pair **A B**
- The space of the left k columns of **Q** has the dominant oriented energy.
- Refold the k columns as **k l-th order dominant tensors E_1.. E_k**

Generalized higher order SVD of a tensor pair

$$A = S \times_1 \mathbf{W} \times_2 \mathbf{U}^{(2)} \ldots \times_N \mathbf{U}^{(N)}$$

$$B = R \times_1 \mathbf{W} \times_2 \mathbf{V}^{(2)} \ldots \times_M \mathbf{V}^{(M)}$$

	single	pair
matrices	SVD	GSVD
tensors	HOSVD	GHOSVD

Conclusions and suggestions for further work

New concepts defined for multidimensional signals oriented energy and oriented signal to signal ratio of tensors

Can be computed with generalization of higher order SVD HOSVD and the GSVD

Many data sets have a multiway structure : bioinformatics, image processing, dynamical systems