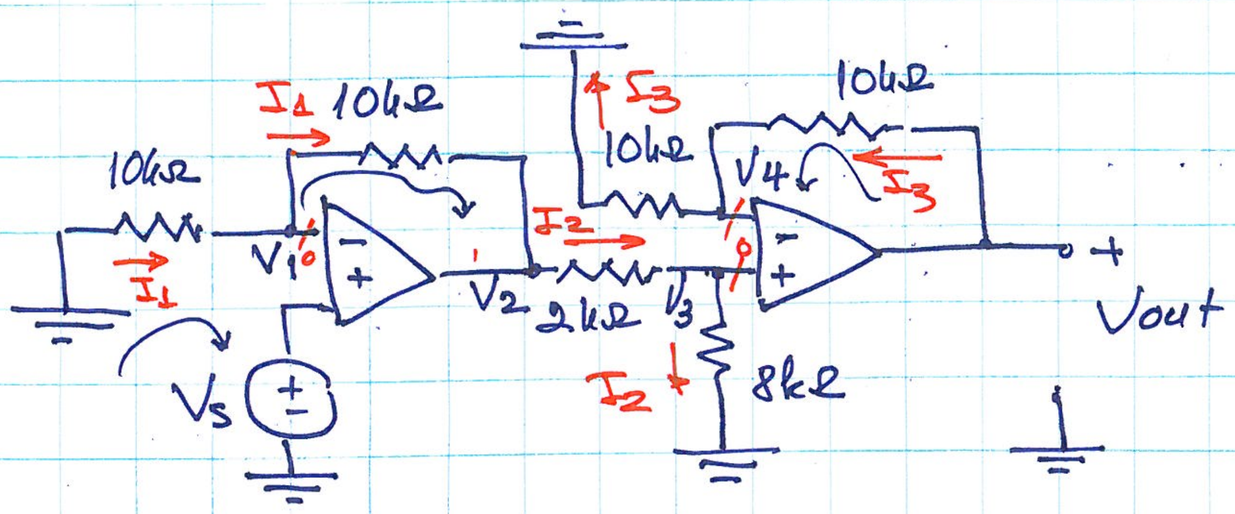
(1)



$V_1 = V_s$

$I_1 \times 10 \times 10^3 + V_s = 0 \Rightarrow I_1 = -\frac{V_s}{10^4}$

$V_2 + 10 \times 10^3 \cdot I_1 - V_s = 0$

$V_2 = V_s - 10^4 \times I_1$

$V_2 = V_s - 10^4 \cdot \left(-\frac{V_s}{10^4}\right)$

$V_2 = 2V_s$

$-V_2 - 2 \times 10^3 \cdot I_2 - 8 \times 10^3 \cdot I_2 = 0$

$V_2 - 10 \times 10^3 \cdot I_2 = 0$

$I_2 = \frac{V_2}{10^4}$; $I_2 = \frac{2 \cdot V_s}{10^4}$

$V_3 = 8 \times 10^3 \cdot I_2$

$V_3 = 8 \times 10^3 \cdot \frac{2V_s}{10^4}$; $V_3 = 1.6 \cdot V_s$

$V_{out} - 10 \times 10^3 \cdot I_3 - V_4 = 0$

$V_4 = V_3$; $V_4 = 1.6\, V_s$

$$V_4 = 10 \times 10^3 \cdot I_3$$

$$I_3 = 10^{-4} \cdot V_4 \; ; \quad I_3 = 10^{-4} \cdot 1.6 \cdot V_s$$

$$I_3 = +1.6 \times 10^{-4} \cdot V_s$$

$$V_4 = 1.6 \cdot V_s$$

Hence;

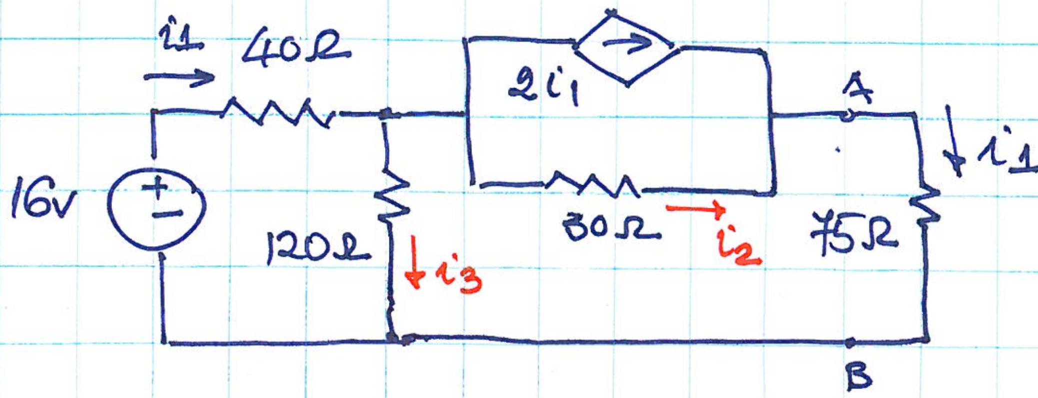
$$V_{out} = 10^4 \cdot I_3 + V_4$$

$$V_{out} = 10^4 \times 1.6 \times 10^{-4} \cdot V_s + 1.6 \cdot V_s$$

$$V_{out} = 3.2 \, V_s$$

$$\frac{V_{out}}{V_s} = 3.2$$

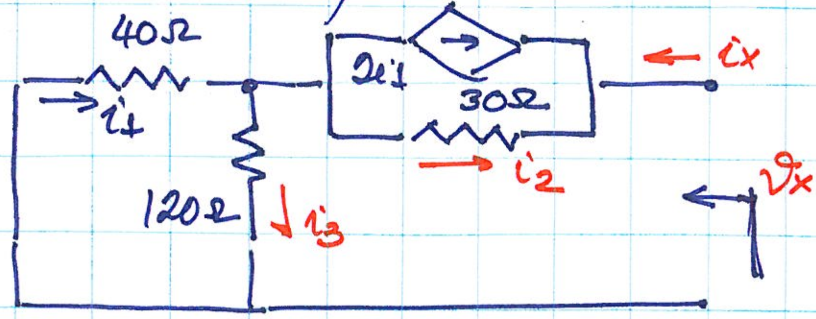
2



Thévenin equivalen:

Rth:

Disconnect the independent sources:



$40 \cdot i_1 + 120 i_3 = 0 \Rightarrow i_1 + 3 i_3 = 0$ (1)

$i_1 - i_3 - 2i_1 - i_2 = 0 \Rightarrow i_1 + i_2 + i_3 = 0$ (2)

$I_x = -i_2 - 2i_1$ (3)

From (1): $i_3 = -\frac{1}{3} i_1$

— (2): $i_2 = -i_1 - i_3$; $i_2 = -i_1 - \left(-\frac{1}{3} i_1\right)$

$i_2 = -\frac{2}{3} i_1$

(3) $I_x = -i_2 - 2i_1 \Rightarrow$

$I_x = -\left(-\frac{2}{3} i_1\right) - 2i_1$

$i_x = -\frac{4}{3} i_1$

Then: $v_x + 30 \cdot i_2 - 120 \cdot i_3 = 0$

$v_x = -30 \cdot \left(-\frac{2}{3} \cdot i_1\right) + 120 \cdot \left(-\frac{1}{3} \cdot i_1\right)$

$v_x = 20\, i_1 - 40\, i_1$

$v_x = -20 \cdot i_1$

since $i_x = -\frac{4}{3}\, i_1$

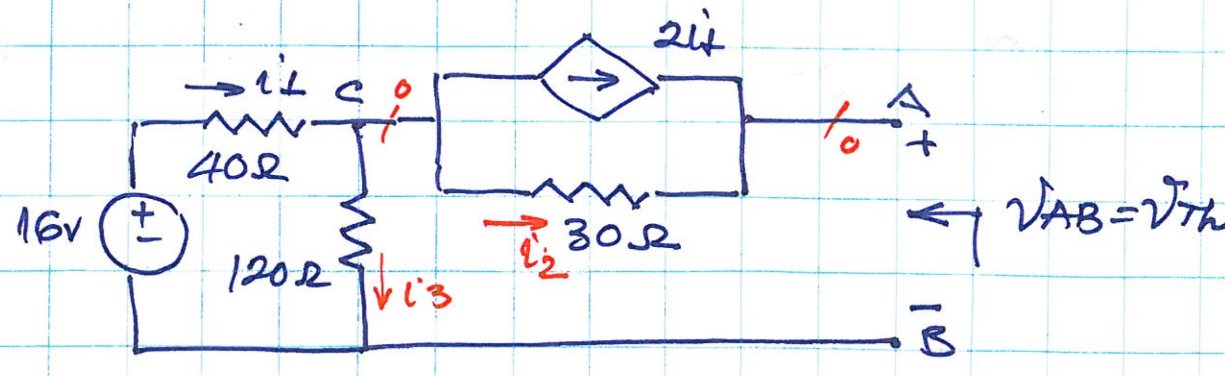
$\Rightarrow i_1 = -3/4\, i_x$

$v_x = -20 \cdot \left(-3/4 \cdot i_x\right)$

$\frac{v_x}{i_x} = 15$; $R_{Th} = 15\,\Omega$

$v_{Th} = ?$

Consider terminals A and B open:



KCL for node A: $2i_1 + i_2 = 0$

$i_2 = -2i_1$

KCL for node c : $i_1 - i_3 = 0$; $i_3 = i_1$

Hence:

$V_{AB} + 30 \cdot i_2 - 120 \cdot i_3 = 0$

$V_{AB} = -30 \cdot (-2i_1) + 120 \cdot (i_1)$

$V_{AB} = 180\, i_1$

KVL : $16 - 40\, i_1 - 120 \cdot i_3 = 0$; $16 - 40\, i_1 - 120\, i_1 = 0$

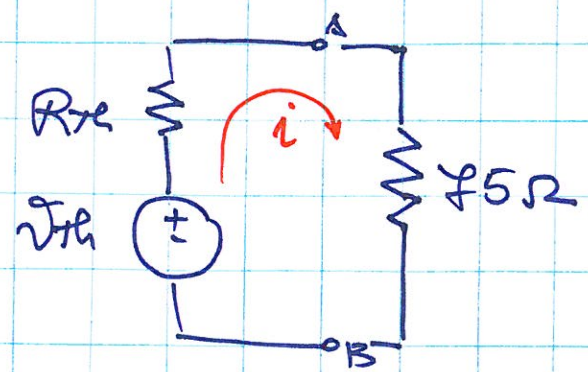
$i_1 = 16/160$; $i_1 = 0.1\,A$

Hence:

$$V_{AB} = 180 \times 0.1$$
$$= 18\,V$$

Thévenin equivalent: $V_{Th} = 18\,V$
$R_{Th} = 15\,\Omega$

Equivalent circuit:

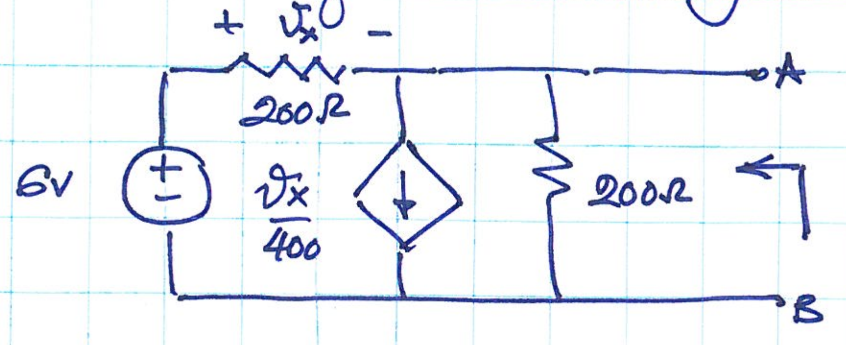


$$V_{Th} - R_{Th} \cdot i - 75 \times i = 0$$

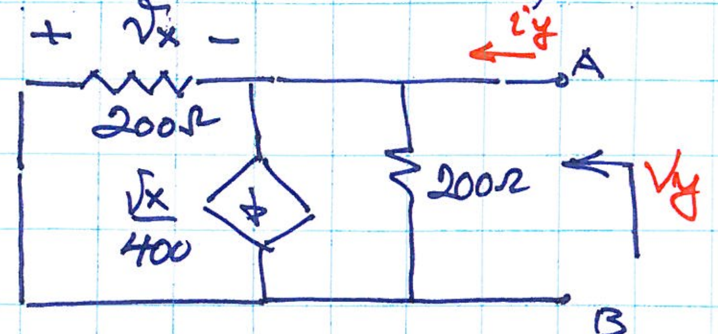
$$i = \frac{V_{Th}}{R_{Th} + 75} \; ; \; i = \frac{18}{15 + 75} \quad i = 0.2\,A$$

$$P = 75 \times i^2$$
$$P = 75 \times (0.2)^2$$
$$P = 3\,W$$

3 Thevenin equivalent seen by the capacitor:



R_{th} − ? Disconnect the independent sources:



$R_{th} = v_y / i_y$

Nodal equation for node A:

$$\frac{v_A}{200} + \frac{v_x}{400} + \frac{v_A}{200} = i_y$$

Since $v_x = -v_A$

$$v_A\left(\frac{1}{200} + \frac{1}{200}\right) - \frac{v_A}{400} = i_y$$

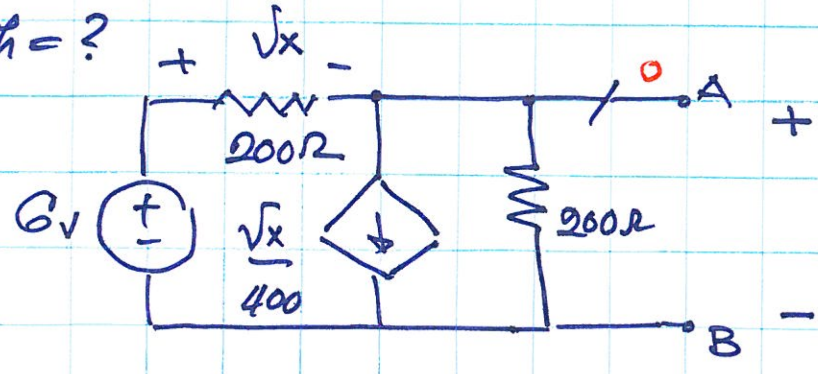
$$-v_A \cdot \frac{3}{400} = i_y; \quad v_A = -\frac{400}{3} \cdot i_y$$

Note that $v_A = -v_y$

Hence:

$$v_y = \frac{400}{3} \cdot i_y \quad R_{th} = \frac{v_y}{i_y}; \; R_{th} = \frac{400}{3}\,\Omega$$

$v_{Th} = ?$



$V_{AB} = V_{Th}$

Node equation

$$\frac{V_A - 6}{200} + \frac{V_x}{400} + \frac{V_A}{200} = 0$$

$$2(V_A - 6) + V_x + 2V_A = 0$$

$$4V_A + V_x = 12 \qquad (1)$$

KVL around the outer Loop:

$$V_A + V_x - 6 = 0$$

$$V_A + V_x = 6 \qquad (2)$$

From (1) : $V_x = 12 - 4V_A$

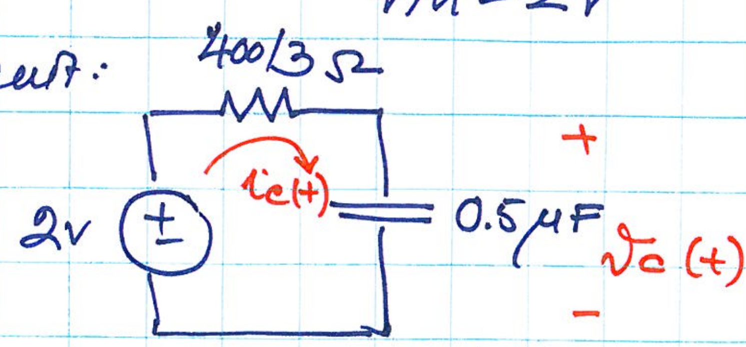
then:

$$V_A + (12 - 4V_A) = 6$$

$$-3V_A = -6 \; ; \; V_A = 2$$

$$v_{Th} = 2v$$

Equivalent circuit:



$$2 - \frac{400}{3} \cdot i_c - V_c = 0 \; ; \; i_c = C\frac{dV_c}{dt}$$

Hence:

$$\frac{400}{3}\cdot 0.5\times10^{-6}\cdot\frac{dv_c}{dt}+v_c=2$$

$$\frac{200}{3}\times10^{-6}\cdot\frac{dv_c}{dt}+v_c=2$$

$$\frac{dv_c}{dt}+\frac{3}{200}\times10^{6}\,v_c=2\times\frac{3}{200}\times10^{6}$$

Solution:

$$v_c(t)=v_p(t)+v_h(t)$$

homogeneous soln.

particular soln.

$$v_p(t)=A\ ;\quad \frac{dv_p(t)}{dt}=0$$

Hence,

$$\frac{3}{200}\times10^{6}\cdot A=2\times\frac{3}{200}\times10^{6}$$

$$A=2$$

$$v_p(t)=2V$$

$$v_h(t)=Ke^{+st}\ ;\ \text{then}\ \frac{dv_c}{dt}=Kse^{st}$$

From the differential equation:

$$K\cdot se^{st}+\frac{3}{200}\times10^{6}\cdot Ke^{st}=0$$

homogeneous portion.

$$s=-\frac{3}{200}\times10^{6}\ ,\ s=-1.5\times10^{4}$$

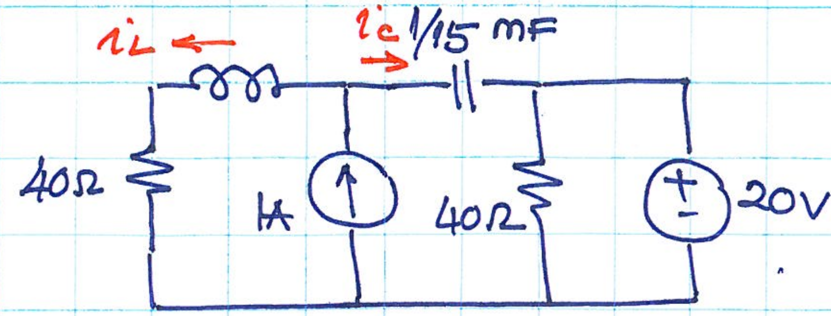
$$v_c(t)=Ke^{-15000t}+2$$

Assumption: $v_c(0-)=-6V\ ;\ v_c(0+)=-6V$

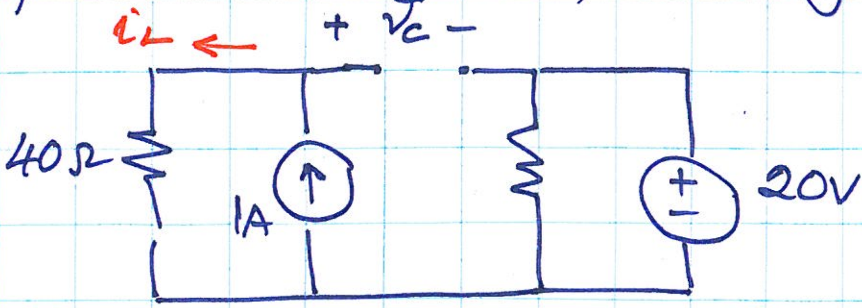
Hence: $-6=K+2\ ;\ K=-8$

$$v_c(t)=(2-8e^{-15000t})u(t)$$

4 The circuit with switch closed:



If the switch was closed for a long time:



$i_L(0-) = 1A$

$20 + V_{c(0-)} - 40 \times i_{L(0-)} = 0$ Hence: $V_c(0-) = 20V$

$i_L(0+) = i_L(0-) \Rightarrow i_L(0+) = 1A$

$V_c(0+) = V_c(0-) \Rightarrow V_c(0+) = 20V$

$i_c(0+) = ?$ In general, $i_c(0+) \neq i_c(0-)$

KCL: $i_c + i_L = 1$

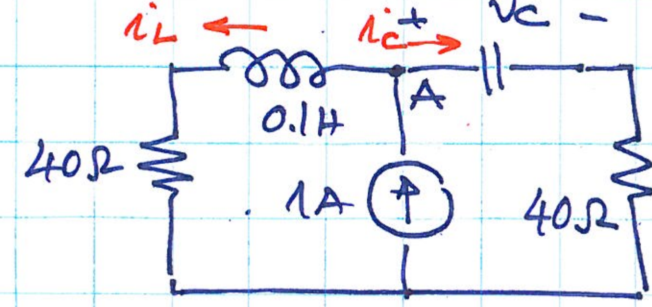
$i_c = 1 - i_L$ (for any t)

$i_c(0+) = 1 - i_L(0+)$

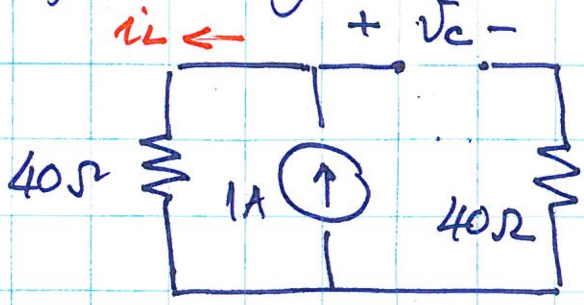
$i_c(0+) = 1 - 1$; $i_c(0+) = 0A$

At $t \to \infty$, the switch has been open for a long time.

The circuit with the switch open:



After a long time ($t \to \infty$), the circuit looks as:



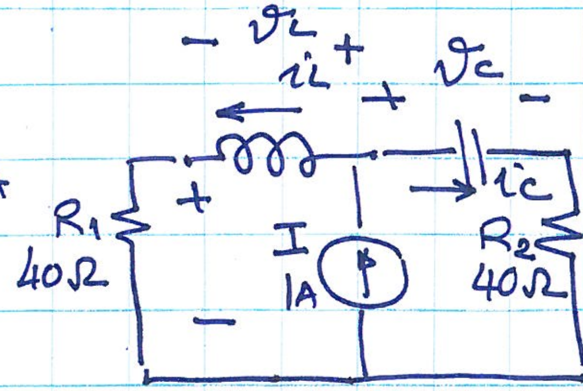
$$i_L(\infty) = 1A$$

$$v_C(\infty) - 40 \times i_L(\infty) = 0 \;;\; \text{Hence: } v_C(\infty) = 40V$$

Differential equations for the circuit with the switch open are:

(1) $i_L + i_C = I$ (KCL) for node A

(2) $v_C - v_L - R_1 \cdot i_L + R_2 \cdot i_C = 0$



$$i_C = C\frac{dv_C}{dt}$$

$$v_L = L\frac{di_L}{dt}$$

$R_1 = 40\Omega$
$R_2 = 40\Omega$

Hence:

$$v_C - L\frac{di_L}{dt} - R_1 \cdot i_L + R_2 C\frac{dv_C}{dt} = 0$$

or
$$R_2 C\frac{dv_C}{dt} - L\frac{di_L}{dt} + v_C - R_1 \cdot i_L = 0$$

Since
$$i_C = I - i_L$$

$$C\frac{dv_C}{dt} = I - i_L$$

Two state equations:

$$\left.\begin{aligned} & C\frac{dv_C}{dt} + i_L = I \\ & R_2 C\frac{dv_C}{dt} - L\cdot\frac{di_L}{dt} + v_C - R_1 \cdot i_L = 0 \end{aligned}\right\} *$$

From the first equation: $i_L = I - C\frac{dv_C}{dt}$

Hence: $\frac{di_L}{dt} = -C\frac{d^2 v_C}{dt^2}$

Substituting into the second equation gives:

$$R_2 C\cdot\frac{dv_C}{dt} - L\cdot\left(-C\frac{d^2 v_C}{dt^2}\right) + v_C - R_1\cdot\left(I - C\frac{dv_C}{dt}\right) = 0$$

$$LC\cdot\frac{d^2 v_C}{dt^2} + C\cdot(R_1 + R_2)\cdot\frac{dv_C}{dt} + v_C = R_1 \cdot I$$

$$\frac{d^2 v_C}{dt^2} + \frac{R_1 + R_2}{L}\cdot\frac{dv_C}{dt} + \frac{1}{LC}\cdot v_C = \frac{R_1}{LC}\cdot I$$

Characteristic equation:

$$s^2 + \frac{R_1 + R_2}{L}\cdot s + \frac{1}{LC} = 0$$

$$s^2 + (40+40)\cdot\frac{1}{0.1}\times s + \frac{1}{0.1\cdot\frac{1}{15}\times 10^{-3}} = 0$$

$$s^2 + 800s + 15\times 10^4 = 0$$

$$s_{1/2} = -400 \pm \sqrt{(400)^2 - 15\times 10^4}$$

$$s_{1/2} = -400 \pm \sqrt{160000 - 150000}$$

$$= -400 \pm 100 \quad ; \quad \begin{matrix} s_1 = -300 \\ s_2 = -500 \end{matrix} \quad \text{Natural frequencie.}$$

General form of the complete solution:

$$v_C(t) = K_1 e^{s_1 t} + K_2 e^{s_2 t} + v_{Cp}(t)$$

$$i_L(t) = K_3 e^{s_1 t} + K_4 e^{s_2 t} + i_{Lp}(t)$$

Particular solution.

$$v_{Cp}(t) = A \; ; \; \frac{dv_{Cp}(t)}{dt} = 0; \; \frac{d^2 v_{Cp}(t)}{dt^2} = 0$$

From the state equation

$$\frac{d^2 v_C}{dt^2} + 800\cdot\frac{dv_C}{dt} + 15\times 10^4 v_C = \frac{40}{0.1\cdot\frac{1}{15}\times 10^{-3}}\cdot 1$$

$$15\times 10^4\cdot v_{Cp}(t) = 40\times 15\times 10^4$$

$$v_{Cp}(t) = 40\text{ V} \quad (\text{as expected. Recall: } v_C(\infty) = 40\text{ V})$$

$$i_{Lp}(t) = B$$

From the state equation:

$$C\frac{dv_C}{dt} + i_L = I$$

$$C\frac{dv_{Cp}(t)}{dt} + i_{Lp}(t) = I \; ; \; C\cdot 0 + B = I;$$

$$B = I$$

Hence, $i_{cp}(t) = 1\,A$ (as expected
Recall: $i_L(\infty) = 1A$)

$$v_c(t) = k_1 e^{s_1 t} + k_2 e^{s_2 t} + 40 \qquad s_1 = -300$$
$$i_L(t) = k_3 e^{s_1 t} + k_4 e^{s_2 t} + 1 \qquad s_2 = -500$$

$v_c(0+) = 20$; $i_L(0+) = 1$

Hence:

$$k_1 + k_2 + 40 = 20 \quad \Rightarrow \quad k_1 + k_2 = -20$$
$$k_3 + k_4 + 1 = 1 \qquad\quad k_3 + k_4 = 0$$

We need two additional equations in order to find all unknown constants

Note that: $\frac{dv_c(t)}{dt} = k_1 s_1 + k_2 s_2$

$$\frac{di_L(t)}{dt} = k_3 s_1 + k_4 s_2$$

From the differential equations: (*):

$$C\frac{dv_c}{dt} + i_L = 1 \quad ; \quad \frac{1}{15}\times 10^{-3}\cdot\frac{dv_c}{dt} + 1 i_L = 1$$

$$40 \times \frac{1}{15}\times 10^{-3}\frac{dv_c}{dt} - 0.1\cdot\frac{di_L}{dt} + v_c - 40.i_L = 0$$

at $t = 0+$:

$$\frac{1}{15}\times 10^{-3}\cdot\left.\frac{dv_c}{dt}\right|_{0+} + i_L(0+) = 1$$

Note that $i_L(0+) = 1$

Hence $\left.\frac{dv_c}{dt}\right|_{0+} = 0$

Then: $40\cdot\frac{1}{15}\times 10^{-3}\cdot 0 - 0.1\cdot\left.\frac{di_L}{dt}\right|_{0+} + v_c(0+) - 40.i_L(0+) = 0$

$$-0.1 \cdot \frac{di_L}{dt}\Big|_{0+} + 20 - 40 \cdot 1 = 0$$

$$-0.1 \cdot \frac{di_L}{dt}\Big|_{0+} = 20 \quad ; \quad \frac{di_L}{dt}\Big|_{0+} = -200$$

Hence:

$$k_1 \cdot (-300) + k_2 \cdot (-500) = 0$$
$$k_3 (-300) + k_4 (-500) = -200$$

Again:

$$k_1 + k_2 = -20$$
$$-300k_1 - 500k_2 = 0 \Rightarrow -3k_1 - 5k_2 = 0$$

$k_1 + k_2 = -20 \; / \overset{*}{5}$ add

$$(5-3)\,k_1 = -100$$
$$k_1 = -50$$
$$k_2 = 30$$

And:

$$k_3 + k_4 = 0$$
$$-300k_3 - 500k_4 = -200$$

OR:

$$k_3 + k_4 = 0$$
$$-3k_3 - 5k_4 = -2 \; ;$$

$$k_3 + k_4 = 0 \; ; \; k_4 = -k_3$$
$$3k_3 + 5k_4 = 2$$
$$3k_3 - 5k_3 = 2 : \; k_3 = -1$$
$$k_4 = 1$$

Finally.

$$v_C(t) = \left(-50e^{-300t} + 30e^{-500t} + 40\right) u(t)$$

$$i_L(t) = \left(-e^{-300t} + e^{-500t} + 1\right) u(t)$$

Check: $v_C(0+) = 20$; $v_C(\infty) = 40$

$i_L(0+) = 1$; $i_L(\infty) = 1$