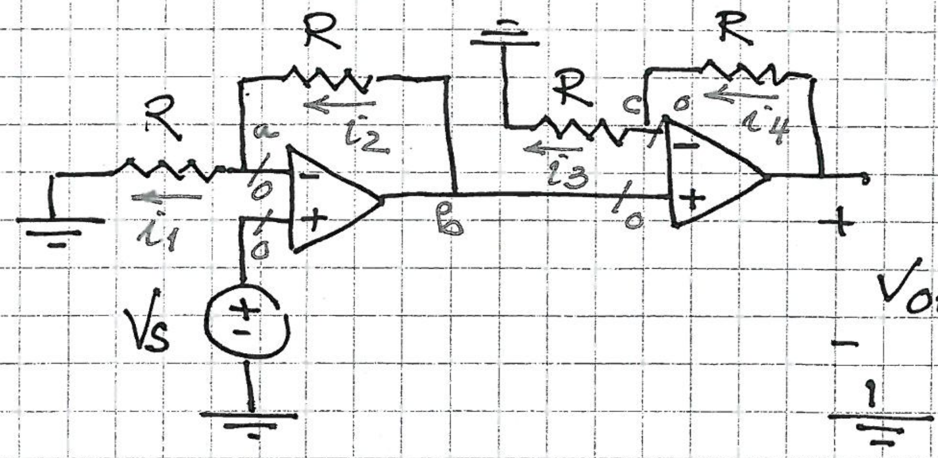
(1)

(1)



Ideal op-amp: $i_- \equiv 0$; $i_+ \equiv 0$

$v_- - v_+ \equiv 0$

$V_a = V_s$

$V_a - R \cdot i_1 = 0$; $i_1 = \frac{V_a}{R}$

$i_2 = i_1$ Hence, $i_2 = \frac{V_a}{R}$

$V_b - V_a = R \cdot i_2$

Hence, $V_b = V_a + R \cdot i_2$

$V_b = V_s + R \cdot \frac{V_a}{R}$

$V_b = V_a + V_a$

$V_b = 2V_a$; $V_b = 2 \cdot V_s$

$V_c = V_b$; $V_c = 2 \cdot V_s$

$V_c - R \cdot i_3 = 0$; $i_3 = \frac{V_c}{R}$; $i_3 = \frac{2V_s}{R}$

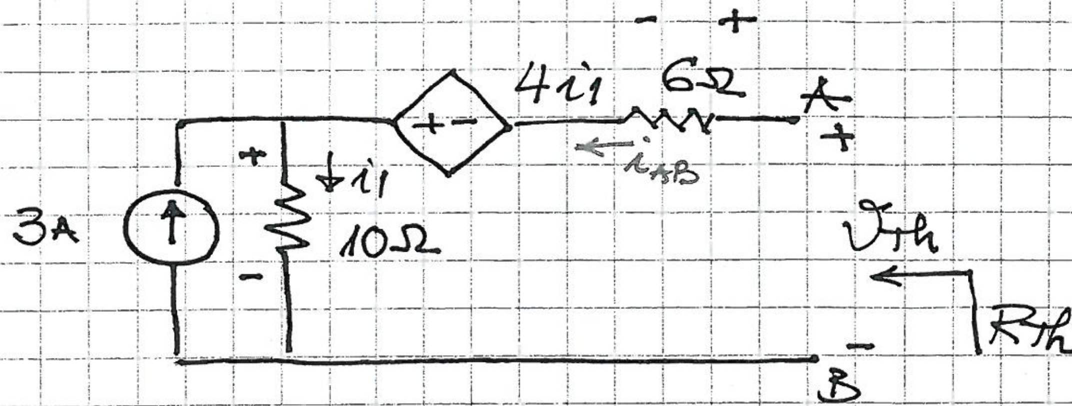
$i_4 = i_3$ Hence, $i_4 = \frac{2V_s}{R}$

$V_c = R \cdot i_3$ Hence, $V_c = R \cdot \frac{2V_s}{R}$; $V_c = 2V_s$

$V_{out} - V_c = R \cdot i_4$; $V_{out} = V_c + R \cdot i_4$

$= 2V_s + R \cdot \frac{2V_s}{R}$; $\frac{V_{out}}{V_s} = 4$

(2)



(a)

Thévenin equivalent:

V_{th} with terminals open-circuited: $i \equiv 0$

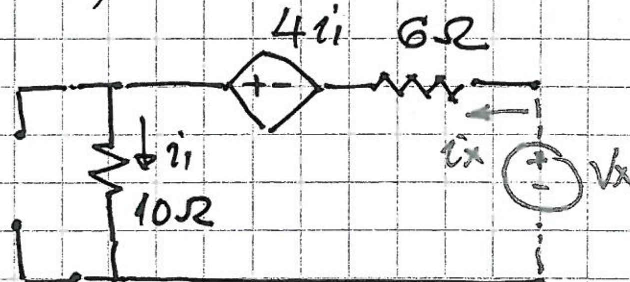
KCL: $3 + i_{AB} = i_1$; $i_1 = 3A$

KVL: $V_{Th} - 6 \cdot i_{AB} + 4 \cdot i_1 - 10 \cdot i_1 = 0$

$$V_{Th} = 6 \times 0 - 4 \cdot i_1 + 10 \cdot i_1$$

$$V_{Th} = 6 \cdot i_1 \quad ; \quad V_{Th} = 6 \times 3 \quad ; \quad V_{Th} = 18V$$

$R_{th} = \frac{V_x}{i_x}$ with all independent sources disconnected.



KCL: $i_x = i_1$

KVL:

$$V_x - 6\, i_x + 4 \cdot i_1 - 10\, i_1 = 0$$

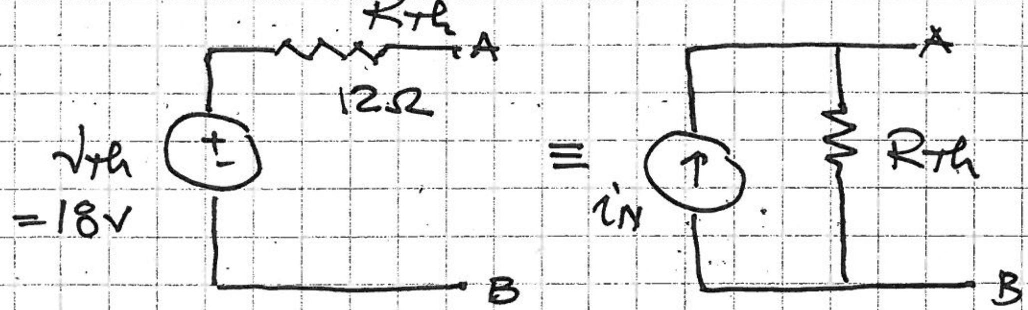
$$V_x = 6 \cdot i_x + 6 \cdot i_1$$

$$V_x = 6 \cdot i_x + 6 \cdot i_x \quad ; \quad V_x = 12\, i_x$$

$$\frac{V_x}{i_x} = 12$$

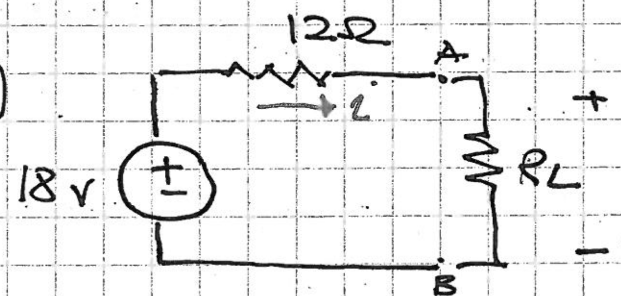
$R_{th} = 12\Omega$

(b) Norton equivalent:



$$i_N = \frac{v_{th}}{R_{th}} \; ; \; i_N = \frac{18}{12} \; ; \; i_N = 1.5A$$

(c)



$$18 - 12 \cdot i - R_L \cdot i = 0$$

$$i = \frac{18}{12 + R_L}$$

Power dissipated in R_L: $P_L = i \cdot v$

$$P_L = \frac{18}{12+R_L} \cdot R_L \cdot \frac{18}{12+R_L}$$

$$P_L = R_L \cdot \left(\frac{18}{12+R_L}\right)^2 \; ; \; \frac{\partial P_L}{\partial R_L} = 18^2 \cdot \frac{\partial}{\partial R_L} \left\{ \frac{R_L}{(12+R_L)^2} \right\}$$

$$\frac{\partial P_L}{\partial R_L} = 324 \cdot \frac{(12+R_L)^2 - R_L \cdot 2 \times (12+R_L)}{(12+R_L)^{2 \times 2}}$$

$$\frac{\partial P_L}{\partial R_L} = 0 \text{ if: } \quad 12 + R_L - 2R_L = 0 \; ; \; R_L = 12\Omega$$

Maximum power transfer.

$$P_{max} = R_L \cdot \left(\frac{18}{12 + R_L} \right)^2 \quad \text{for } R_L = 12\Omega$$

$$P_{max} = 12 \times \frac{18^2}{(2 \times 12)^2}$$

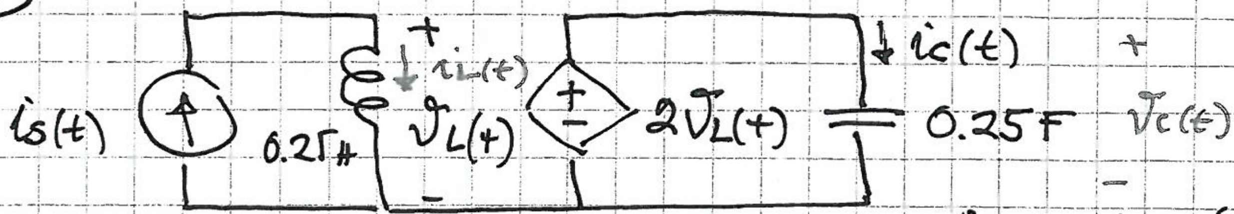
$$= \frac{324}{4 \times 12}$$

$$= \frac{81}{12}$$

$$= \frac{27}{4}$$

$$= 6.75 \text{ Watts}$$

3



$i_s(t) = 4\sin(4t)$ A

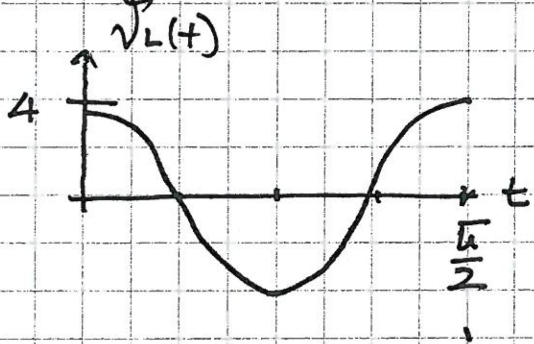
(a): $v_L(t) = ?$

$$i_L = i_s$$

$$v_L = L\frac{di_L}{dt} \; ; \; v_L = 0.25 \times \frac{d}{dt}\left(4\sin(4t)\right)$$

$$v_L = 0.25 \times 4 \times 4 \times \cos(4t)$$

$$v_L = 4\cos(4t) \text{ V}$$



$$\omega = 4 \text{ rad/sec}$$

$$\omega = 2\pi \frac{1}{T} \; ; \; T = \frac{2\pi}{\omega}$$

$$T = \frac{2\pi}{4}$$

$$T = \pi/2$$

(b) $i_c(t) = ?$

KVL: $2v_L(t) - v_c(t) = 0$

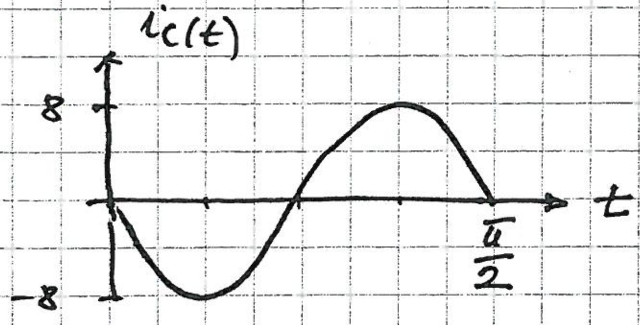
$$v_c = 2 \cdot v_L \; ; \; v_c = 8\cos(4t)$$

$$i_c = C\frac{dv_c}{dt} \; ; \; i_c = 0.25 \times \frac{d}{dt}\left(8\cos(4t)\right)$$

$$i_c = 0.25 \times 8 \cdot 4 \cdot \left(-\sin(4t)\right)$$

$$i_c = -8\sin(4t) \text{ A}$$

6



(c) Energy stored:

- inductor: $P_L = i_L \cdot v_L$

$$P_L = i_s(t) \cdot V_L(t)$$

$$P_L = 4 \cdot \sin 4t \cdot 4 \times \cos 4t$$

$$P_L = 16 \cdot \sin 4t \cdot \cos 4t$$

Note: $2 \sin\alpha \cdot \cos\alpha = \sin 2\alpha$

$$P_L = 8 \sin 8t$$

$$W_L = \int_0^t 8 \sin 8x \cdot dx \; ; \; W_L = 8 \cdot \left(-\frac{1}{8}\right) \cdot \cos 8x \Big|_0^t$$

$$W_L = -(\cos 8t - 1)$$

$$W_L = (1 - \cos 8t) \text{ Watts}$$

- capacitor: $P_C = i_C \cdot v_C$

$$P_C = (-8 \sin(4t)) \cdot 8 \cos(4t)$$

$$P_C = -64 \cdot \sin(4t) \cdot \cos(4t)$$

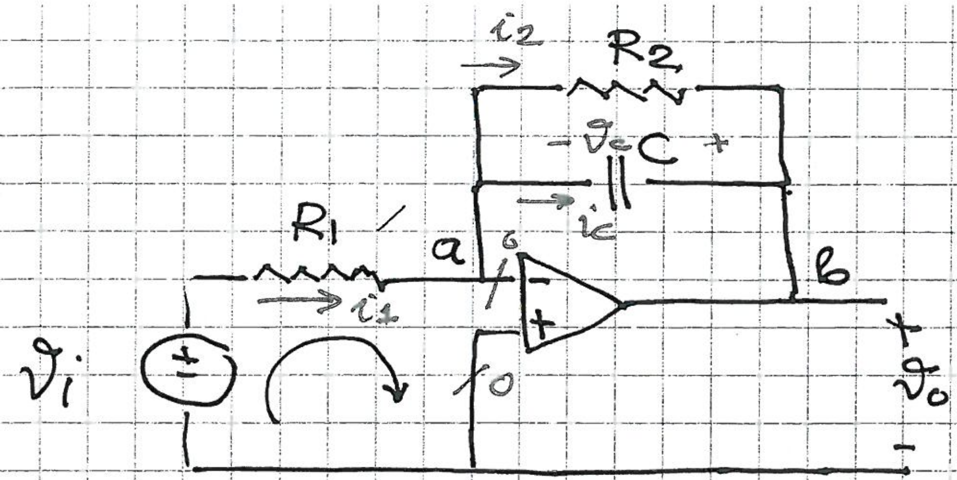
$$P_C = -32 \cdot \sin(8t)$$

$$W_c = \int_0^t (-32 \times \sin(8x)) \, dx$$

$$W_c = -32 \times \frac{1}{8} \cdot \left(-\cos 8x\right)\Big|_0^t$$

$$W_c = 4\left(\cos 8t - 1\right) \quad \text{Watts}$$

4

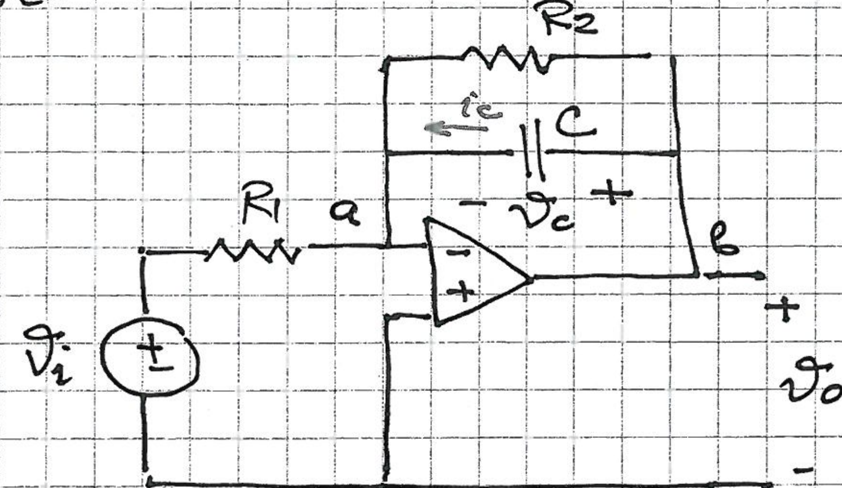


Ideal op amp: $v_- - v_+ \equiv 0$; $v_- = 0$

KCL: $i_1 - i_2 - i_C = 0$

KVL:

4



(a)

Nodal equations: $\frac{v_a - v_i}{R_1} + \frac{v_a - v_b}{R_2} - i_C = 0$

$v_a \equiv 0$

Ideal op amp: $v_- - v_+ = 0$
$v_- \equiv 0$
$v_a \equiv 0$

$-\frac{v_i}{R_1} - \frac{v_b}{R_2} - i_C = 0$

Note that $v_b = v_C$

Hence, $-\frac{v_i}{R_1} - \frac{v_C}{R_2} - i_C = 0$

$$i_c = C\frac{dv_c}{dt}$$

$$C\frac{dv_c}{dt} + v_c \cdot \frac{1}{R_2} = -v_i \cdot \frac{1}{R_1}$$

or:
$$\frac{dv_c}{dt} + \frac{1}{CR_2} \cdot v_c = \frac{-1}{CR_1} \cdot v_i$$

Since $v_0 \equiv v_b$

(b) $v_0 = v_c$

Differential equation:

$$\frac{dv_0}{dt} + \frac{1}{CR_2} \cdot v_0 = -\frac{1}{CR_1} \cdot v_i$$

(c) $R_1 = 4\Omega$; $R_2 = 10\Omega$, $C = 0.5F$

$v_i(t) = 6u(t)$

$v_c(0-) = 4V$

$$\frac{dv_0}{dt} + \frac{1}{0.5 \times 10} \cdot v_0 = -\frac{1}{0.5 \times 4} \cdot v_i$$

$$\frac{dv_0}{dt} + \frac{1}{5} v_0 = -\frac{1}{2} \times 6u(t)$$

$$\frac{dv_0}{dt} + \frac{1}{5} v_0 = -3u(t) \quad \text{for } t \geq 0$$

$$\frac{dv_0}{dt} + \frac{1}{5} v_0 = -3$$

Homogeneous equation: $\frac{dv_0}{dt} + \frac{1}{5} v_0 = 0$

$$v_{oh} = Ke^{st}$$

$$\frac{dv_{oh}}{dt} = Kse^{st}$$

Hence:
$$K \cdot se^{st} + \frac{1}{5} \cdot Ke^{st} = 0 \quad ; \quad s = -1/5$$
natural frequency

$$v_{oh} = Ke^{-t/5}$$

Particular solution:

$$v_{op}(t) = A \quad \text{(constant)}$$

$$\frac{dv_{op}(t)}{dt} = 0$$

Hence, differential equation becomes:

$$0 + \frac{1}{5}A = -3 \quad A = -15$$

$$v_{op}(t) = -15$$

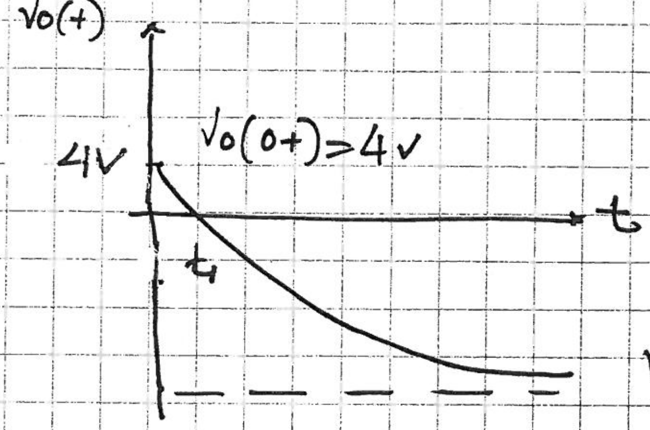
Complete solution:

$$v_o(t) = Ke^{-t/5} - 15$$

Initial conditions: $v_C(0-) = 4V$; $v_C(0+) = 4V$
or $v_o(0+) = 4V$

Hence:
$$4 = K \cdot e^{-0/5} - 15 \quad ; \quad K = 19$$

$$v_o(t) = 19e^{-t/5} - 15$$



$$19e^{-t_1/5} - 15 = 0$$

$$e^{-t_1/5} = \frac{15}{19}$$

$$t_1 = 5\ln 19/15$$

(d) $$i_c(t) = C \cdot \frac{dv_c}{dt}$$

$$i_c(t) = C \cdot \frac{dv_o}{dt}$$

$$i_c = 0.5 \times \frac{d}{dt}\left(19e^{-t/5} - 15\right)$$

$$i_c = 0.5 \times 19 \cdot \left(-\frac{1}{5}\right)e^{-t/5}$$

$$i_c = -1.9e^{-t/5}$$