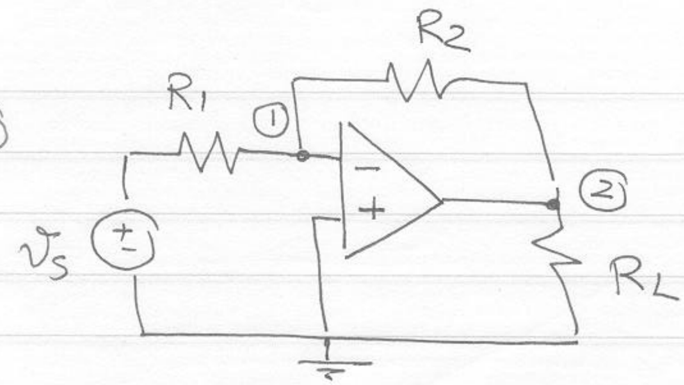
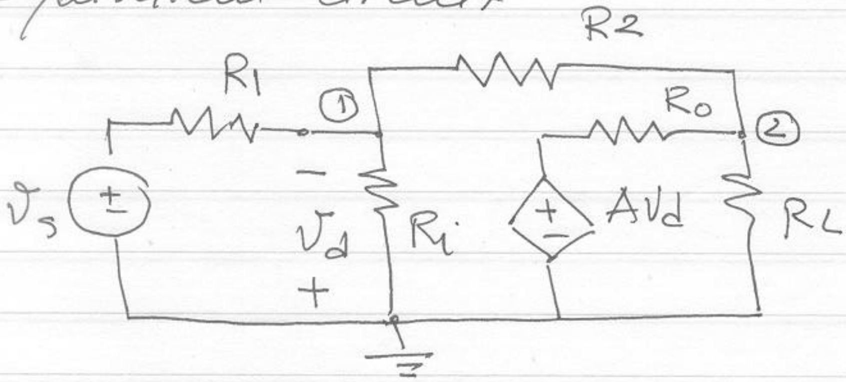
(1)



Op-amp is not ideal.
Equivalent circuit



Nodal equations:

(1)
$$\frac{v_1 - v_S}{R_1} + \frac{v_1}{R_i} + \frac{v_1 - v_2}{R_2} = 0$$

$$\frac{v_2 - v_1}{R_2} + \frac{v_2 - Av_d}{R_o} + \frac{v_2}{R_L} = 0$$

Note: $v_d = -v_1$

Hence: $v_1 \cdot \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_i}\right) - \frac{1}{R_2} \cdot v_2 = \frac{1}{R_1} \cdot v_S$

$$\left(\frac{A}{R_o} - \frac{1}{R_2}\right) v_1 + \left(\frac{1}{R_2} + \frac{1}{R_L} + \frac{1}{R_o}\right) v_2 = 0$$

In matrix form:

$$\begin{bmatrix} \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_i} & -\frac{1}{R_2} \\ \frac{A}{R_o}-\frac{1}{R_2} & \frac{1}{R_2}+\frac{1}{R_L}+\frac{1}{R_o} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 1/R_1 \\ 0 \end{bmatrix} v_s$$

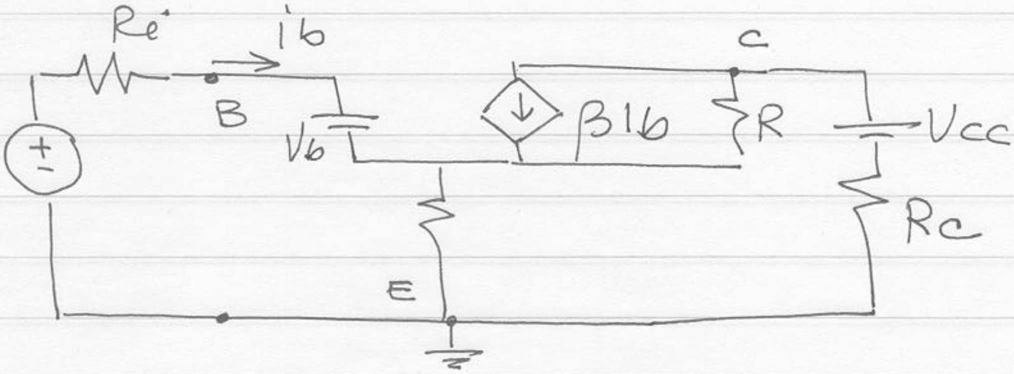
① Note: if $A=0$ (no dependent source)
the nodal-admittance matrix is symmetric because it represents a circuit consisting of resistors only.

② All diagonal elements positive

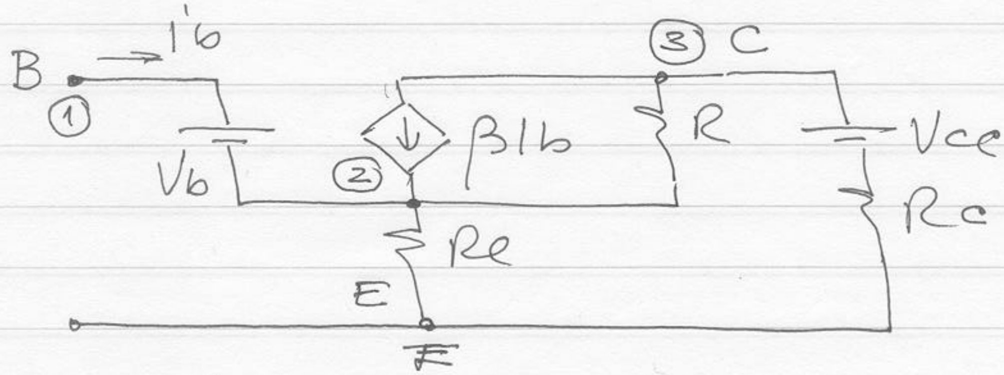
③ Off diagonal elements negative, except for $\frac{A}{R_o}$ that comes from the dependent source.
If $A=0$, all off-diagonal elements are negative.

Output voltage: $v_O = v_2$.

2



The circuit used to find Thévenin's equivalent between B and E is:



To find V_{th}, nodes B and E should be kept open-circuited. Hence

$$i_b = 0$$

Nodal equations are:

$$V_1 - V_2 = V_b$$

$$\frac{V_2}{R_e} - \beta \cdot i_b + \frac{V_2 - V_3}{R} = 0$$

$$\beta i_b + \frac{V_3 - V_2}{R} + \frac{V_3 - V_{cc}}{R_c} = 0$$

Since $I_b = 0$, the equations become:

$$V_1 - V_2 = V_b$$

$$V_2 \cdot \left(\frac{1}{Re} + \frac{1}{R} \right) - \frac{V_3}{R} = 0$$

$$-\frac{1}{R} \cdot V_2 + V_3 \cdot \left(\frac{1}{Rc} + \frac{1}{R} \right) = \frac{Vcc}{Rc}$$

Note that $V_{th} = V_1$

Since $V_1 = V_2 + V_b$, it is sufficient to find V_2 from the nodal equations

Since $V_3 = R \cdot \left(\frac{1}{Re} + \frac{1}{R} \right) \cdot V_2$

then

$$-\frac{1}{R} \cdot V_2 + R \cdot \left(\frac{1}{Re} + \frac{1}{R} \right) \cdot \left(\frac{1}{Rc} + \frac{1}{R} \right) V_2 = \frac{Vcc}{Rc}$$

And

$$V_2 = \frac{Vcc}{Rc} \cdot \frac{1}{-\frac{1}{R} + \frac{R}{Re}\left(\frac{1}{Rc} + \frac{1}{R} \right) + \frac{1}{Rc} + \frac{1}{R}}$$

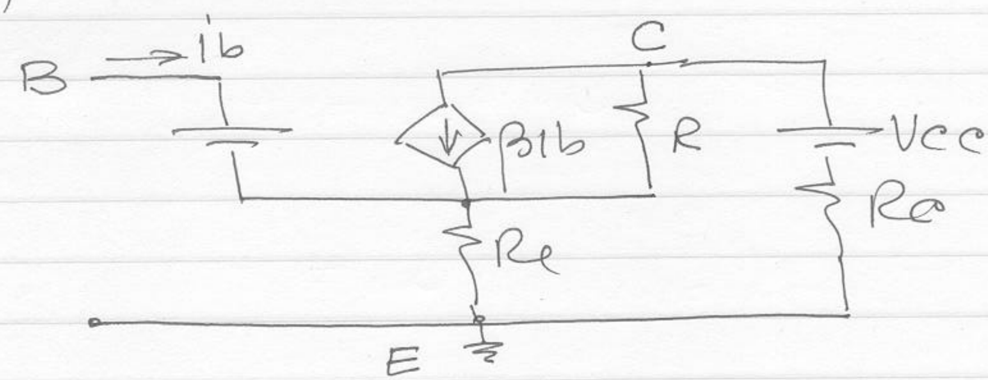
$$V_2 = \frac{Vcc}{Rc} \cdot \frac{1}{\frac{R}{Re \cdot Rc} + \frac{1}{Re} + \frac{1}{Rc}}$$

Or: $$V_2 = V_{cc} \cdot \frac{R_c}{R+R_e+R_c}$$

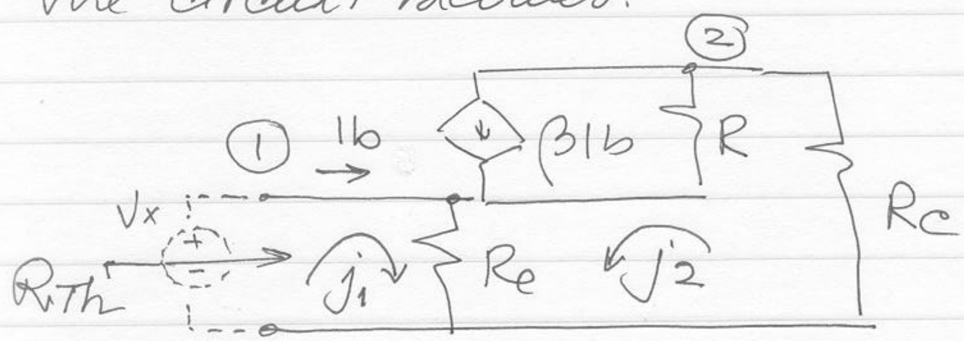
Finally, $V_{th} = V_2 + V_b$

$$V_{th} = V_b + \frac{R_e}{R+R_e+R_c} V_{cc}$$

3 The same circuit, but with all independent sources properly disconnected, is used to find Thevenin's resistance between B-E:

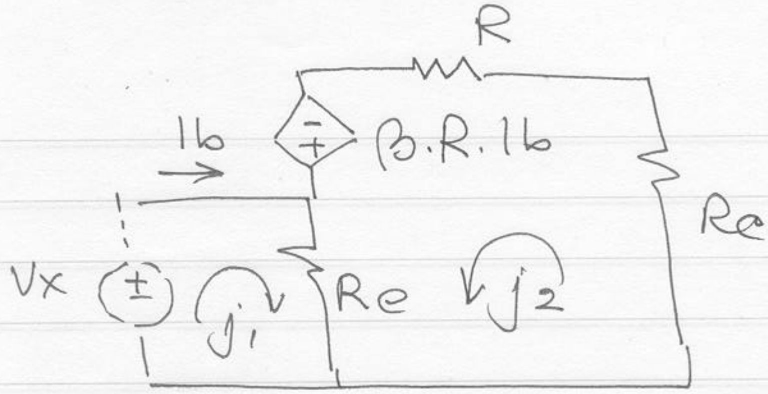


When independent sources are disconnected, the circuit becomes:



To find Rth, assume a voltage source Vs is connected between

Note that the parallel connection of the dependent current source and resistor can be transformed into a series connection of a dependent voltage source and the same resistor. (This will reduce the number of meshes.)



Mesh equation:

$$Vx - Re \cdot (j_1 + j_2) = 0$$

$$\beta R \cdot ib - Re(j_1 + j_2) - (R + Rc) j_2 = 0$$

Note that: $ib = j_1$

Hence: $Re \cdot j_1 + Re \cdot j_2 = Vx$

$$-(Re - \beta R) j_1 - (Re + R + Rc) j_2 = 0$$

Note that $R_{Th} = \dfrac{Vx}{j_1}$

From above equation: $j_2 = -j_1 + \dfrac{Vx}{Re}$

and $-(Re - \beta R) j_1 - (Re + R + Rc)\left(-j_1 + \dfrac{Vx}{Re}\right) = 0$

$$j_1 \cdot (\beta R + R + Rc) = \frac{Re + R + Rc}{Re} \cdot Vx$$

Hence,

$$J_1 = \frac{Re+R+Rc}{Re(\beta R+R+Rc)} \cdot V_x$$

And

$$R_{th} = \frac{V_x}{J_1}$$

$$R_{th} = \frac{Re\cdot(\beta R+R+Rc)}{Re+R+Rc}$$

or:

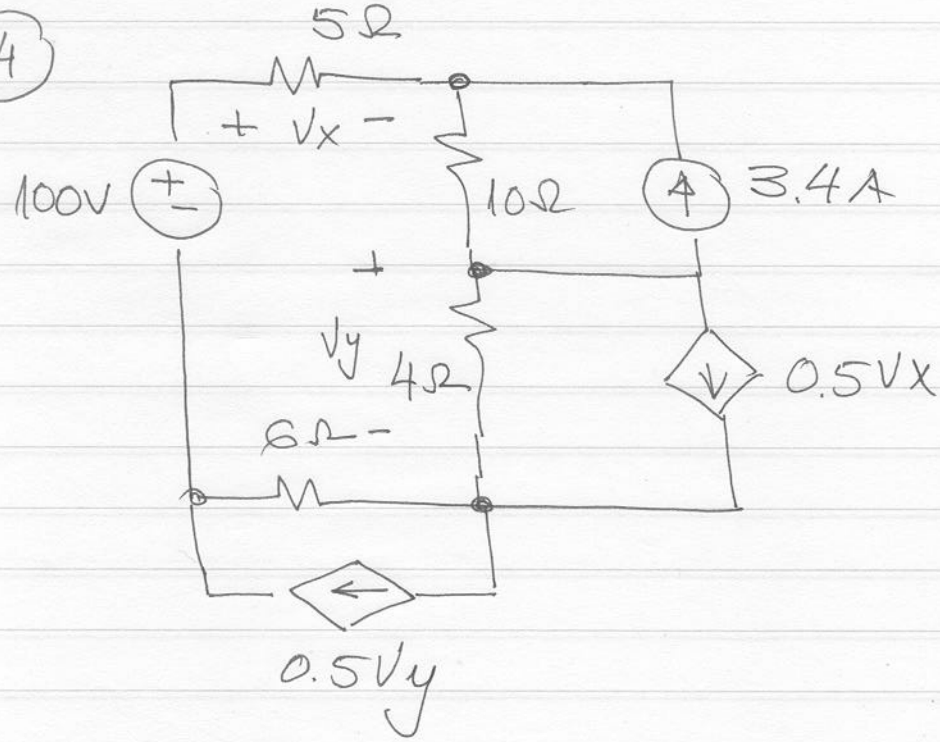
$$R_{th} = Re \cdot \frac{(1+\beta)R+Rc}{Re+R+Rc}$$

$$\beta \to 0: \quad R_{th} = Re \cdot \frac{R+Rc}{Re+R+Rc}$$

From the circuit: R and Rc in series, and connected in parallel to Re.

$$\beta \to 0,\ R \to \infty: \quad R_{th} = Re$$ (the rest of the circuit is open!)

4



$n = 4$ nodes

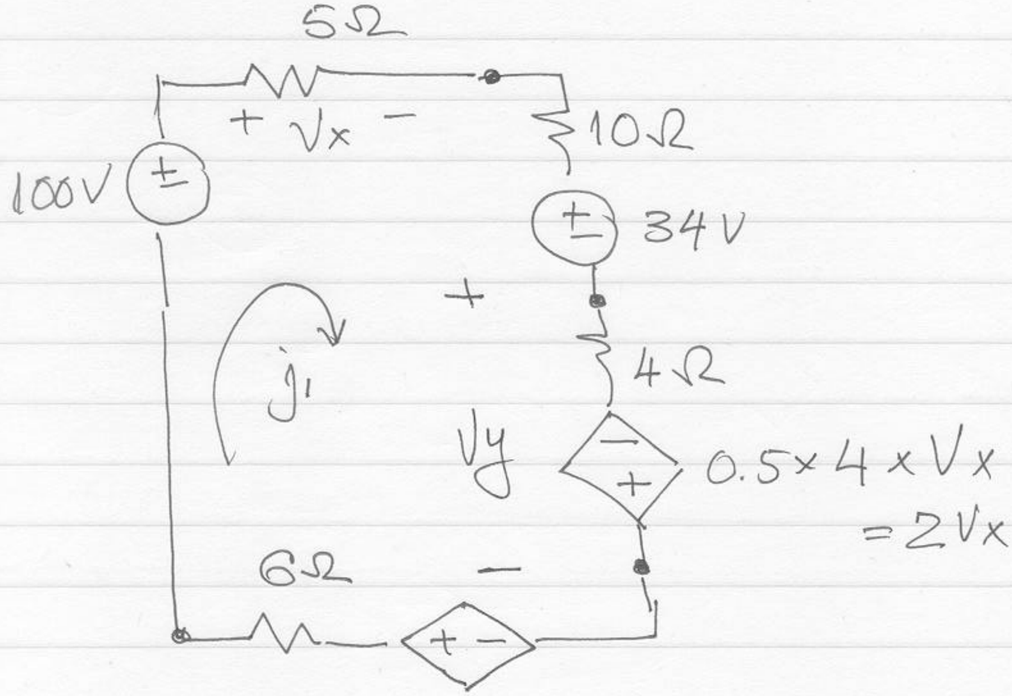
$b = 4$ (generalized) branches

No. of independent node equatn.: $n-1 = 3$

No. of — " — mesh — " — $b-(n-1) = \underline{1}$

Hence, mesh equations produce a smaller set of equation. (one equation):

Transform all current sources into voltage sources:



$0.5 \cdot 6 \times V_y = 3V_y$

$100 - 5 \cdot j_1 - 10 \cdot j_1 - 34 - 4 \cdot j_1 + 2V_x + 3V_y - 6j = 0$

Since $V_x = 5j_1$

$$V_y = 4j_1 - 2V_x$$
$$= 4j_1 - 2 \cdot (5j_1)$$
$$= -6j$$

Hence: $66 - 25j + 2 \cdot 5j_1 + 3 \cdot (-6j) = 0$

$\underline{j = 2A}$