Surge Impedance of Transmission-Line Tower

C. A. Jordan's Formula

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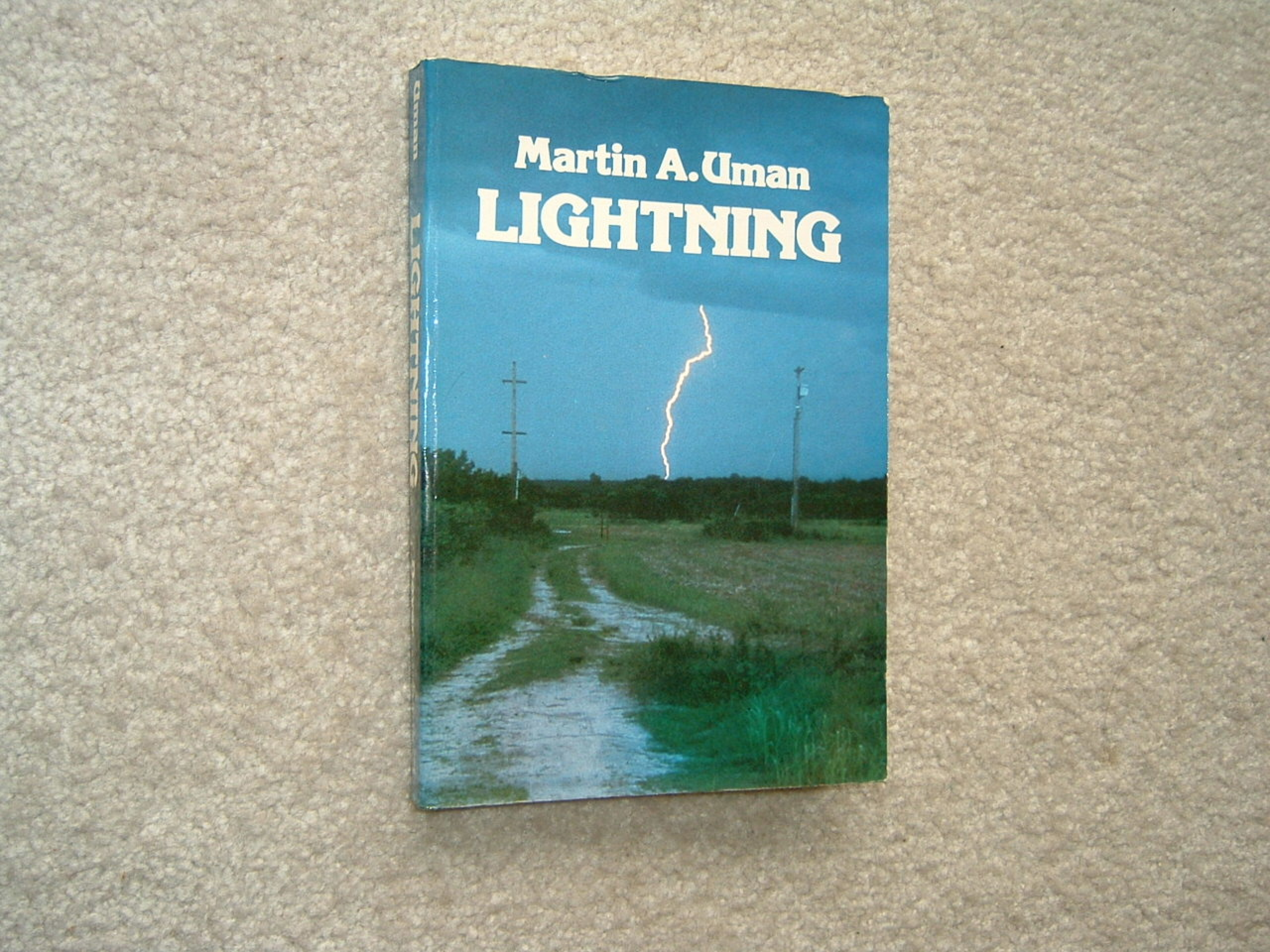
Martin A. Uman
LIGHTNING
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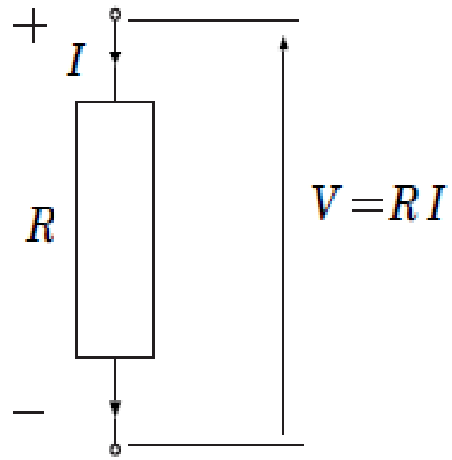
LIGHTNING

What is the surge impedance of transmission-line tower?

Ohm's law

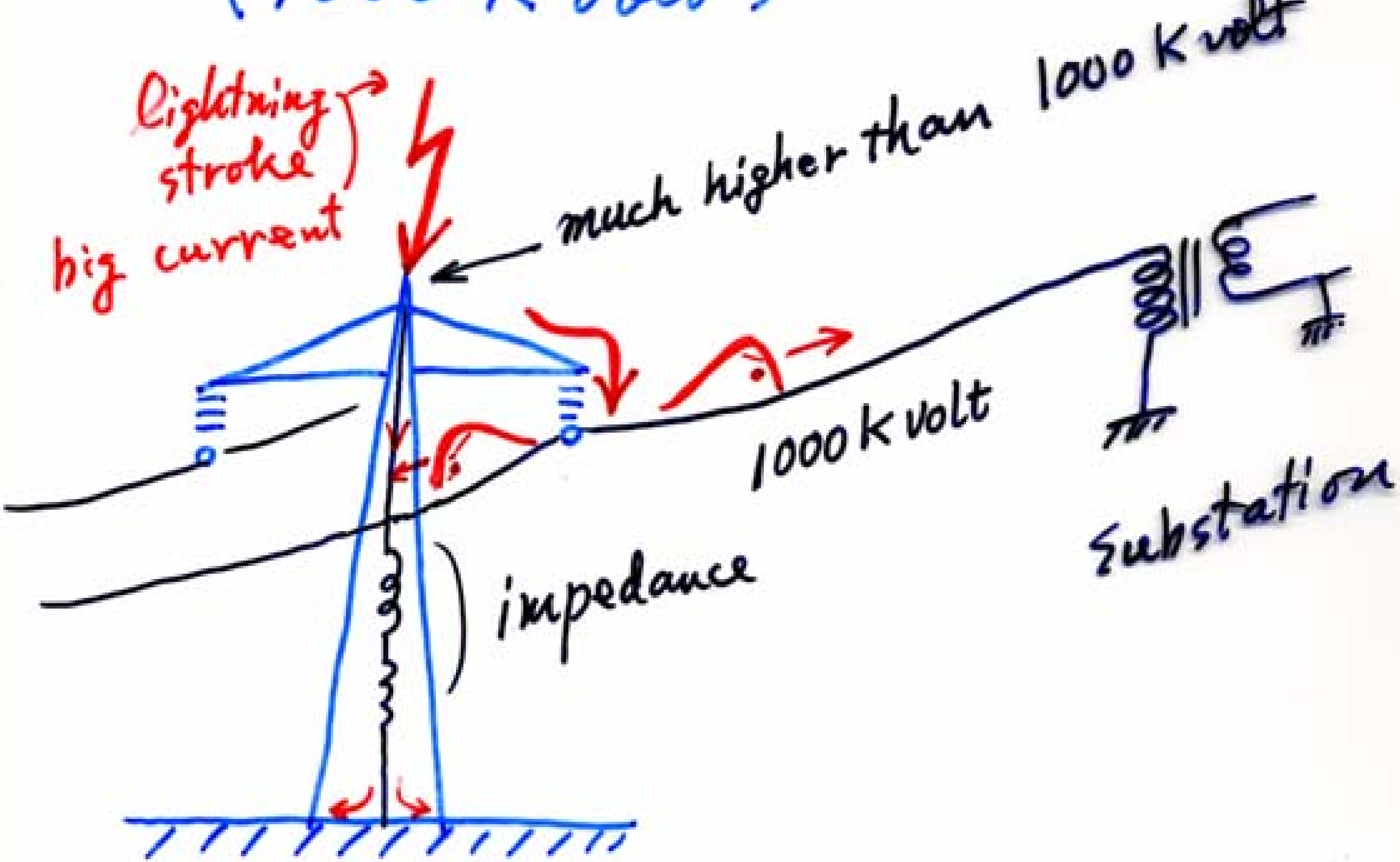


$$R = 1\Omega, \quad I = 1\text{A} \ \rightarrow \quad V = 1\text{V}$$

$$R = 100\Omega, \quad I = 10\text{kA} \ \rightarrow \quad V = 1,000\text{kV}$$

Ultra High Voltage Transmission

(1000 K volt)

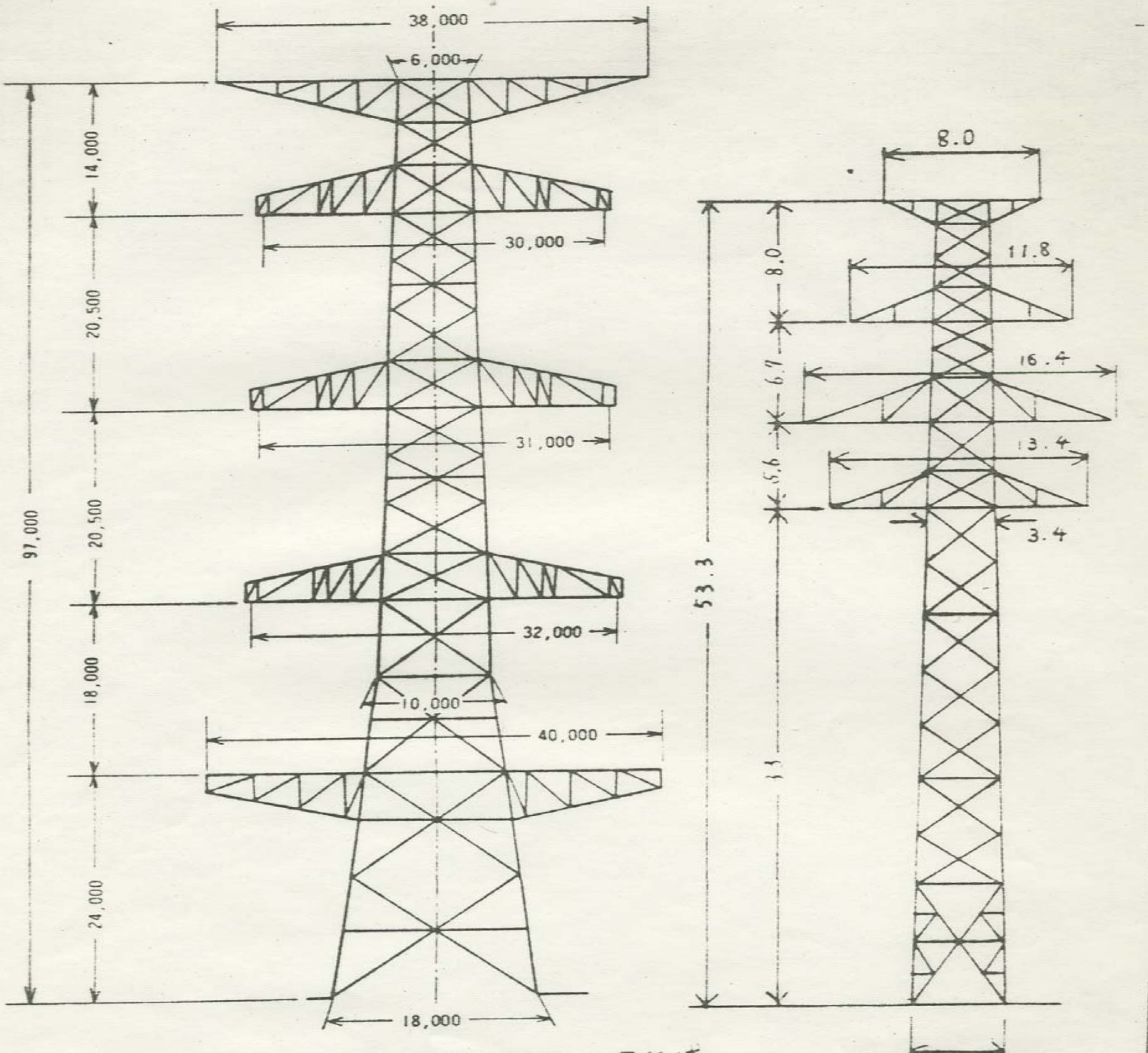


Necessity for estimation of the tower surge impedance

Why do we need to estimate the tower surge impedance? Because we should know how high the tower-top voltage is when the top is struck by lightning.

Background

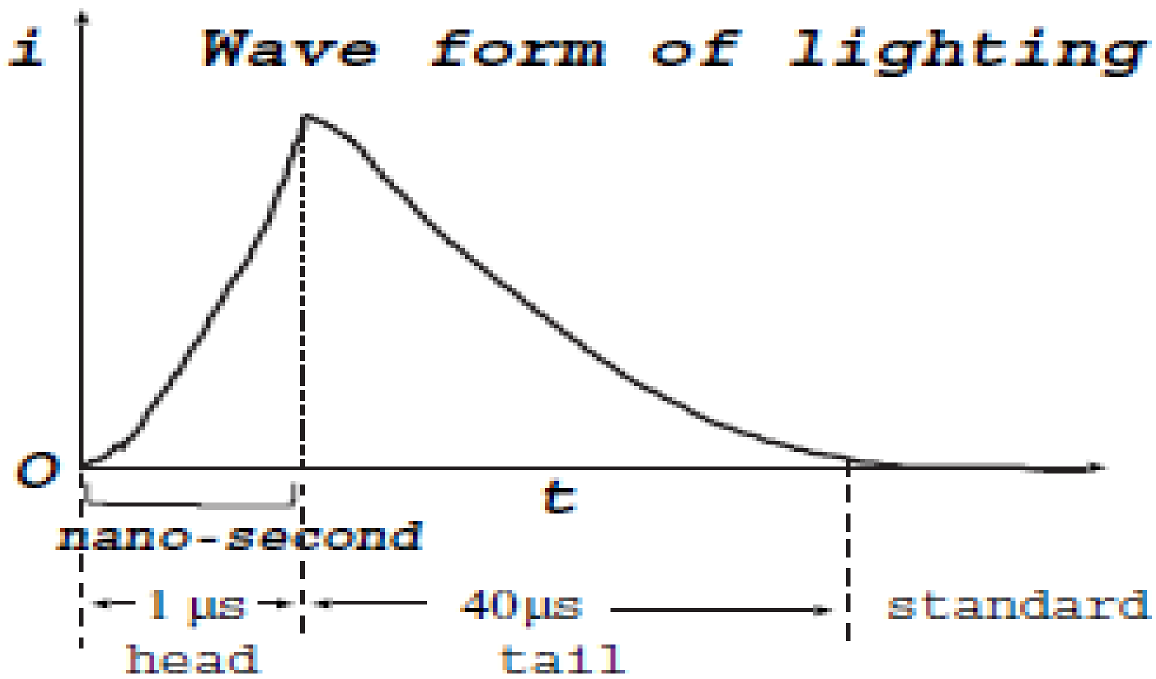
- Japan needed to transport large electric energy to Tokyo (12 million people) from Japan-sea side.
- Construction of 1,000 kV (UHV) transmission line begun.



第2図 UHV赤城実規模試験線 2号鉄塔

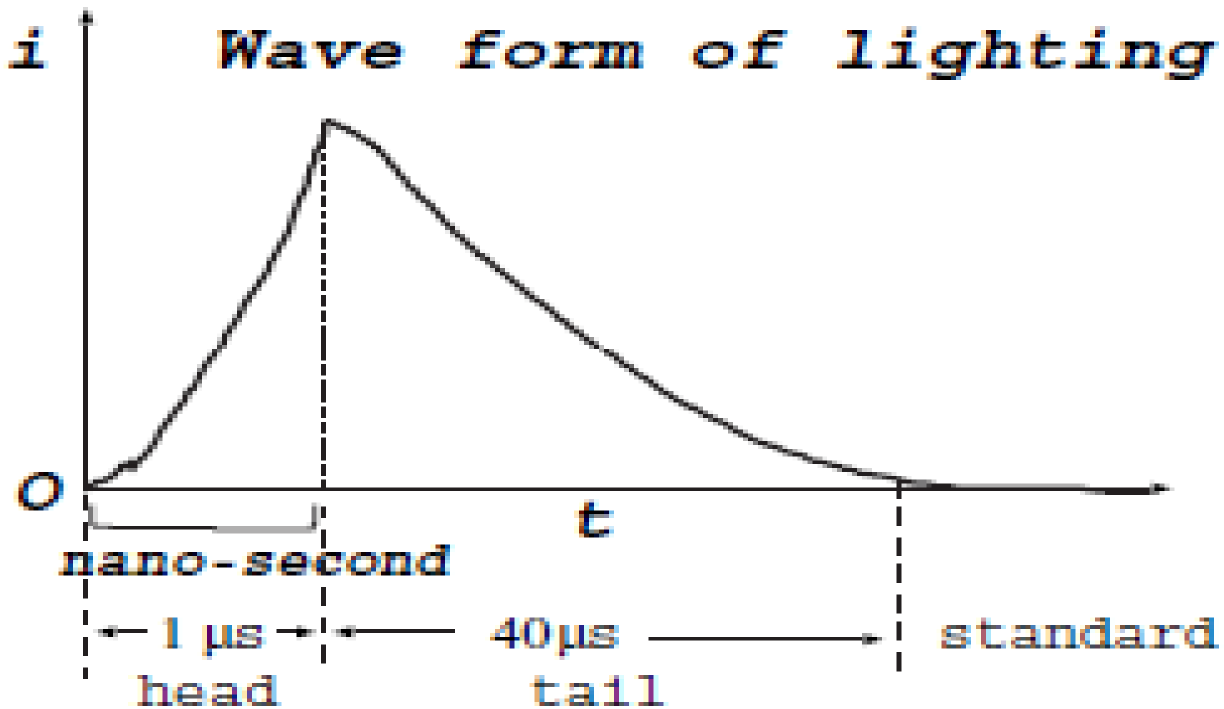
Technology development

- Observation of lightning has been undertaken.
- Measurement of the surge impedance of the model-tower for 1,000 kV has been developed.



Technology development

- Observations of lightning were undertaken.
- Measurements of the surge impedance of the model-tower for 1,000 kV were performed.



History of estimation of tower surge impedance

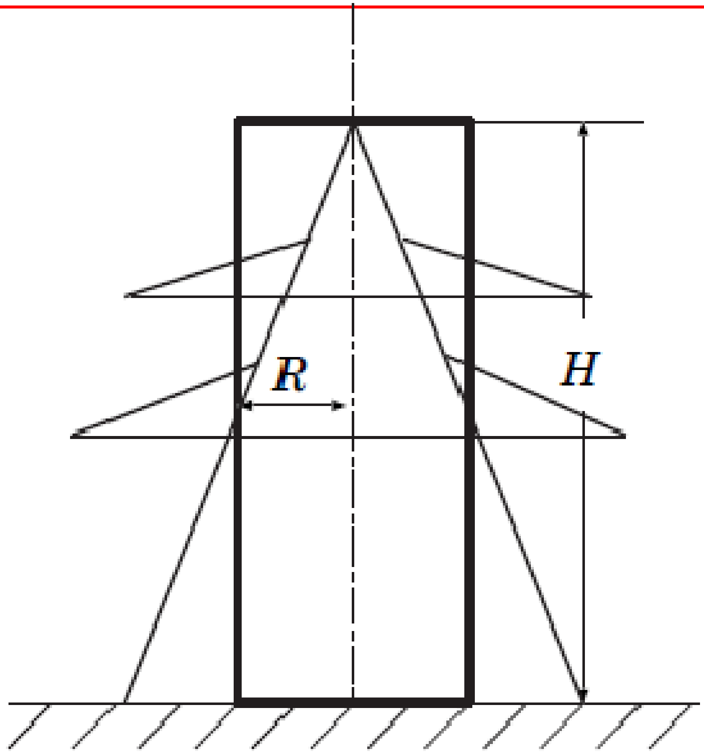
(1) C. A. Jordan's formula (1934)
(2) S. Hayashi, K. Iwamoto and B. Kondo (1954)
(3) R. Lundholm (1957)
(4) C. F. Wagner and A. R. Hileman (1959)
(5) M. Kawai (1959)
(6) M. A. Sargent and D. Darveniza (1969)
(7) K. Okumura and A. Kishima (1985)

C . A. Jordan's formula (1934)

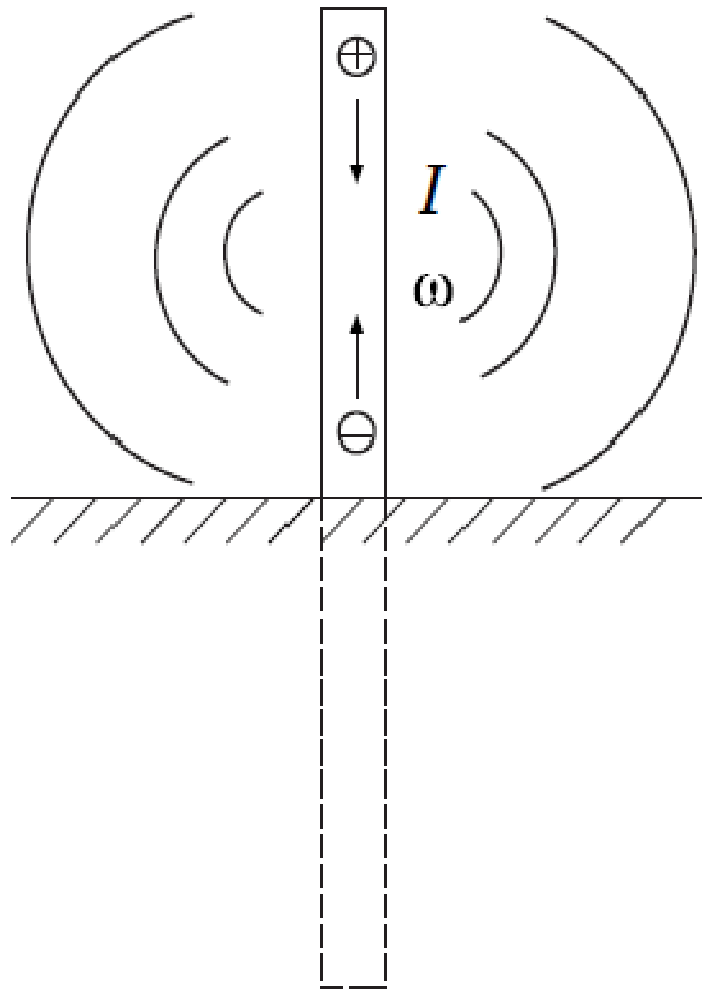
C. A. Jordan, "Lightning computation for transmission lines with overhead wires," *G. E. Rev.*, 37, No. 4, p. 180, 1934.

- The formula is well-known and has been used by many engineers and researches for over 50 years. It has been cited even in textbooks.
- This formula for the tower surge impedance matches well the experimental data.
- The cylinder model was used for the body of towers.

C . A. Jordan's formula: cylinder model

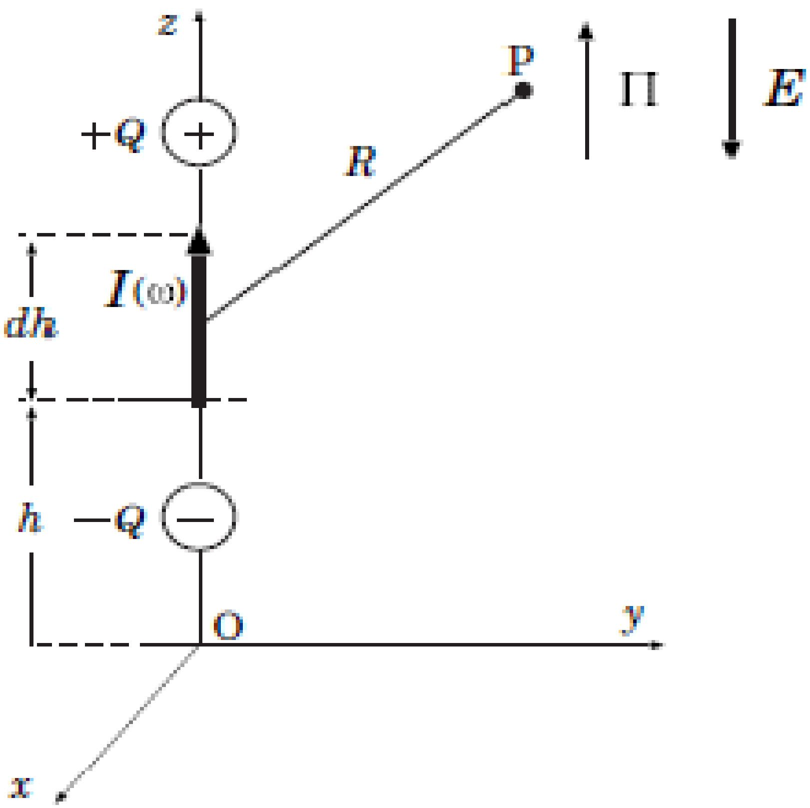


Dipole antenna



How to obtain Jordan's formula: Hertz vector

Hertz dipole

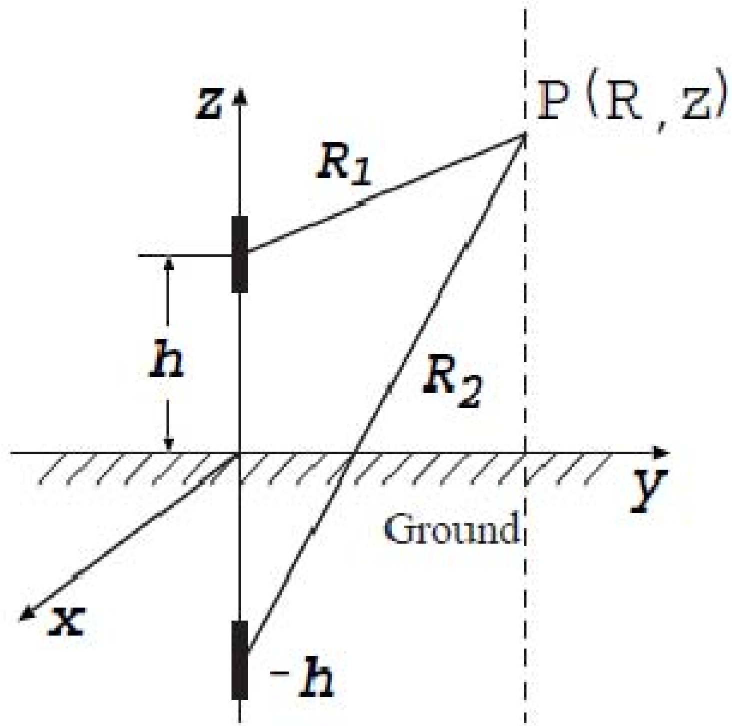


$$\Pi = \frac{j\omega\mu_0}{4\pi\gamma_0^2} I\, dh \frac{e^{-\gamma_0 R}}{R}$$

$$\gamma_0 = j\omega\sqrt{\varepsilon_0\mu_0}$$

$$j = \sqrt{-1}$$

$$E_z \simeq -\gamma_0^2 \Pi_z$$


z
P (R , z)
R_1
h
R_2
y
Ground
x
-h

Derivation of C. A. Jordan's formula

Hertz vector Π at $\mathsf{P}(R, z)$ by small current Idh with height h on z-axis.

$$\begin{aligned}\mathbf{\Pi}_z &= j\omega\frac{\mu_0}{4\pi\gamma_0^2}Idh\left(\frac{e^{-\gamma_0 R_1}}{R_1}-\frac{e^{-\gamma_0 R_2}}{R_2}\right)\\ &\simeq j\omega\frac{\mu_0}{4\pi\gamma_0^2}Idh\left(\frac{1}{R_1}-\frac{1}{R_2}\right)\end{aligned}$$

$R_1 = \sqrt{R^2+(z-h)^2},\ R_2 = \sqrt{R^2+(z+h)^2}$

$|\gamma_0 R_1|\,,\ \ |\gamma_0 R_2| \ll 1, \quad \gamma_0 = j\omega\sqrt{\varepsilon_0\mu_0} = j\omega/c_0,\ \ j = \sqrt{-1}$

μ_0: permeability, ε_0: permittivity, c_0: light velocity

I: source current, ω: source angular frequency

Derivation of C. A. Jordan's formula

Hertz vector Π at P(R, z) generated by small current Idh with height h on z-axis:

$$\begin{aligned} \Pi_z &= j\omega\frac{\mu_0}{4\pi\gamma_0^2}Idh\left(\frac{e^{-\gamma_0 R_1}}{R_1} - \frac{e^{-\gamma_0 R_2}}{R_2}\right) \\ &\simeq j\omega\frac{\mu_0}{4\pi\gamma_0^2}Idh\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \end{aligned}$$

$R_1 = \sqrt{R^2 + (z-h)^2}$, $R_2 = \sqrt{R^2 + (z+h)^2}$

$|\gamma_0 R_1|$, $|\gamma_0 R_2| \ll 1$, $\gamma_0 = j\omega\sqrt{\varepsilon_0\mu_0} = j\omega/c_0$, $j = \sqrt{-1}$

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μ_0: permeability, ε_0: permittivity, c_0: light velocity

I: source current, ω: source angular frequency

Derivation of C. A. Jordan's formula (cont. 1)

Electric field : $E_z \simeq -\gamma_0^2 \Pi_z$

By taking line integral of E_z along z axis, we obtain the voltage V. The impedance Z_e of the cylinder is calculated as V/I:

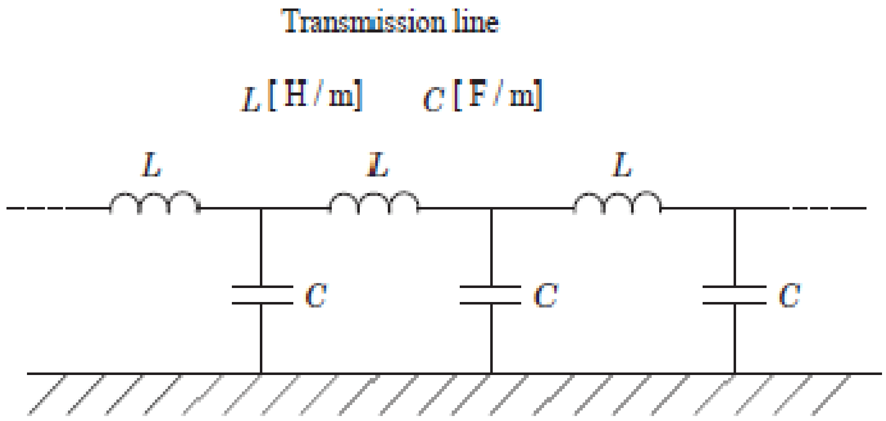
$$
\begin{aligned}
Z_e &= j\omega \frac{\mu_0}{4\pi} \int_0^H dz \int_0^H \left\{ \frac{1}{\sqrt{R^2 + (z-h)^2}} - \frac{1}{\sqrt{R^2 + (z+h)^2}} \right\} dh \\
&\simeq j\omega \underbrace{\frac{60}{c_0} H \left\{ \log\left(\frac{H}{R}\right) + \frac{3}{2}\left(\frac{R}{H}\right) - 1 \right\}}_{L_J} \; (R \ll H) \equiv j\omega L_J
\end{aligned}
$$

Derivation of C. A. Jordan's formula (cont. 2)

He borrowed from transmission-line theory:

Line surge impedance $z = \sqrt{\frac{L}{C}}$, $\quad c_0 = \frac{1}{\sqrt{LC}} \rightarrow z = c_0 L$,

where L, C are inductance and capacitance per meter of the line.



Derivation of C. A. Jordan's formula (cont. 3)

We obtain C. A. Jordan's formula:

$$Z_J = c_o \times \frac{L_J}{H} = 60\log(\frac{H}{R}) + 90(\frac{R}{H}) - 60$$

$$= 138.2\log_{10}(\frac{H}{R}) + 90(\frac{R}{H}) - 60\ [\Omega]$$

1. L. V. Bewley, "*Travelling waves on transmission systems*," John Wiley and Sons, 1951.
2. J. G. Anderson and J. H. Hagenguth, "Magnetic field around a transmission line tower," *AIEE Trans.* 78, p. 1644, 1959.
3. M. A. Sargent and M. Darveniza, "Tower surge impedance," *IEEE Trans. PAS-88*, p. 680, 1969.

このような経験から，鉄塔のサージインピーダンスを考慮すべきであるという方向と，雷遮へいに関する新しい考え方の必要性が認識された．前節の A-W 理論もその結果生まれてきたものである．

進行波理論にしろ電磁界理論にしろ，鉄塔を接地抵抗に比べてかなり大きいインピーダンスとして表現すると，雷道インピーダンス Z_0 を∞とするのは適切とはいえない．それなら，200 Ω と 400 Ω のどちらが正しいのかということを判定できるほど現在でも雷研究が進んでいるとはいえない．一応 400 Ω が使われるのは，200 Ω よりは鉄塔電位が高くなって安全側であること，気球から金属線をつり下げた実験では 300〜700 Ω 程度の値が得られていること〔26〕，通常の導体は（半径がよほど大きくない限り）400〜500 Ω 程度のサージインピーダンスを示すこと，電磁波が伝搬するときの空間のインピーダンスは約 400 Ω であること，などが理由と思われる．

鉄塔のサージインピーダンス Z_t についてはわが国では 100 Ω 程度，進行波の伝搬速度を光速の約 70% ととっている〔10〕．これについても1930 年代から研究があり，Jordan は次の式を導いた〔26〕．

$$Z_t = 138.2 \log_{10} \frac{h}{r} + 90 \frac{r}{h} - 60 \; [\Omega] \tag{3-13}$$

鉄塔を半径 r，高さ h の垂直円筒と考えてインダクタンスを計算し，進行波の伝搬速度 v を光速と考えると，式 (3-10) と次の

$$v = \frac{1}{\sqrt{LC}} \tag{3-14}$$

から，Z が求められる．式 (3-13) はこのような仮定で導出されたが，110〜130 Ω くらいになり，今日用いている値に近い．

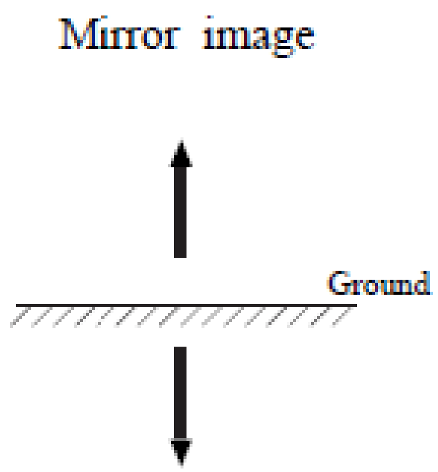
さきに述べた C.F. Wagner 氏の一連の論文は，電磁界理論から送電線雷撃現象を解析したものであるが，これを簡易化し進行波理論に等価的に結びつけた場合の Z_t として，次式が導かれた〔27〕．

$$Z_t = 138.2 \log_{10} \frac{2\sqrt{2}\, h}{r} \; [\Omega] \tag{3-15}$$

この式は，理論的筋道としては正しそうなのであるが，300 Ω 近い値となっ

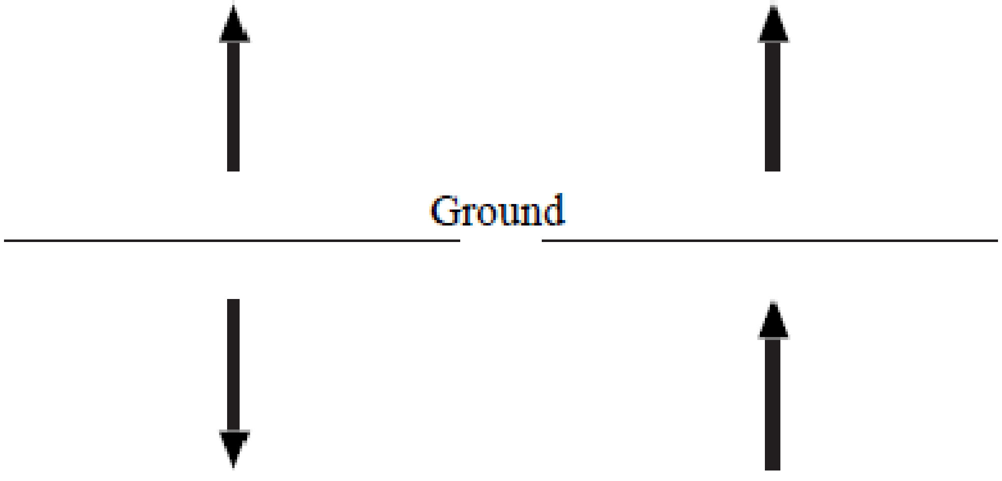
However, C. A. Jordan's formula is erroneous.

What mistake did C. A. Jordan make ?



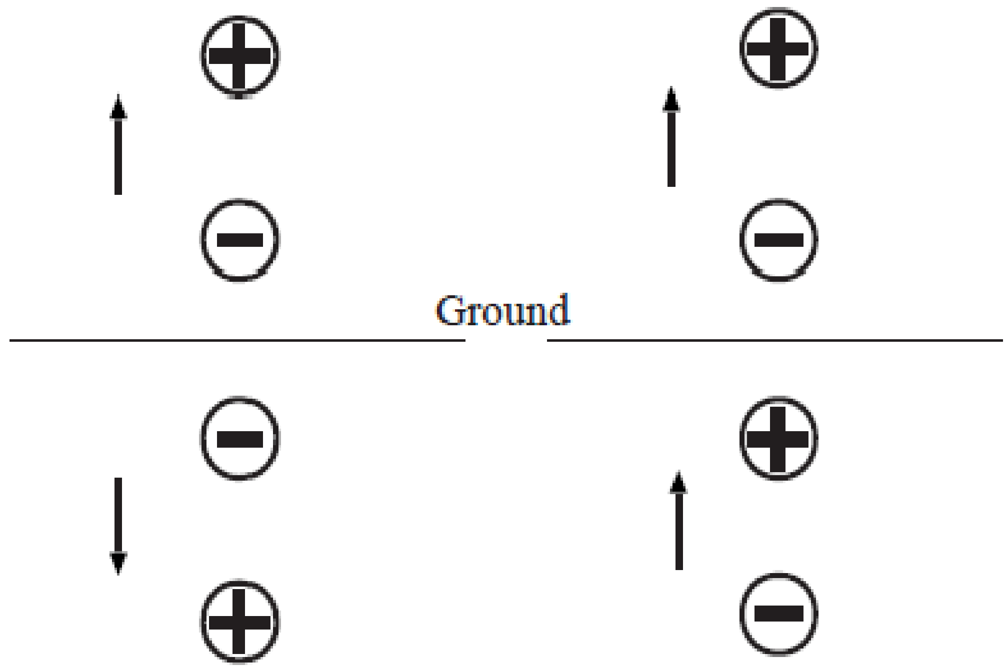
• He took erroneous electric image and had taken the + sign instead of − sign. The terminology 'mirror image' caused a confusion.

Mirror image



C. A. Jordan

Mirror image correction



C. A. Jordan   Wrong   Correct

Correction, let us take the + sign!

$$\Pi_z = j\omega \frac{\mu_0}{4\pi\gamma_0^2} I\,dh\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \quad j = \sqrt{-1}$$

$$\begin{aligned}
Z_e &= j\omega \frac{\mu_0}{4\pi} \int_0^H dz \int_0^H \left\{ \frac{1}{\sqrt{R^2 + (z-h)^2}} + \frac{1}{\sqrt{R^2 + (z+h)^2}} \right\} dh \\
&= j\omega \underbrace{\frac{\mu_0}{2\pi} H \left\{ \log\left(\frac{2H}{R} + \sqrt{\left(\frac{2H}{R}\right)^2 + 1}\right) - \sqrt{1 + \left(\frac{R}{2H}\right)^2} + \frac{R}{2H} \right\}}_{L} \\
&= j\omega L \\
Z &= c_0 \times \frac{L}{H} \simeq 60\left\{\log\left(\frac{4H}{R}\right) - 1\right)\}[\Omega] \quad (R \ll H)
\end{aligned}$$

Correction: let us take the + sign!

$$\Pi_z = j\omega\frac{\mu_0}{4\pi\gamma_0^2} I\,dh\left(\frac{1}{R_1}+\frac{1}{R_2}\right) \quad j=\sqrt{-1}$$

$$\begin{aligned}
Z_e &= j\omega\frac{\mu_0}{4\pi}\int_0^H dz\int_0^H\left\{\frac{1}{\sqrt{R^2+(z-h)^2}}+\frac{1}{\sqrt{R^2+(z+h)^2}}\right\}dh \\
&= j\omega\underbrace{\frac{\mu_0}{2\pi}H\left\{\log\left(\frac{2H}{R}+\sqrt{\left(\frac{2H}{R}\right)^2+1}\right)-\sqrt{1+\left(\frac{R}{2H}\right)^2}+\frac{R}{2H}\right\}}_{L} \\
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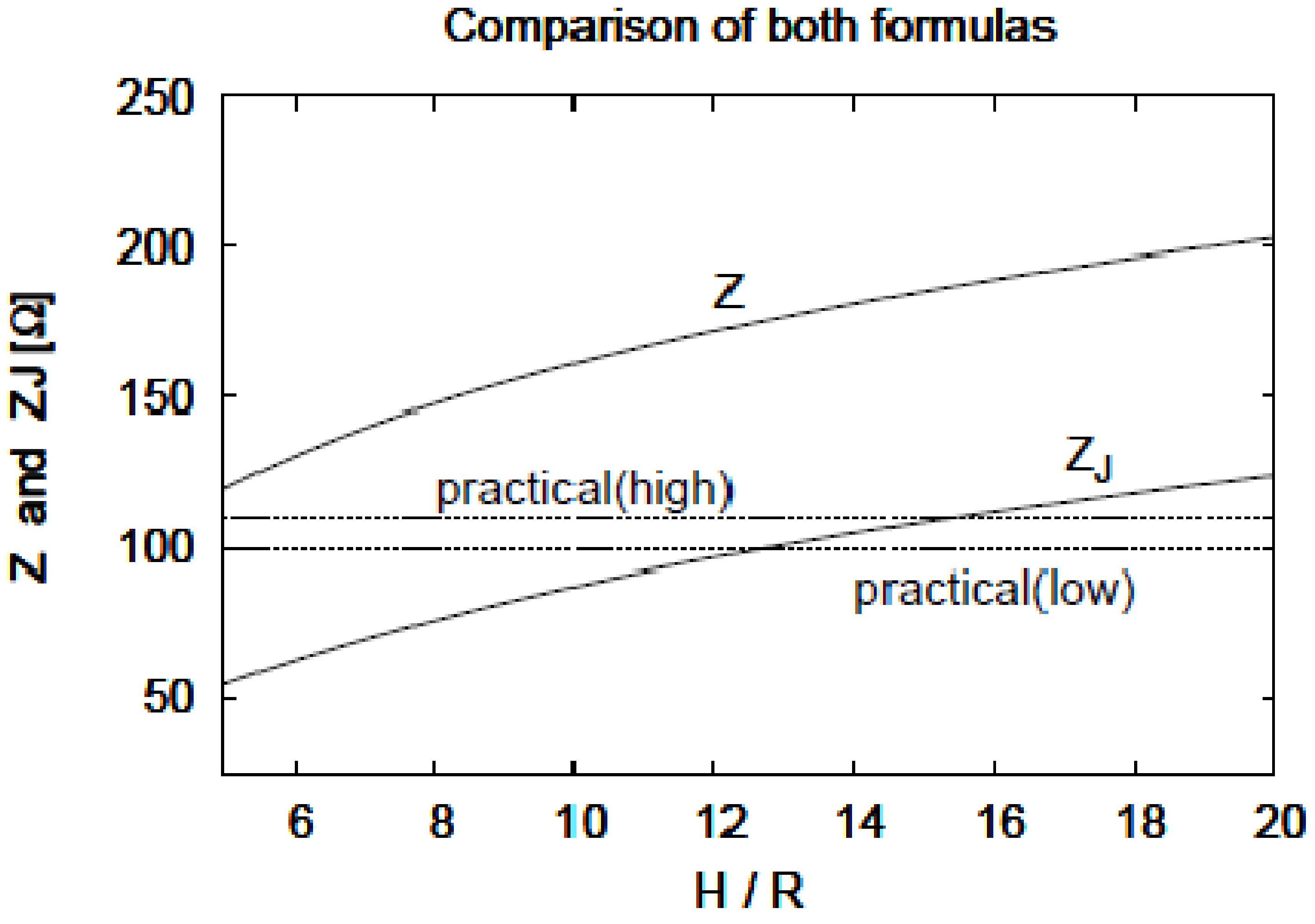


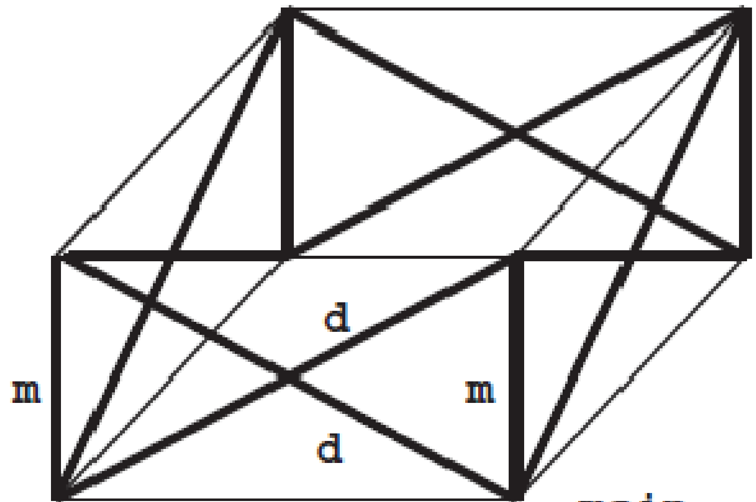
Comparison of both formulas
250
200
150
100
50
Z and ZJ [Ω]
6
8
10
12
14
16
18
20
H / R
Z
Z_J
practical(high)
practical(low)

Improvement by introducing lattice structure

- The corrected formula Z does not produce good values for the tower surge impedance. The values are always far larger than the experimental data.
- C. A. Jordan did not consider the structure of the tower.
- We introduce the lattice structure of the tower to improve the formula.
- The lattice structure is composed of piles of several unit sections.

Lattice structure and unit section

3D

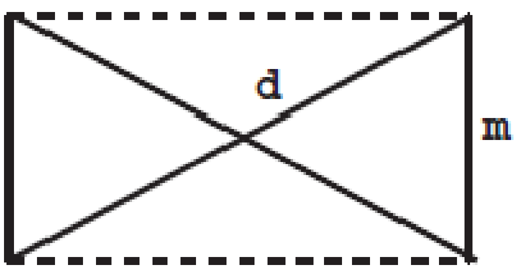


lattice structure

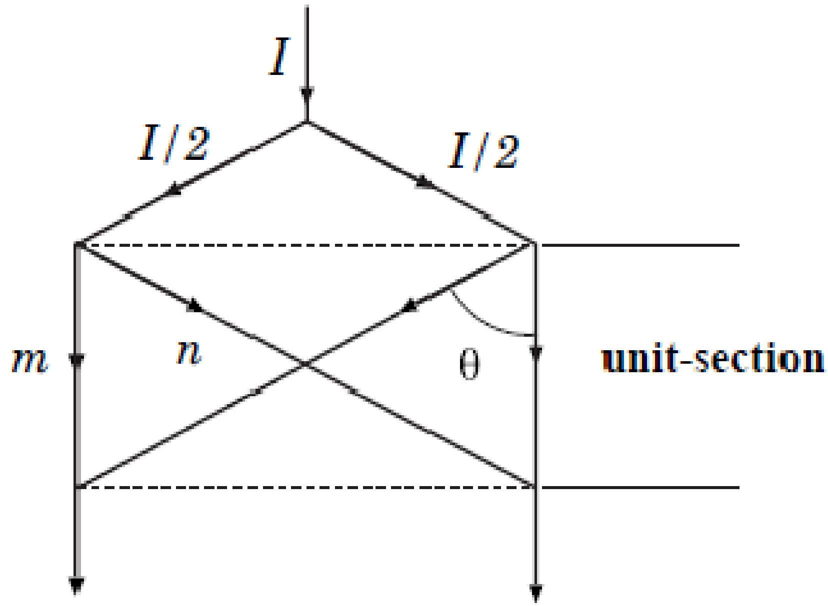
main = 4

diagonal = 8

2D



unit section



Shunt ratio：

$$a = \frac{n}{m} = \frac{n}{m+n} : \frac{m}{m+n}$$

$50 < \theta < 60$ deg

Improvement of the formula

Introducing lattice structure:

At the top of tower, current I is equally divided into two currents I_0 $(I_0 = I/2)$.

In the unit section, I_0 is divided with the ratio $n : m$.

Current through main beam: $\dfrac{m}{m+n} I_0$

Current through diagonal beam: $\dfrac{n}{m+n} I_0$

$$I_0 = \left(\frac{m}{m+n} + \frac{n}{m+n} \right) I_0$$

Improvement of the formula (cont. 1)

Currents through 2 unit-sections:

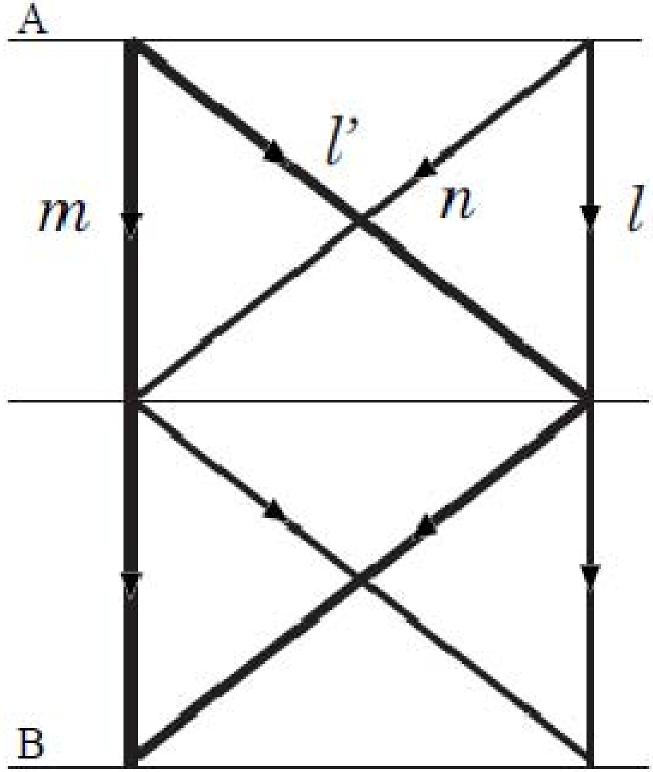
$$I(2) = \left(\frac{m}{m+n} + \frac{n}{m+n}\right)^2 I_0$$

$$= \underbrace{\left(\frac{m}{m+n}\right)^2 I_0}_{I_0(2)} + \underbrace{2\left(\frac{m}{m+n}\right)\left(\frac{n}{m+n}\right) I_0}_{I_1(2)} + \underbrace{\left(\frac{n}{m+n}\right)^2 I_0}_{I_2(2)}$$

$$= I_0(2) + I_1(2) + I_2(2)$$

Current I_0 is divided into flows through: main beams only ($I_0(2)$), main and diagonal beams ($I_1(2)$), and diagonal beams only ($I_2(2)$).

Piling up 2 unit-sections



$l \to l$

$l' \to l'$

$l \to l'$

$l' \to l$

Improvement of the formula (cont. 2)

Currents through k unit-sections are given by:

$$I(k) = \left(\frac{m}{m+n} + \frac{n}{m+n}\right)^{k} I_0 = \sum_{i=0}^{k} \underbrace{{}_kC_i \left(\frac{m}{m+n}\right)^{k-i} \left(\frac{n}{m+n}\right)^{i}}_{D_i(k)} I_0$$

$$= \sum_{i=0}^{k} I_i(k) \quad \text{where } I_i(k) = D_i(k) I_0$$

There are $k+1$ components $I_0(k), \cdots, I_k(k)$. To the flow of i th component, main beams contribute $k-i (i = 0, \cdots, k)$ times and diagonal beams contribute i times.

Improvement of the formula (cont. 3)

The current in the unit section has the diagonal beams. The calculation of electric field E_z perpendicular to the ground is very complicated. Hence, we introduce weight function defined by:

$$W_i(k) = (k - i + i\cos\theta)/k$$

Including the image, there are $2N$ unit sections for N unit sections. Since $I = 2I_0$, we approximate the current vertical to the ground as:

$$\hat{I}_{2N} = \sum_{i=0}^{2N} W_i(2N) I_i(2N) = \underbrace{\{\sum_{i=0}^{2N} W_i(2N) D_i(2N)\}}_{\delta} I$$

Improvement of the formula (cont. 4)

$$
\begin{aligned}
\delta &= \sum_{i=0}^{2N} W_i(2N) D_i(2N) \\
&= \sum_{i=0}^{2N} \frac{1}{2N}(2N - i + i\cos\theta)_{2N}C_i \left(\frac{m}{m+n}\right)^{2N-i} \left(\frac{n}{m+n}\right)^{i} \\
&= \frac{1 + a\cos\theta}{1+a} \qquad a = \frac{n}{m}
\end{aligned}
$$

If currents flow equally through beams, $a = 8/4 = 2.$

Improvement of the formula (cont. 5)

The improved formula:

$$Z_T = \delta 60\{\log(\frac{4H}{R}) - 1\} = \delta Z \ [\Omega] \quad (R \ll H)$$

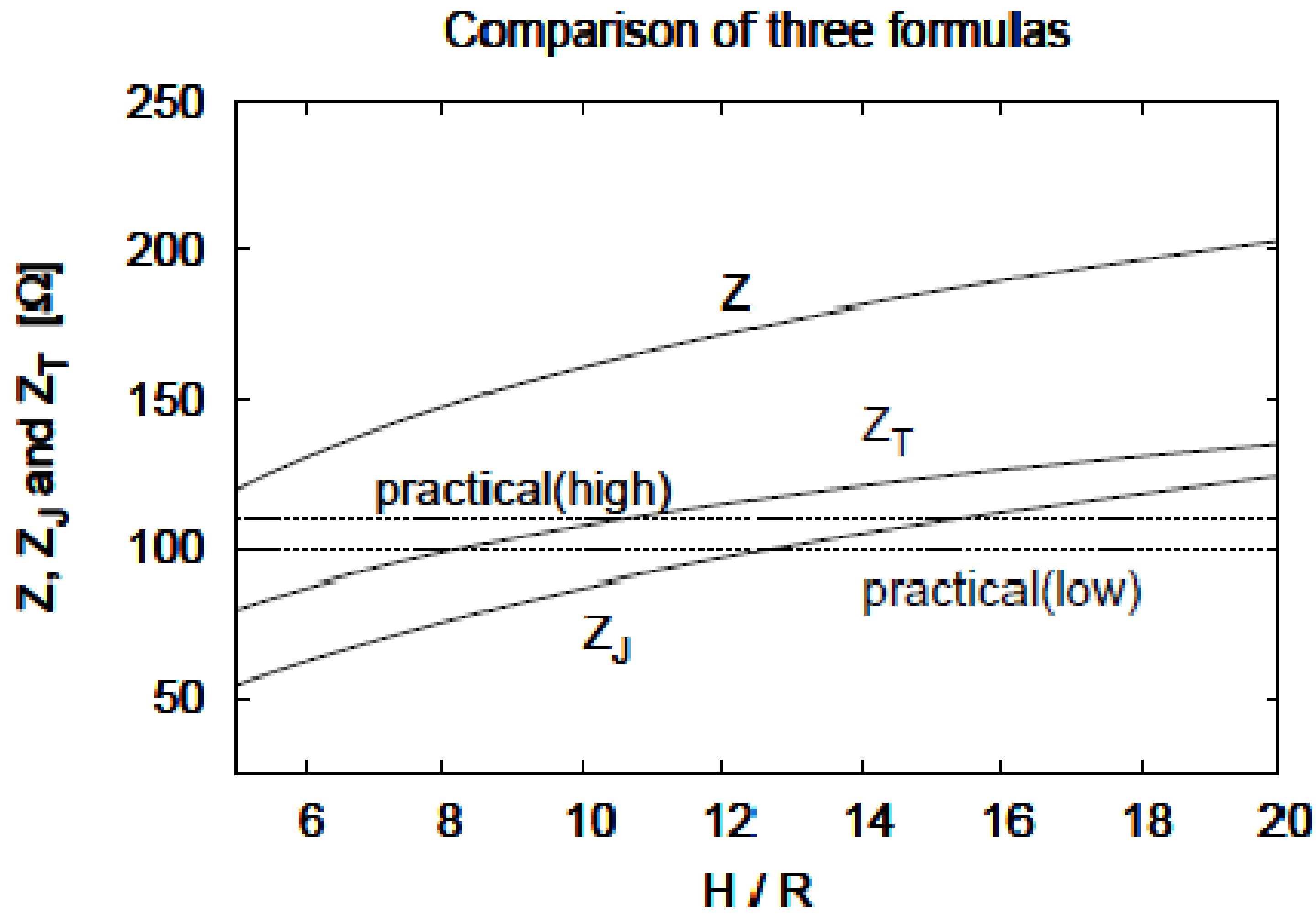
Corrected formula:

$$Z = 60\{\log(\frac{4H}{R}) - 1\} \ [\Omega]$$

C. A. Jordan's formula:

$$Z_J = 60\log(\frac{H}{R}) + 90(\frac{R}{H}) - 60 \ [\Omega]$$

We now compare these formulas with the experimental data.



Comparison of three formulas
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H / R
Z, Z_J and Z_T [Ω]
Z
Z_T
practical(high)
practical(low)
Z_J

Concluding remarks

- C. A. Jordan's formula has been found to be erroneous. However, he suggested the cylinder model as an easy way to calculate tower surge impedance.

- Main reasons for disagreement with experimental data are:

1. Current source $I(\omega)$ is not the wave form of lightning current.
2. Using the theory of transmission line parallel rather than perpendicular to the ground.
3. Current velocity in the tower $(0.7 - 0.9) \times c_0$ is experimental data and could not be theoretically proved.

Acknowledgement

I appreciate the kindness of Director Veselin Jungic and Professor Ljiljana Trajkovic who invited me to the IRMACS Centre and provided an opportunity for this talk.

Also I give my sincere thanks to Professor Emeritus Akira Kishima at Kyoto Univesity, who supported my research continuously since I was the undergraduate student at the same university.