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Digital Signal Processing Group

- Design of quadrature mirror filter banks
- Signal reconstruction from non-uniformly distributed samples
- Location of objects in a JPEG image
- Watermarking

Estimation of Discrete Wavelet Transform Coefficients of Non-Uniformly Sampled Signals

C. Ford and D. M. Etter,
“Wavelet Basis Reconstruction of Nonuniformly Sampled Data,”
IEEE Transactions on Circuits and Systems II,
vol. 45, pp. 573-576, August 1995.

A filterbank is used for estimation of DWT coefficients.

The overdetermined linear systems of equations are solved by the conjugate gradient method.

Sparse matrix techniques are applied to reduce the number of floating point operations.

How to reconstruct the uniformly sampled signal

$\mathbf{x} = [\, x_0, x_1, \ldots , x_{N-1}]^{\mathrm{T}}$

from a given irregularly sampled signal

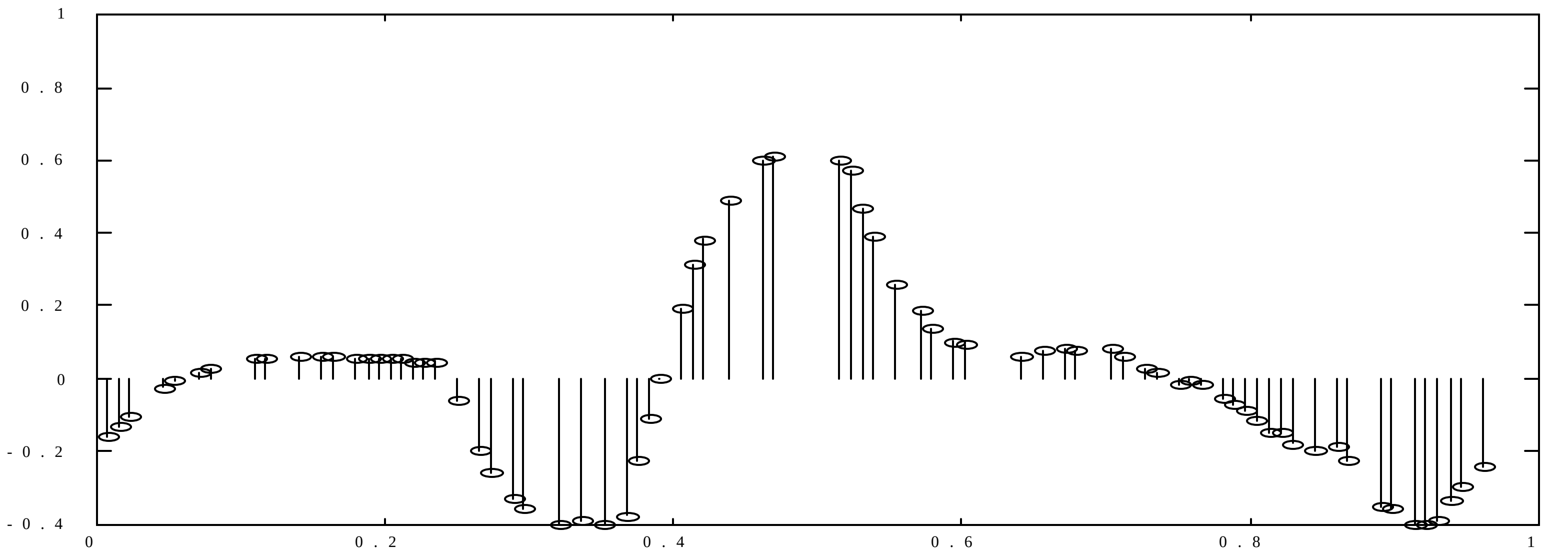
$\mathbf{y} = [\, y_0, y_1, \ldots , y_{P-1}]^{\mathrm{T}}, \quad N/2 < P < N$

$y_i \in \{\, x_0, x_1, \ldots , x_{N-1}\}, \quad i = 0, 1, \ldots , P-1?$

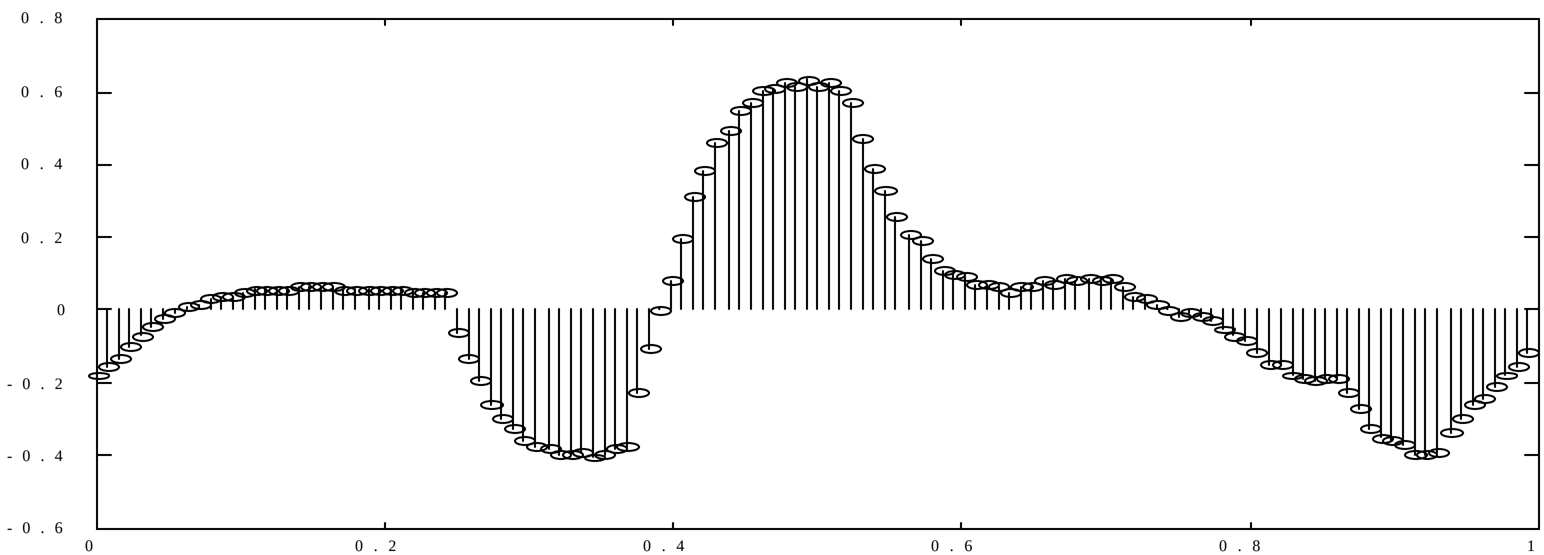
The goal is to obtain a signal

$\mathbf{x'} = [\, x'_0, x'_1, \ldots , x'_{N-1}]^{\mathrm{T}}$

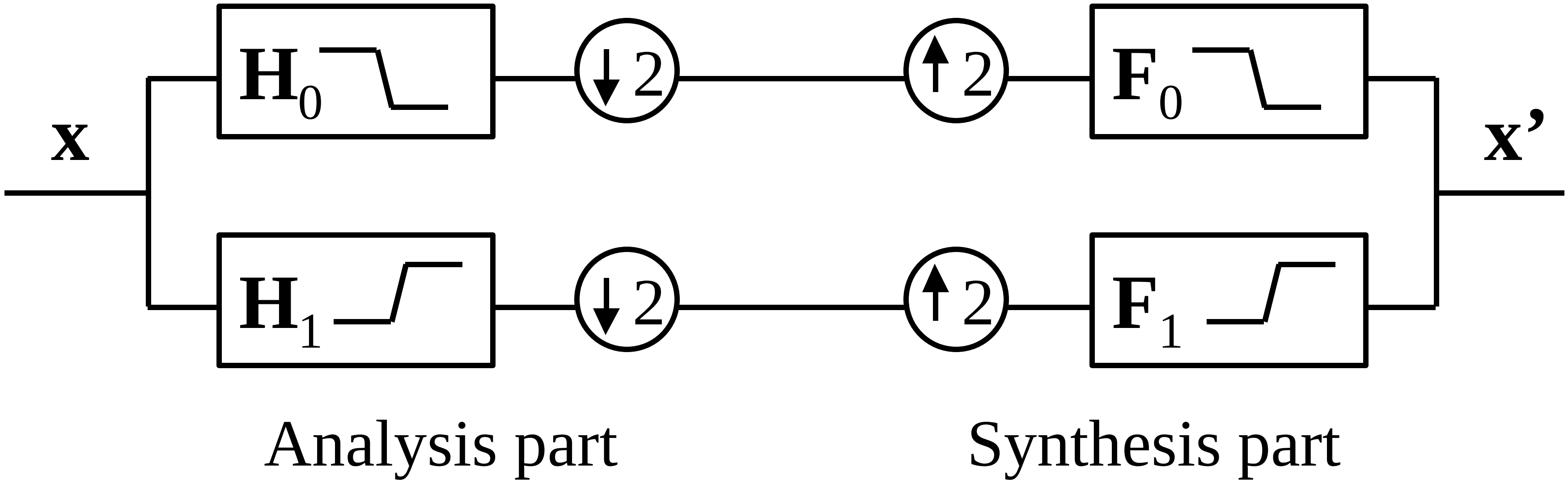
that is the best approximation of the uniformly sampled signal $\mathbf{x}$.



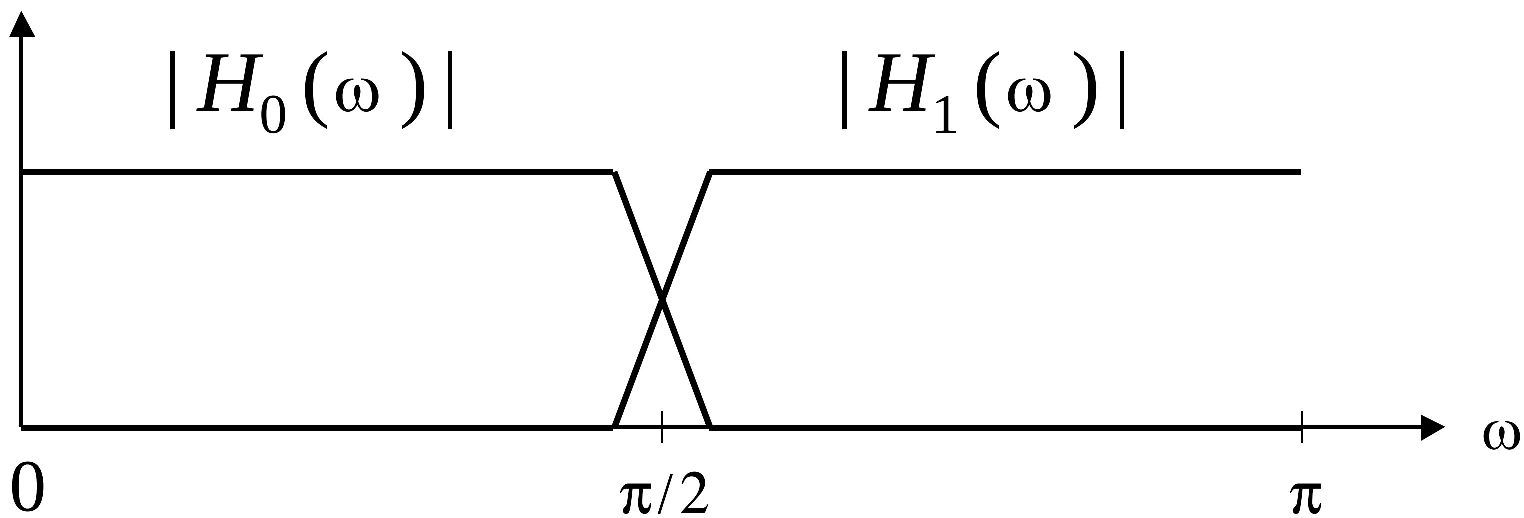
Non-uniformly sampled signal **y** (79 samples)

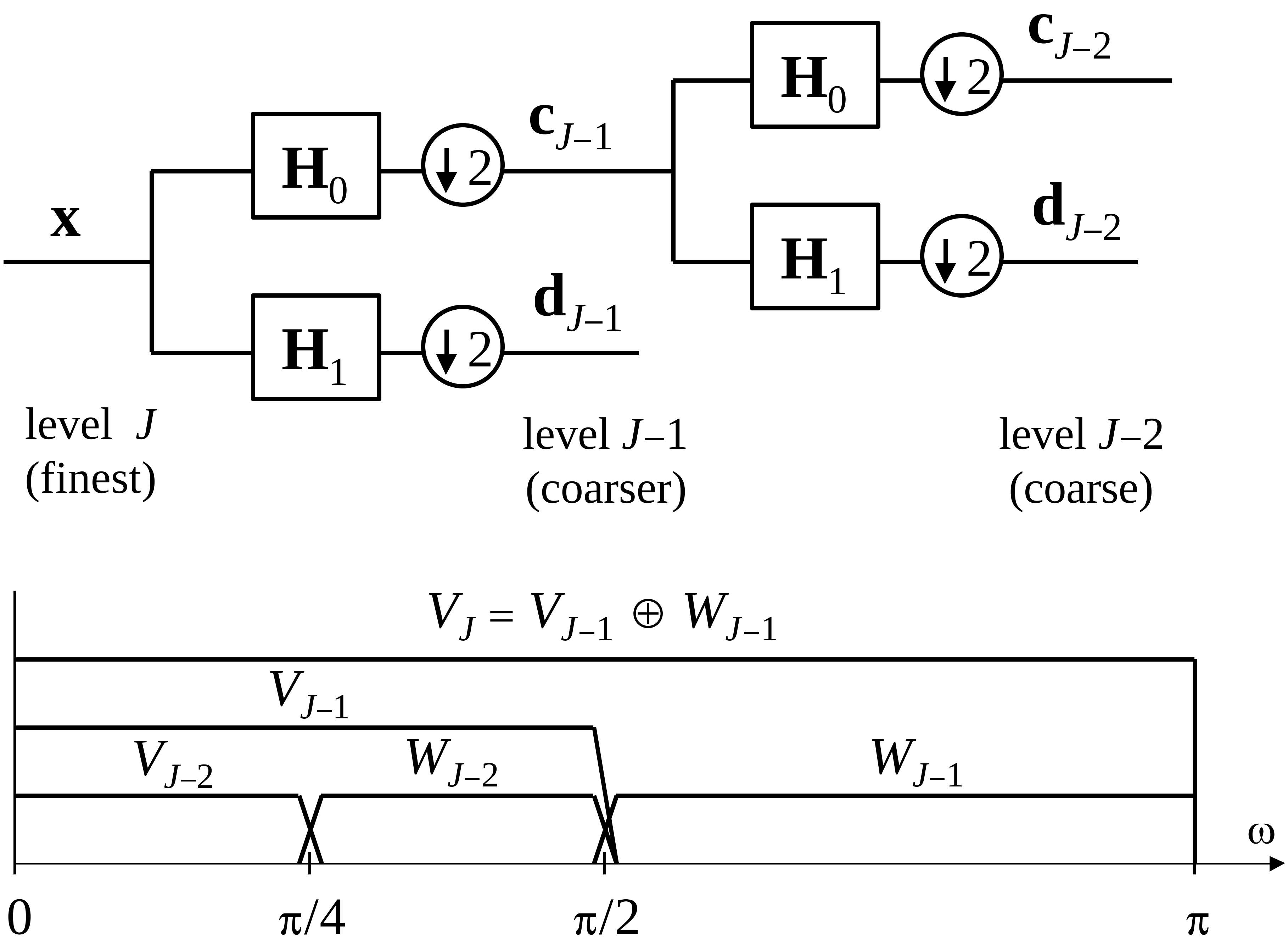


Uniformly sampled signal **x** (128 samples)

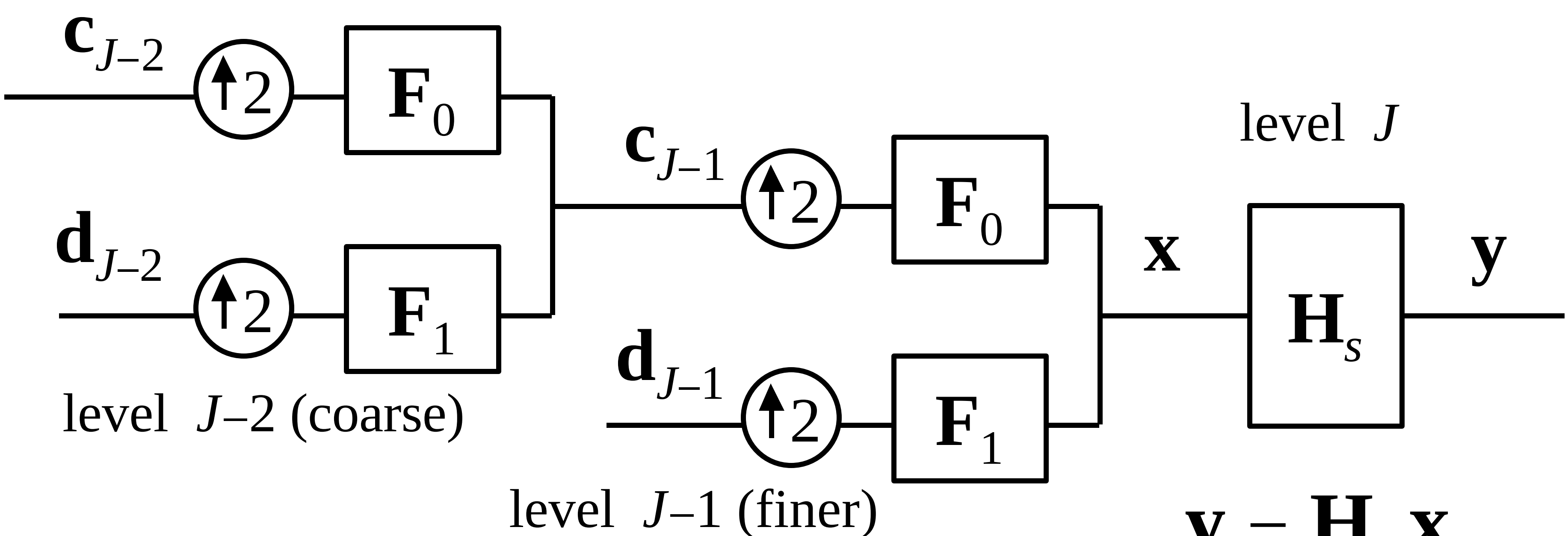


Quadrature Mirror Filter (QMF) bank





x
H0
H1
↓2
cJ–1
dJ–1
cJ–2
dJ–2
level J
(finest)
level J–1
(coarser)
level J–2
(coarse)
VJ = VJ–1 ⊕ WJ–1
VJ–1
VJ–2
WJ–2
WJ–1
ω
0
π/4
π/2
π



$$\mathbf{y} = \mathbf{H}_s\,\mathbf{x}$$

($P \times N$ matrix $\mathbf{H}_s$ with entries 0 and 1, very sparse)

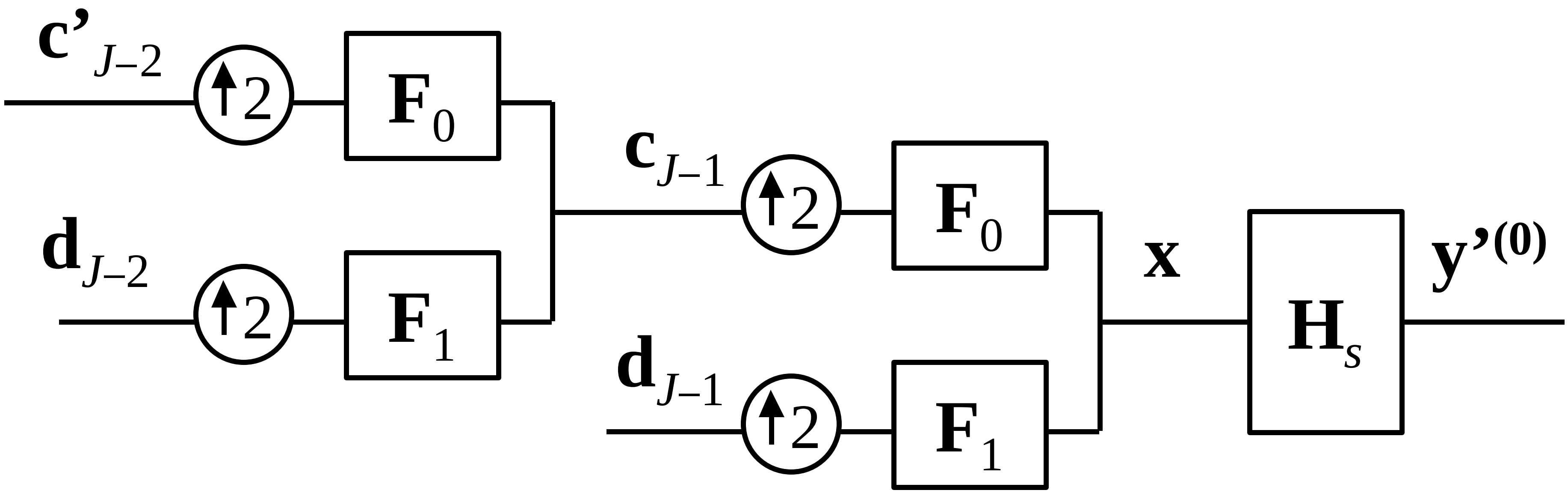
finer approximation of $\mathbf{y}$ (level J–2)

coarse approx. at level J–2 + details at level J–2

$$\mathbf{y} = \overbrace{\overbrace{\mathbf{H}_s\,\mathbf{F}_0\,(\uparrow 2)\,\mathbf{F}_0\,(\uparrow 2)\,\mathbf{c}_{J-2}} + \overbrace{\mathbf{H}_s\,\mathbf{F}_0\,(\uparrow 2)\,\mathbf{F}_1\,(\uparrow 2)\,\mathbf{d}_{J-2}}}$$

$$+\ \mathbf{H}_s\,\mathbf{F}_1\,(\uparrow 2)\,\mathbf{d}_{J-1}$$

details at level J–1

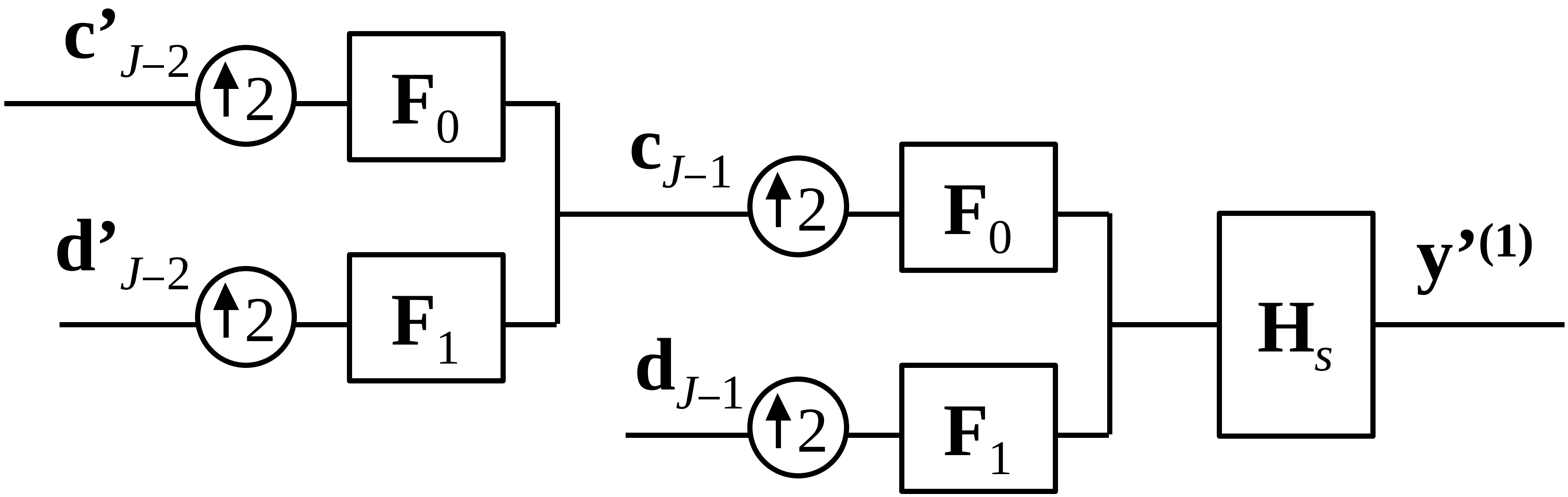


$\mathbf{H}_s\,\mathbf{F}_0\,(\uparrow 2)\,\mathbf{F}_0\,(\uparrow 2)\,\mathbf{c}_{J-2} = \mathbf{y}$ (overdetermined system)

(all details are ignored)

$\Rightarrow$ estimate $\mathbf{c}'_{J-2}$

$\Rightarrow$ first estimate $\mathbf{y}'^{(0)} = \mathbf{H}_s\,\mathbf{F}_0\,(\uparrow 2)\,\mathbf{F}_0\,(\uparrow 2)\,\mathbf{c}'_{J-2}$



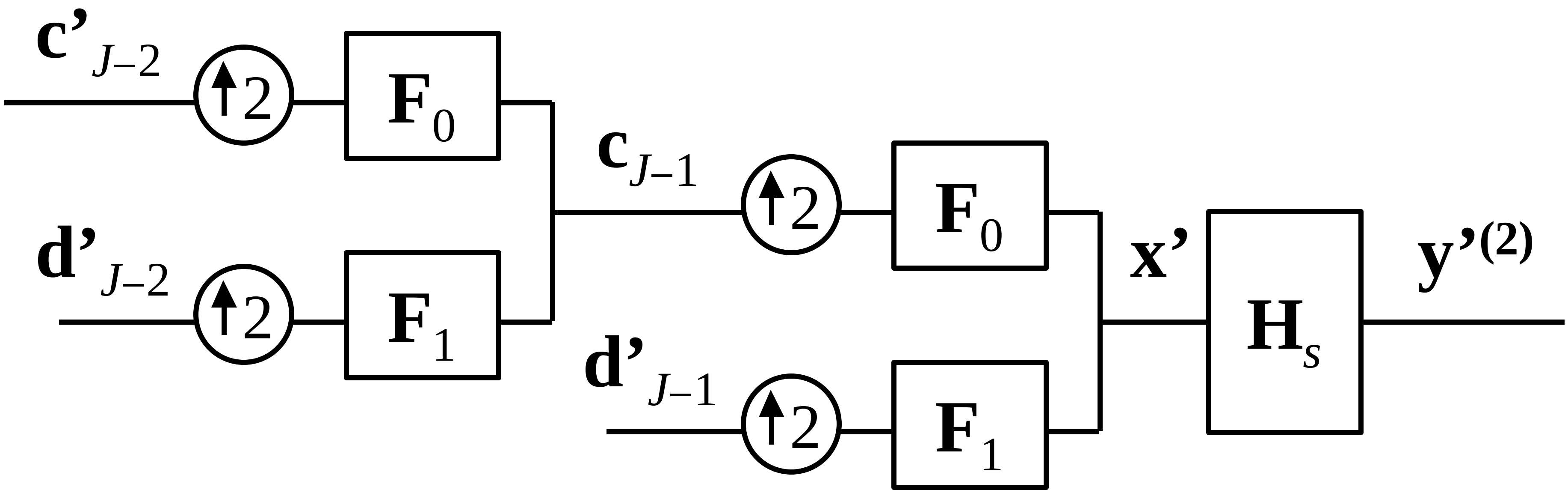
$\mathbf{e}^{(0)} = \mathbf{y} - \mathbf{y}'^{(0)}$ error (at each available sample)

$\mathbf{H}_s\, \mathbf{F}_0\, (\uparrow 2)\, \mathbf{F}_1 (\uparrow 2)\, \mathbf{d}_{J-2} = \mathbf{e}^{(0)}$ (overdetermined system)

(details at level J–1 are ignored)

$\Rightarrow$ estimate $\mathbf{d}'_{J-2}$ (details at level J–2)

$\Rightarrow$ first refinement $\mathbf{y}'^{(1)} = \mathbf{H}_s\, \mathbf{F}_0\, (\uparrow 2)\, \mathbf{F}_1\, (\uparrow 2)\, \mathbf{d}'_{J-2}$



$$\mathbf{e}^{(1)} = \mathbf{e}^{(0)} - \mathbf{y'}^{(1)} = \mathbf{y} - \mathbf{y'}^{(0)} - \mathbf{y'}^{(1)}$$

$$\mathbf{H}_s\,\mathbf{F}_1(\uparrow 2)\,\mathbf{d}_{J-1} = \mathbf{e}^{(1)} \quad \text{(overdetermined system)}$$

$\Rightarrow$ estimate $\mathbf{d'}_{J-1}$ (details at level J–1)

$$\mathbf{x'} = \mathbf{F}_0(\uparrow 2)\,\mathbf{F}_0(\uparrow 2)\,\mathbf{c'}_{J-2} + \mathbf{F}_0(\uparrow 2)\,\mathbf{F}_1(\uparrow 2)\,\mathbf{d'}_{J-2} + \mathbf{F}_1(\uparrow 2)\,\mathbf{d'}_{J-1}$$

Searching for solutions of overdetermined systems

Let $\mathbf{H}\,\mathbf{x} = \mathbf{c}$ is an overdetermined linear system

$$\mathbf{H}^{\mathrm{T}}\mathbf{H}\,\mathbf{x} = \mathbf{H}^{\mathrm{T}}\mathbf{c}$$

$\mathbf{A}\,\mathbf{x} = \mathbf{b}$ is a symmetric positive definite system

Conjugate Gradient (CG) method

extremely effective method when the coefficient matrix is symmetric positive definite

small storage requirements

converges in at most n iterations (n is the order of the system)

$$\mathbf{A}\,\mathbf{x} = \mathbf{b}$$

$\mathbf{x}^{(i)}$ iterates

$\mathbf{r}^{(i)} = \mathbf{b} - \mathbf{A}\,\mathbf{x}^{(i)}$ residuals

$\mathbf{p}^{(i)}$ search directions

α_i, β_i, update scalars

$$\mathbf{x}^{(i)} = \mathbf{x}^{(i-1)} + \alpha_i\,\mathbf{p}^{(i)}$$

$$\mathbf{r}^{(i)} = \mathbf{r}^{(i-1)} - \alpha\;\mathbf{A}\,\mathbf{p}^{(i)} \qquad (*)$$

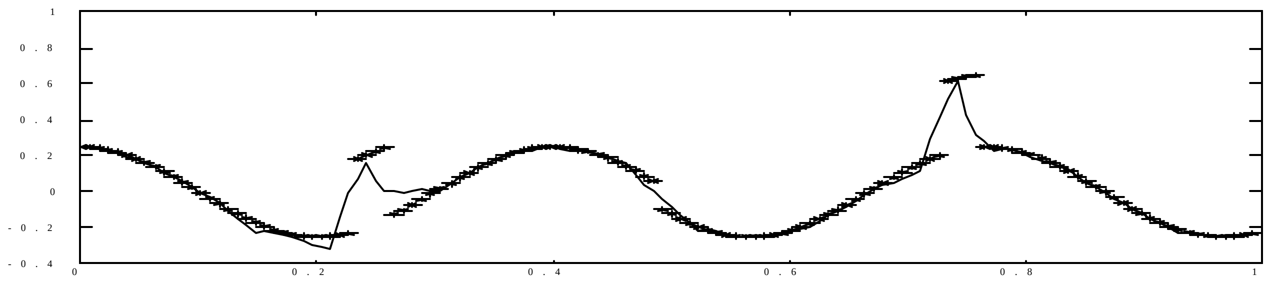
$$\alpha = \alpha_i = \frac{\langle\, \mathbf{r}^{(i-1)}, \mathbf{r}^{(i-1)} \rangle}{\langle\, \mathbf{p}^{(i)}, \mathbf{A}\,\mathbf{p}^{(i)} \rangle}$$ minimizes $\langle \mathbf{r}^{(i)}, \mathbf{A}^{-1}\mathbf{r}^{(i)} \rangle$

over all possible choices for α in the equation (*).

$$\mathbf{p}^{(i)} = \mathbf{r}^{(i-1)} + \beta_{i-1} \mathbf{p}^{(i-1)}$$

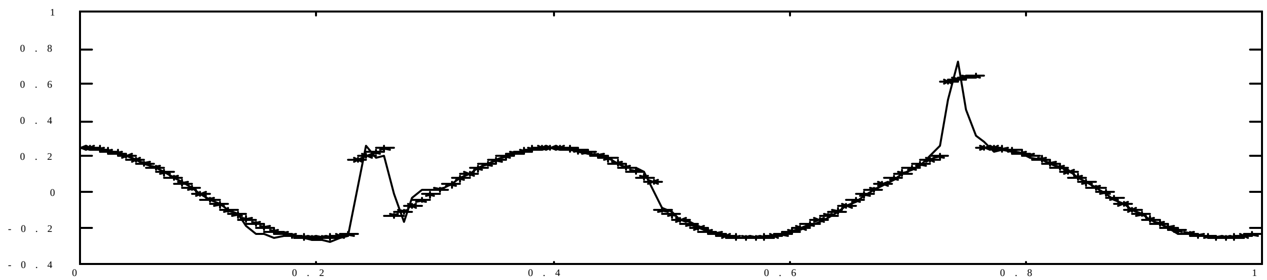
The choice $\beta_i = \dfrac{\langle \mathbf{r}^{(i)}, \mathbf{r}^{(i)} \rangle}{\langle \mathbf{r}^{(i-1)}, \mathbf{r}^{(i-1)} \rangle}$ ensures that $\mathbf{p}^{(i)}$ and $\mathbf{r}^{(i)}$ are orthogonal for <u>all</u> previous $\mathbf{A}\,\mathbf{p}^{(j)}$ and $\mathbf{r}^{(j)}$ respectively.

Convergence is achieved in at most *n* steps of iteration, where *n* is the dimension of the system.

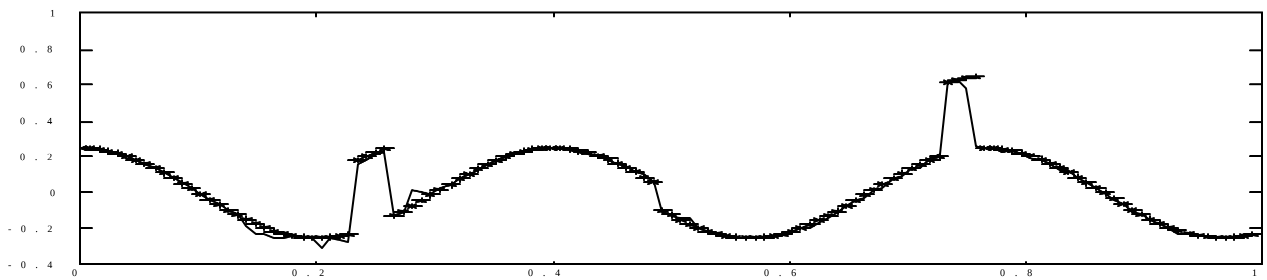


MCG

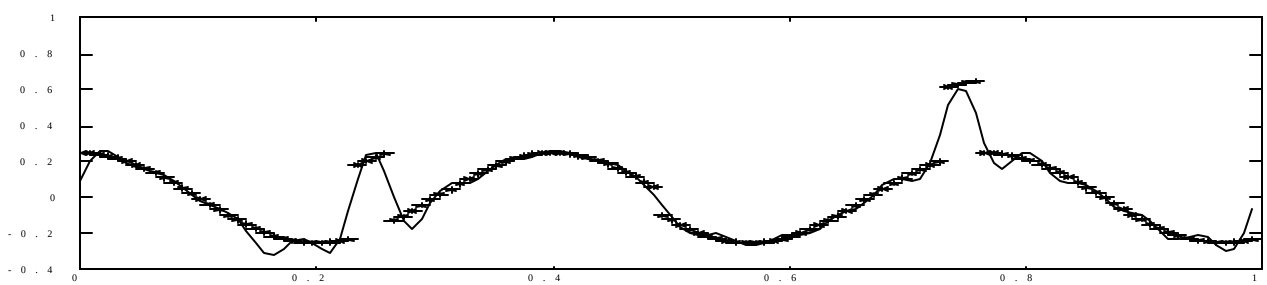
approximation
(level *J*–2)



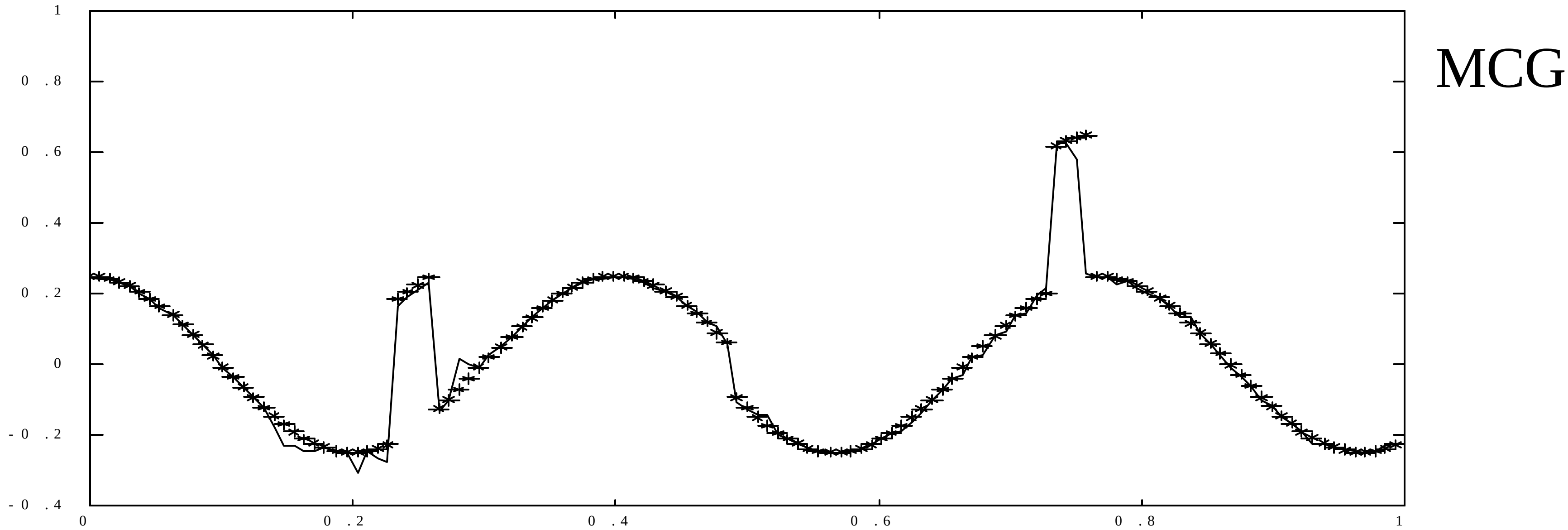
approximation
+ details at *J*–2



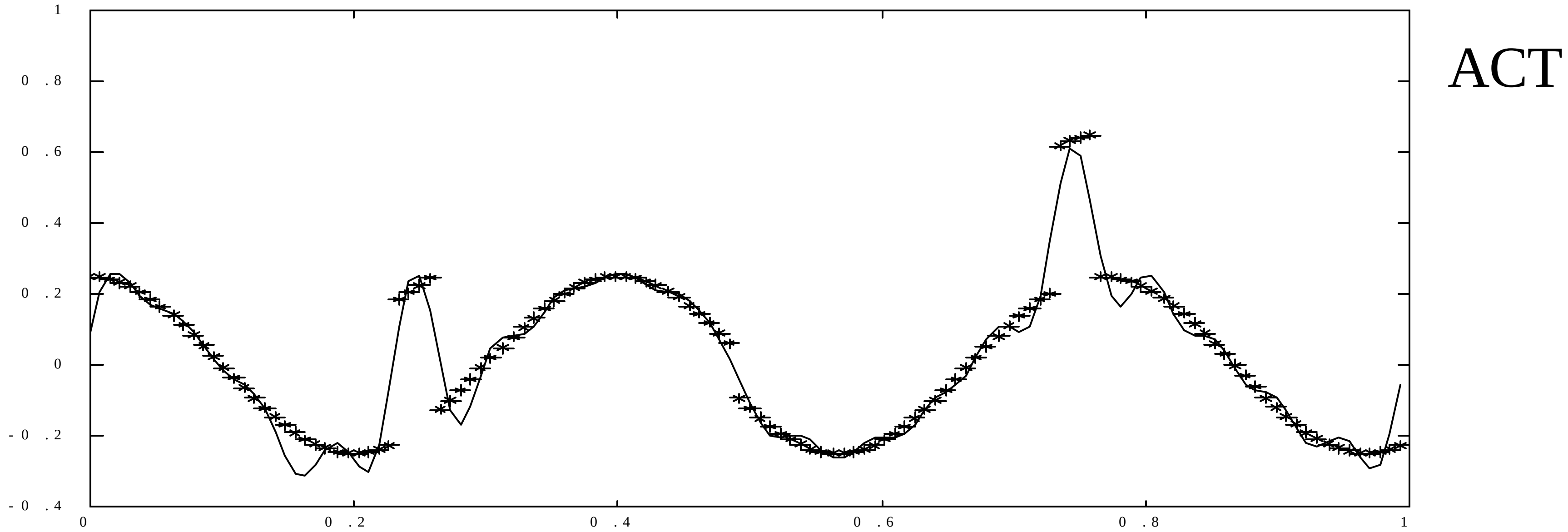
approximation
+ details at *J*–2
+ details at *J*–1



ACT



1
0 .8
0 .6
0 .4
0 .2
0
- 0 .2
- 0 .4
0
0 .2
0 .4
0 .6
0 .8
1
MCG



1
0 .8
0 .6
0 .4
0 .2
0
- 0 .2
- 0 .4
0
0 .2
0 .4
0 .6
0 .8
1
ACT

	$\|\mathbf{x}\|$	$\|\mathbf{x'}\|$	$\|\mathbf{x}-\mathbf{x'}\|$	flops
MCG	2.384	2.315	0.4316	94 184
MBFR	2.384	2.263	0.4044	4 858 788
ACT	2.384	2.320	0.5395	577 615

MCG Multiresolutional Based Conjugate Gradients

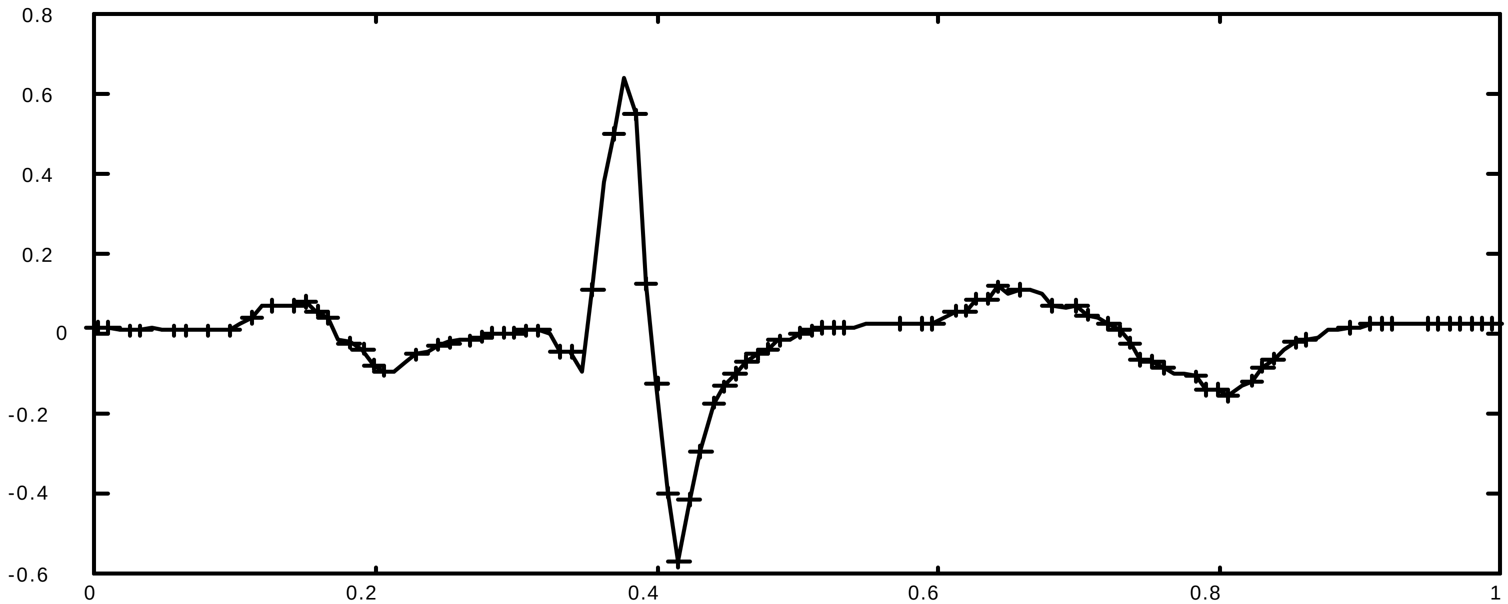
MBFR Multiresolutional Basis Fitting Reconstruction (Ford & Etter)

ACT Adaptive weights Conjugate gradient Toeplitz

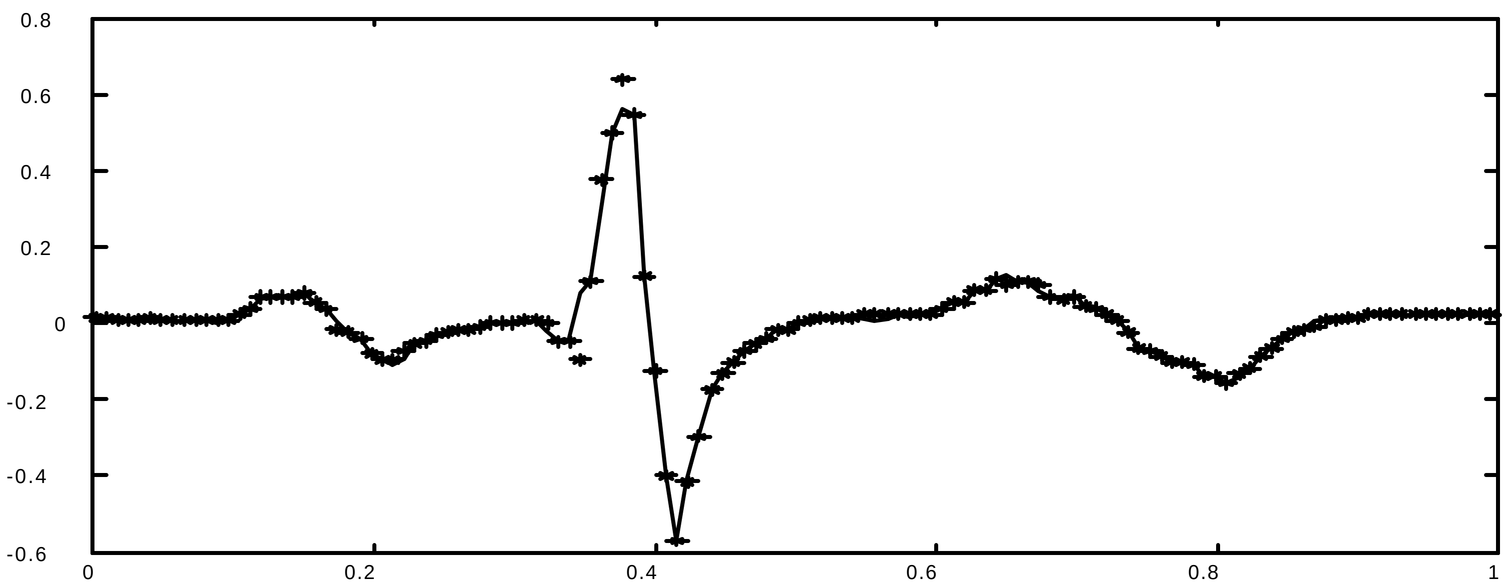
Fourier basis to set a linear system

CG method for solving the system

the system matrices have Toeplitz structure

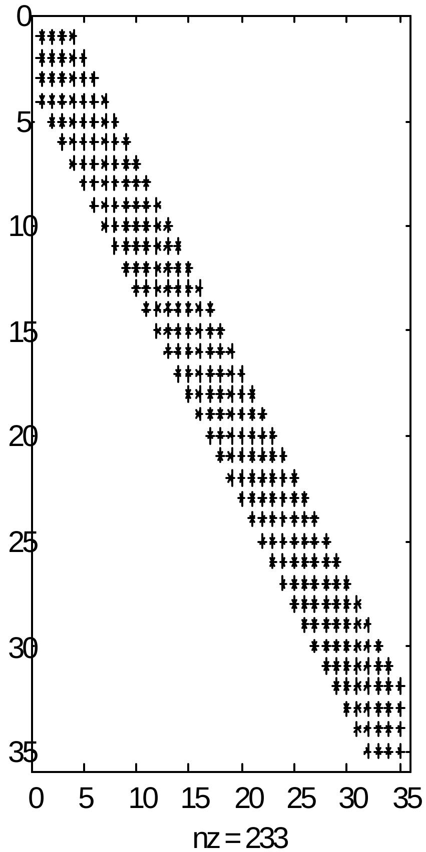

0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
0
0.2
0.4
0.6
0.8
1

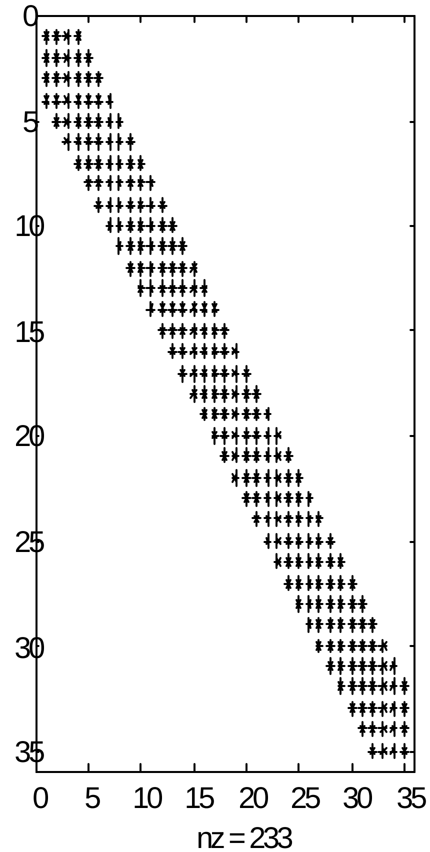
y

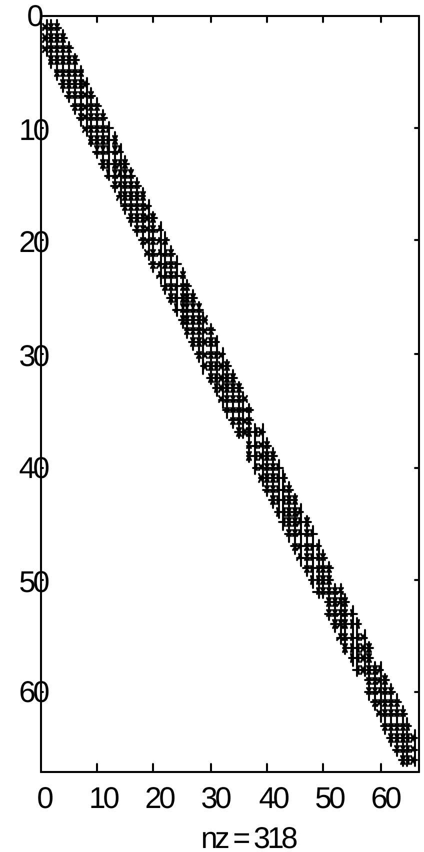

0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
0
0.2
0.4
0.6
0.8
1

x’

Sparse matrices

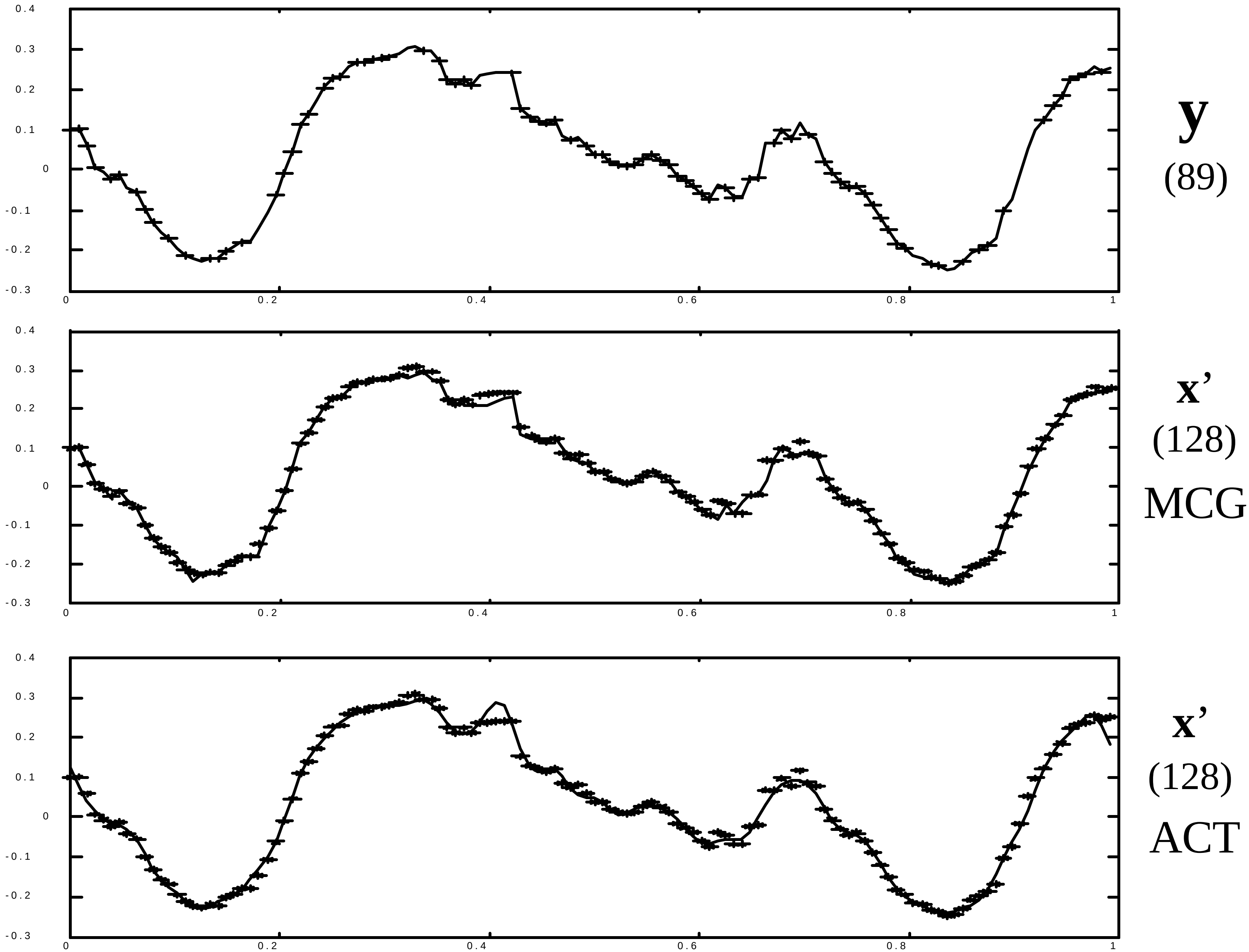






35 x 35,
233 non-zeros

70 x 70,
318 non-zeros



y
(89)
x'
(128)
MCG
x'
(128)
ACT

y length: 89 samples

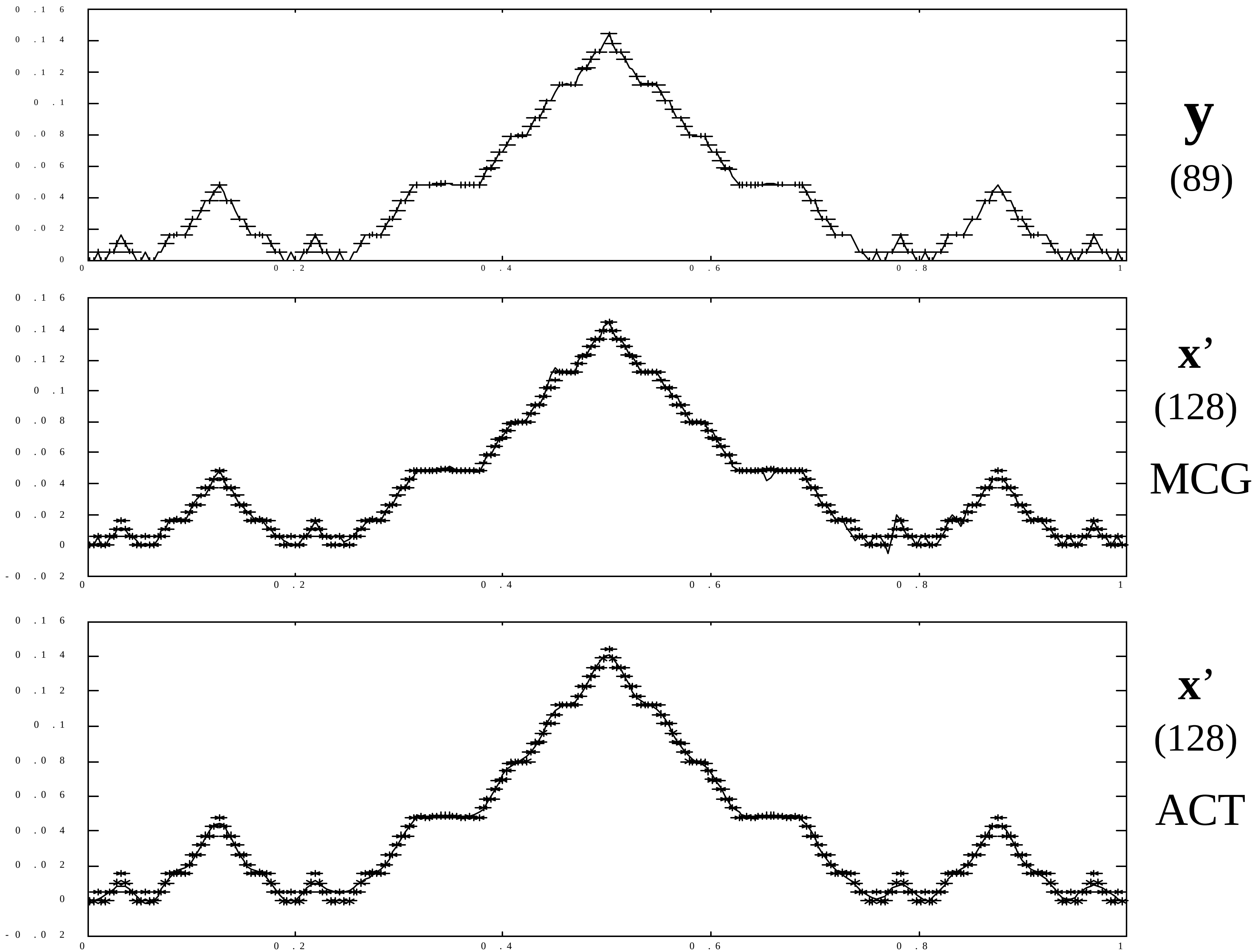
x' length: 128 samples

	$\|\mathbf{x}\|$	$\|\mathbf{x}'\|$	$\|\mathbf{x}-\mathbf{x}'\|$	flops
MCG	1.872	1.847	0.1216	13 047 125
MCG + ST	1.872	1.847	0.1216	85 319
ACT	1.872	1.901	0.1568	870 431

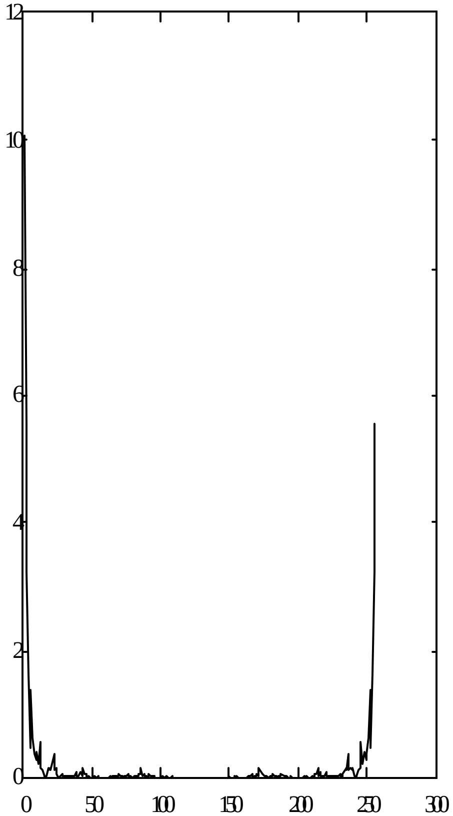
MCG Multiresolutional Conjugate Gradients

MCG + ST Multiresolutional Conjugate Gradients + Sparse Techniques

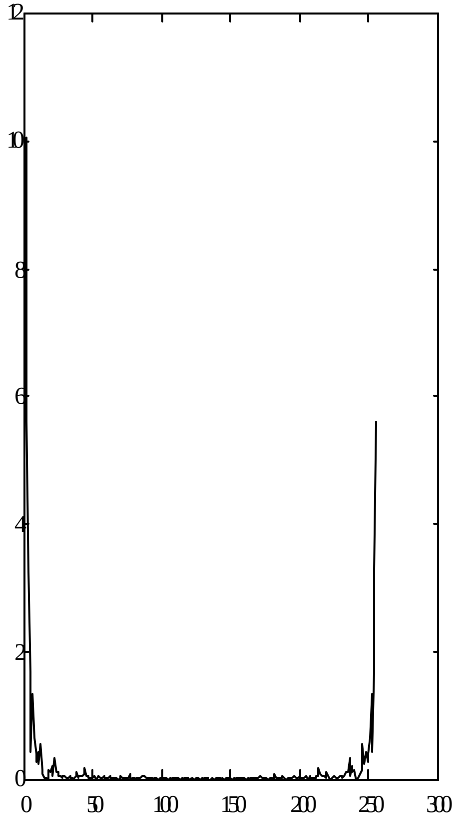
ACT Adaptive weights Conjugate gradient Toeplitz


y
(89)
x'
(128)
MCG
x'
(128)
ACT
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
-0.02
0
0.2
0.4
0.6
0.8
1

The spectra

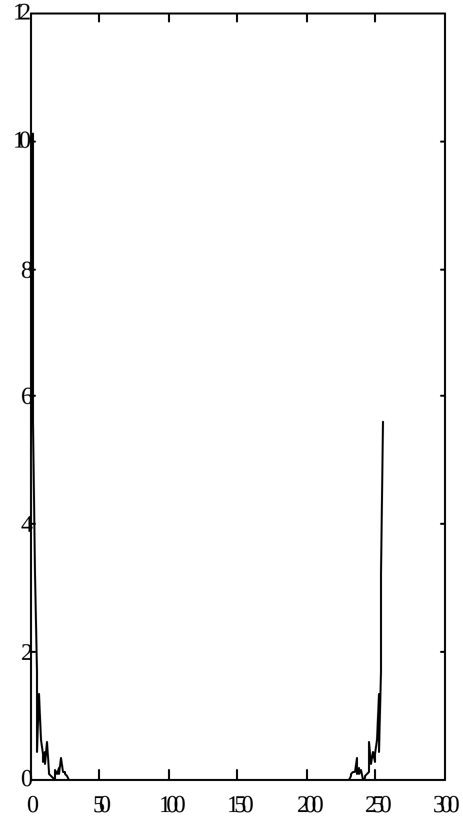


$X(e^{j\omega})$



$|X'(e^{j\omega})|$

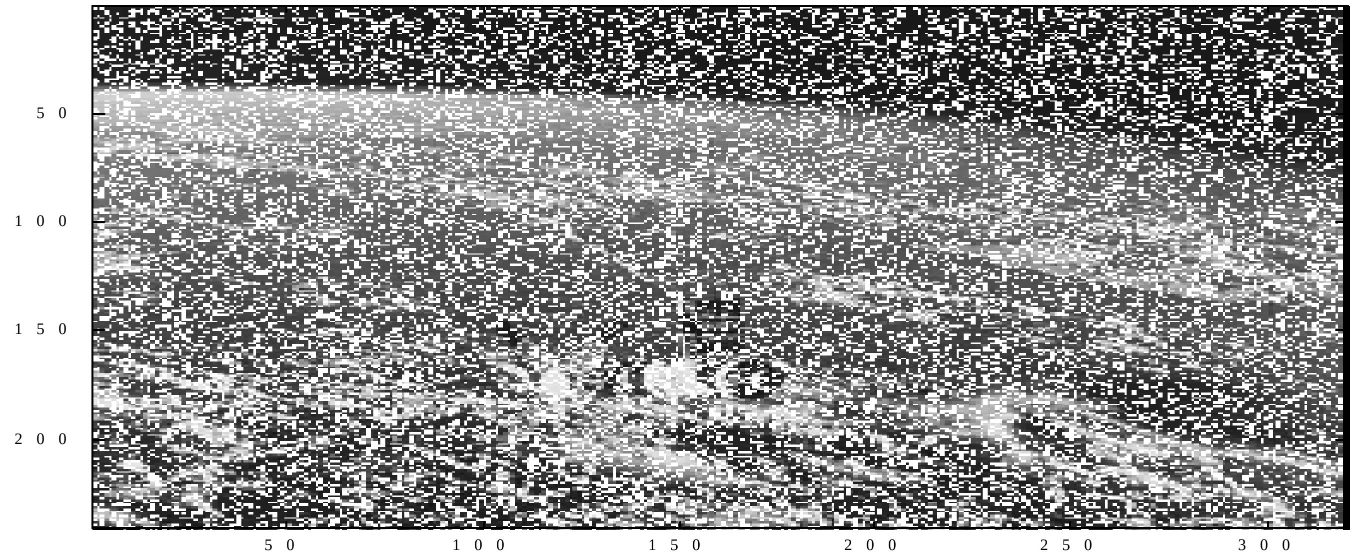
MCG



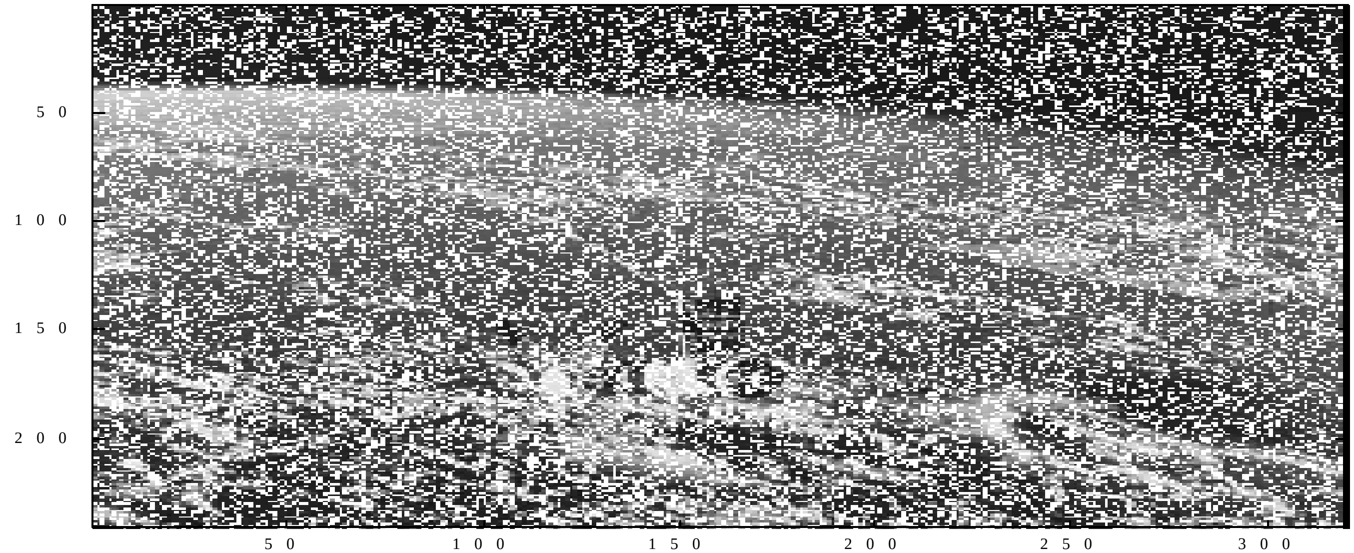
$|X'(e^{j\omega})|$

ACT

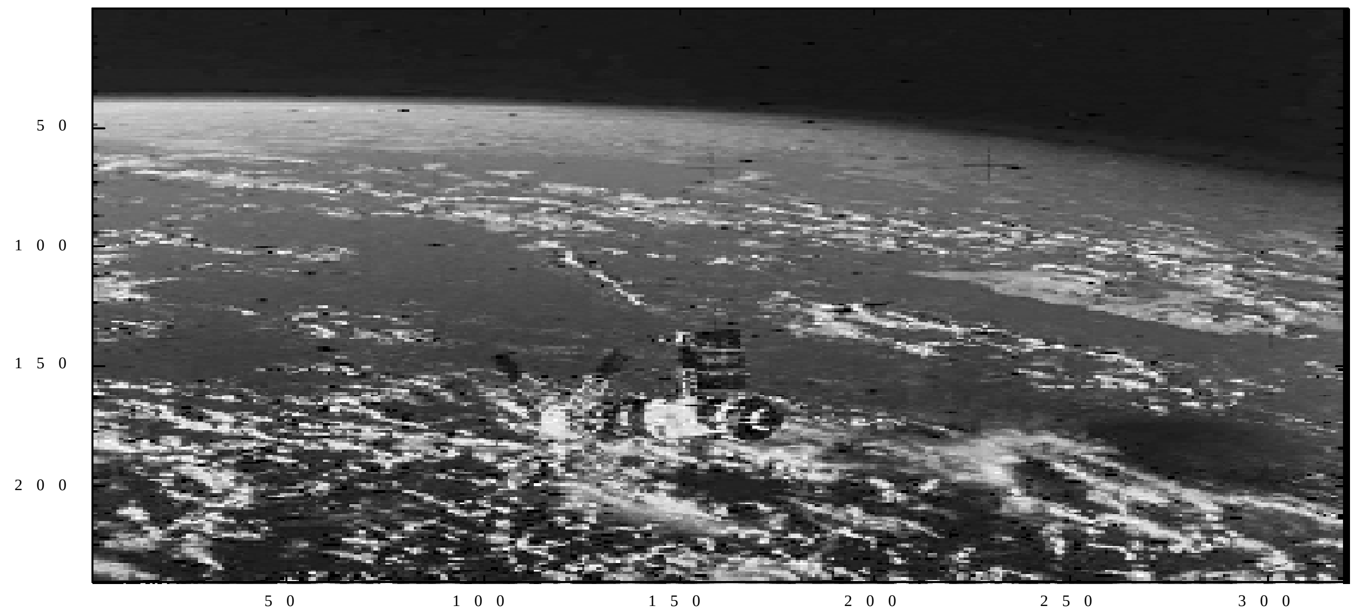
a) Original image uniformly sampled



b) Non-uniformly randomly sampled image (70% of a)



b) Non-uniformly randomly sampled image (70% of a)



c) Reconstructed image a) using MCG

Image magnifying using MBFR method of Ford & Etter

Small 64x64 picture (starting picture)

a) b) c)

Magnified 256x256 images using different wavelet basis:
(a) Haar; (b) Daubechies db2; (c) Daubechies db3.

M. Petkovski and M. Bogdanov, “Estimation of Discrete Wavelet Transform Coefficients of Non-Uniformly Sampled Signals,” *Proc. Int. Symp. on Signals, Circuits and Systems, SCS 2001*, pp. 9–14, July 11–12, 2001, Iaºi, Romania.

S. Mitevski and M. Bogdanov, “Application of Multiresolutional Basis Fitting Reconstruction In Image Magnifying,” *Proc. 9th Telecommunications Forum TELFOR 2001*, pp. 565–568, November 20–22, 2001, Belgrade, Yugoslavia.