Consistent Realignment of 3D Diffusion Tensor MRI Eigenvectors

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ABSTRACT

Diffusion tensor MR image data gives at each voxel in the image a symmetric, positive definite matrix that is denoted as the diffusion tensor at that voxel location. The eigenvectors of the tensor represent the principal directions of anisotopy in water diffusion. The eigenvector with the largest eigenvalue indicates the local orientation of tissue fibers in 3D as water is expected to diffuse preferentially up and down along the fiber tracts. Although there is no anatomically valid positive or negative direction to these fiber tracts, for many applications, it is of interest to assign an artificial direction to the fiber tract by choosing one of the two signs of the principal eigenvector in such a way that in local neighborhoods the assigned directions are consistent and vary smoothly in space.

We demonstrate here an algorithm for realigning the principal eigenvectors by flipping their sign such that it assigns a locally consistent and spatially smooth fiber direction to the eigenvector field based on a Monte-Carlo algorithm adapted from updating clusters of spin systems. We present results that show the success of this algorithm on 11 available unsegmented canine cardiac volumes of both healthy and failing hearts.

Keywords: Diffusion tensor MR processing, Monte Carlo, Spatial smoothing, consistent alignment

1. INTRODUCTION

Diffusion weighted MR imaging allows the imaging of tissue architecture by measuring the attenuation in MR signal due to the random thermal diffusion of water molecules along fiber tracts in tissue. Measuring the attenuation along several (atleast six) directions, a 3x3 symmetric, positive definite diffusion tensor matrix D is constructed at each voxel location. The principal eigenvector e_1 of this tensor indicates the direction of fastest diffusion and is taken to indicate the orientation of the underlying tissue fibre tracts. Such a tensor valued image (denoted as the diffusion tensor MR image) provides information regarding the local tissue architecture and the eigenvectors of the tensor at each voxel location provide a local coordinate system that is based on the local tissue fiber tract orientation.

For vectors $v \in \mathbb{R}^n$ we can speak of motion directed along the vector $\alpha v,\ \alpha > 0$ or opposed to the vector $-\alpha v,\ \alpha > 0$. In either case, the orientation of the vector v inherently provides an ordered coordinate positive and negative "direction" with reference to v. However, for anatomical fiber tracts, one does not have the notion of an intrinsic direction; diffusion occurs up and down the fiber tracts and thus the principal eigenvector is merely oriented along the fiber tracts. This distinction between direction (referring to a particular $+v$ or $-v$) versus orientation (referring to $\pm v$, the general local alignment of the fiber tract) is reflected in the estimated principal eigenvector and eigenvalue pair; principal eigenvalues α_i are positive valued, however, the eigenvectors can be either $+e_i$ or $-e_i$ leading to non-smooth spatially varying eigenvector field. This complicates the calculation of statistics (such as means, variances etc.) for eigenvectors in a region of interest. For several applications it is also of interest to enforce a consistent direction to the principal eigenvector field (over the available orientation) such that eigenvectors from adjacent locations point in the same "direction" and thus vary smoothly in space. Doing so will in effect enforce a locally consistent direction, albeit artificial, to the underlying fiber tracts as well.

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Then, one can speak of local motion along the positive and negative directions along the fiber tract. In many applications, such as measuring fiber helical angle, mid-wall calculations and some visualization techniques in computational cardiac anatomy, this preprocessing step can be very useful in developing new algorithms and to simplify existing algorithms and to calculate consistent statistics of eigenvectors in local regions.

One proposed approach to making local statistical calculations on eigenvectors insensitive to inherent sign ambiguity relies on using a second order dyadic tensor representation[1] given by $\alpha_i e_i e_i^t$ and the degree of similarity between two dyadic tensors is then found to be the dyadic tensor product $\alpha_i e_i e_i^t : \alpha_i^{'} e_i e_i^{'t} = Trace(\alpha_i e_i e_i^t \alpha_i^{'} e_i e_i^{'t}) = \alpha_i \alpha_i^{'} (e_i \cdot e_i^{'})^2$. We describe here an algorithm that, instead of developing calculations that are insensitive to the sign-ambiguity, re-assign a locally consistent direction to each eigenvector so as to remove the sign-ambiguity from the data itself. This is done by flipping the eigenvectors (with a sign change) to change direction if it points "away" from its neighbors. This process involves only a sign change of the original vector and thus preserves its orientation.

2. THEORY

The realignment procedure we have developed involves flipping the sign of those vectors that are not pointing in the same direction as their local neighbors. We cast this problem in the spirit of the well-known Ising model, where given a location x, and a vector $e_i(x)$, the direction in which the vector $e_i(x)$ points relative to its neighbors is akin to the spin ± 1 on 3D voxel lattice. When adjacent vectors point in the same direction, it is analogous to adjacent "spins" on the lattice taking the same value, either +1 or -1. The goal that the vectors (coming from the eigenvectors of diffusion tensor matrix data) form a smoothly varying vector-field is then analogous to all the spins equilibrating to the same value of +1 or -1 over the entire volume. These "artificial" spins on neighboring lattice sites in the eigenvector field interact via an interaction energy that is defined to be low for vectors pointing in the same direction and high if they point in opposite direction. We define a simple interaction energy that is of the form

$$E(e_i, x) = -\sum_{y \in S_x} e_i(x)^t e_i(y) \tag{1}$$

where S_x is a neighborhood of lattice positions connected to the central lattice position x and thus interacting with and contributing energy to it. We have experimented with several neighborhood definitions, and in the end, found that choosing the nearest six neighbors in 3D is computationally the best for solving this problem i. e. $S_x = \{(x_1 \pm 1, x_2, x_3), (x_1, x_2 \pm 1, x_3), (x_1, x_2, x_3 \pm 1)\}$. The global energy is then defined as being $E = \sum_{x \in \Omega} E(e_i, x)$. We seek the minimum of this energy via a modification to the Swendson-Wang (SW)[2] type algorithm proposed by Wolff.[3] Both SW and Wolff algorithms are cluster update algorithms. They are similar to simulated annealing methods, but instead of performing single spin updates following a Metropolis procedure, they build clusters and update all the spins in the cluster simultaneously. The SW algorithm assigns all the spins in the volume to connected clusters of like spins and then these clusters are flipped as needed; this is shown to have extraordinarily small critical slowing down in equilibration of the system that the single spin update methods suffer from. The modification of the SW algorithm proposed by Wolff builds and updates a single cluster at a time of like spins which is then flipped. Upon completion, it exits with the largest cluster of "aligned" eigenvectors pointing in the same direction.

3. METHOD

Our adaptation of the Wolff algorithm is given in the flowchart in Figure 1 and was implemented in C++. An initial temperature of $T = 1/\beta = 5.0$ and a cooling rate of $\delta T = 2 \times 10^{-4}$ were used. Random numbers were generated (uniformly distributed in $[0, 1]$) using the Boost C++ Library[4] with machine-time used as the required seed. The same random number generator was also used to generate a random location in the volume from where to start a cluster to grow by appropriate scaling to be in the range of voxel coordinates followed by a cast as integer.

We have applied this method on diffusion tensor MR images generated from 11 normal and the failing canine hearts. Each excised canine heart was placed in a low-MR-signal medium. Images were acquired with a 4-element knee phased array coil on a 1.5 T GE CV/I MRI Scanner, using a 3D fast spin echo diffusion tensor MRI pulse

sequence. Diffusion weighted data was acquired along 16-17 gradient directions each from which the tensor and its eigenvectors were calculated at each voxel location using standard methods. The size of each image was on the order of $140 \times 256 \times 256$. Since the data came from excised canine hearts sitting in a jar, the datasets were preprocessed to remove background "noise" by summing the eigenvalues and using a binary threshold on this summation. Then a connectedness filter was applied and connected regions with volume below a threshold were removed. The resulting binary volume was used as a mask to remove background from the input dataset.

4. RESULTS

We have performed the realignment of eigenvectors from 11 available DT-MRI 3D volumes of both healthy and failing deceased canine hearts. The images in Figures 2 , 3, 4 visualize the interaction energy for the realignment of the principal, secondary and tertiary eigenvector at each voxel calculated from Equation 1 and normalized over the size of the neighborhood i. e. the interaction energy value at each voxel location is normalized to be in the range $[-1, 1]$, -1 indicating all vectors in the neighborhood point in the same direction, $+1$ indicating that all vectors in the neighborhood point in the opposite direction. This range $[-1, 1]$ is mapped to the grayscale values $[0, 255]$ with -1 mapping to black (grayscale value 0) and 1 mapping to white (grayscale value 255). Locations where all neighborhood vectors are pointing in opposing directions show as a white pixel in these images and where they are all pointing in the same direction show as black pixels. Figure 5 shows the interaction energy $E = \sum_{x \in \Omega} E(e_i, x)$ over the entire volume before and after processing. Figure 6 shows the size of the largest connected cluster of "like" spins, which in our case are eigenvectors pointing in the same direction, as compared to the number of voxel locations in the data. From these figures, we see that there is a significant lowering of energy over the whole volume concurrent with the growth of a single cluster encompassing almost all the voxels of the data indicating an increase of spatial smoothness of the vector field. It is not necessary to reorient the tertiary eigenvector independently as it can always be calculated from the reoriented primary and secondary eigenvectors at each voxel location. This algorithm has been observed to be robust across all 11 whole heart DTMRI datasets available to us. The algorithm is also reasonably fast, taking on the order of one hour on a single CPU 2.8 Ghz desktop computer and uses less than 1G of RAM. If the heart datasets were pre-segmented to remove the papillary muscles and background noise, then the run-time reduces to the order of ten to twenty minutes for each dataset.

5. CONCLUSION

We have described here a straightforward method for re-orienting the eigenvectors of the diffusion tensor to enforce a locally consistent direction, albeit artificial, to the underlying anatomical fiber tracts. With this method, adjacent eigenvectors are flipped with a sign change if needed so that the angle between them falls in $[-\pi/2, \pi/2]$. This leads to a spatially smooth vector field for each of the principal, secondary and tertiary eigenvectors and is likely to be useful as a preprocessing step for several applications such as calculation of statistics of the eigenvectors and development of new algorithms for diffusion tensor based tractography and visualization of anatomical fiber architecture. We demonstrate here our method on diffusion tensor MR data from the heart. We expect that due to its general nature, this method can also be applied to brain diffusion tensor MR data or similar vector fields generated from other applications.

ACKNOWLEDGMENTS

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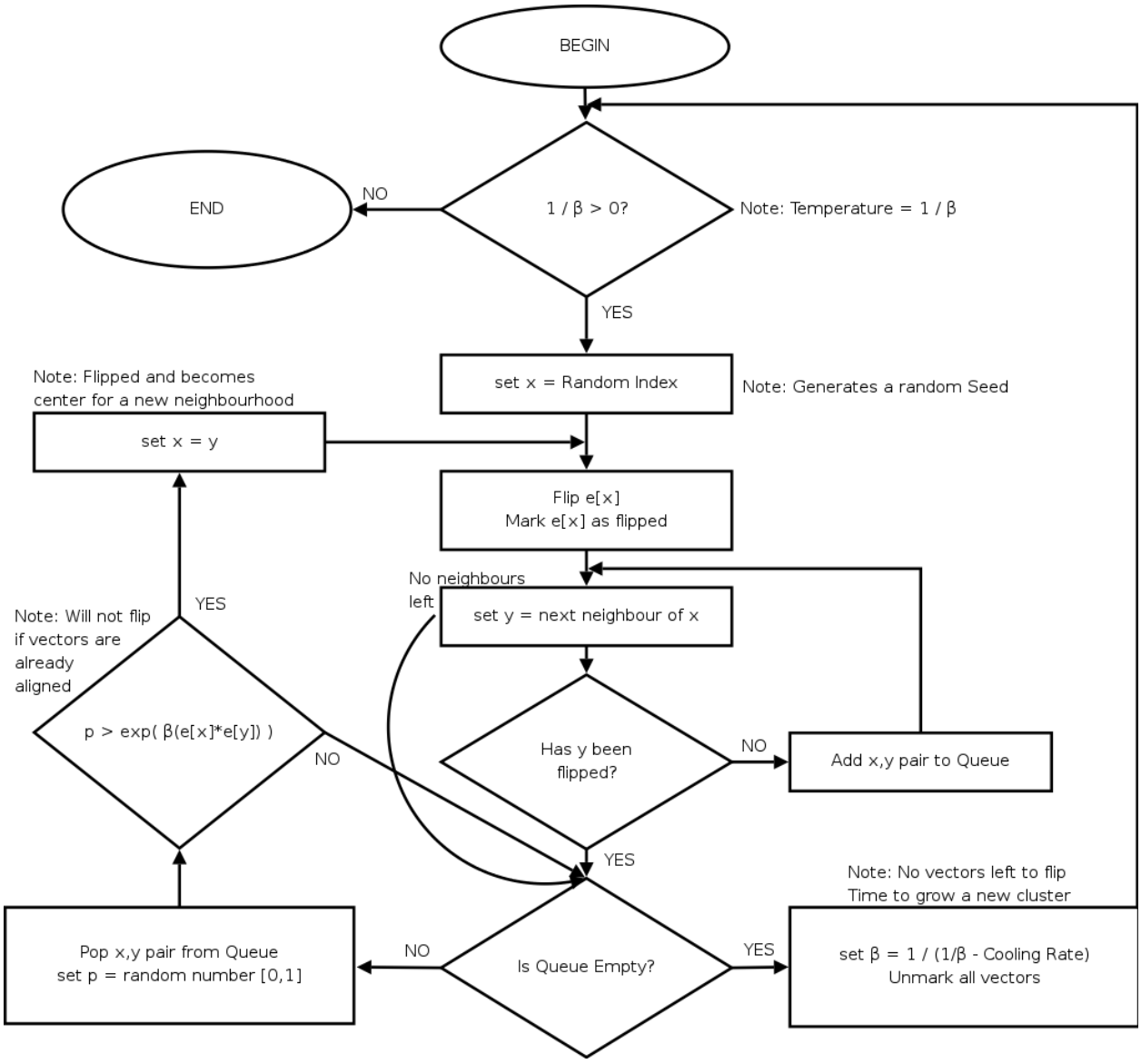
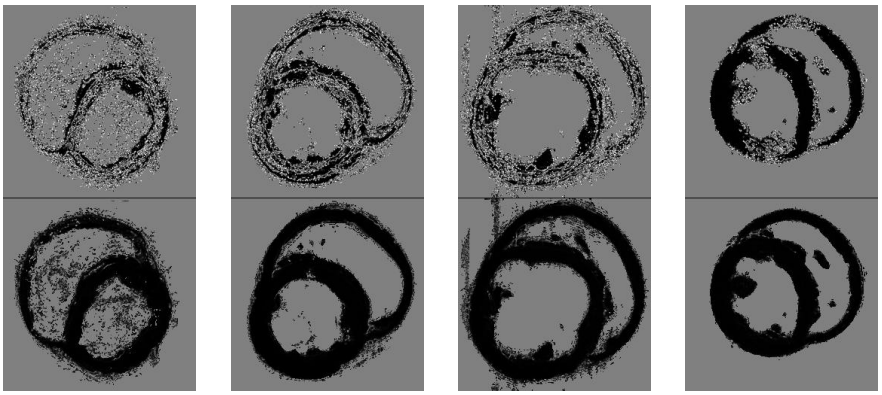
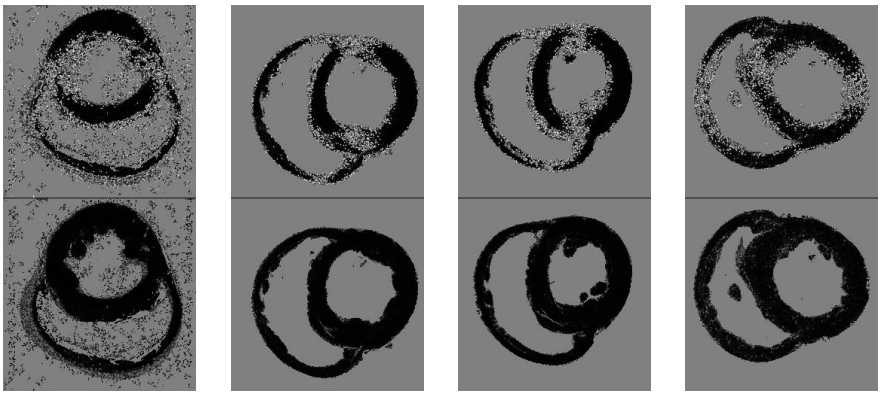


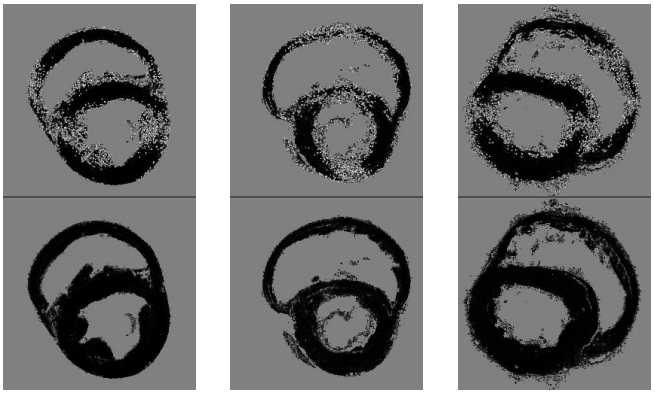
Figure 1. The flowchart above gives an overview of our adaptation of the Wolff algorithm for cluster growth and updating to provide local alignment of diffusion-tensor generated eigenvector fields. The algorithm visits random locations in the image and tries to enlarge the size of the cluster of vectors starting from that location such that adjacent neighbors are pointing in the "same" direction i. e. flipping vectors if needed so that the angle between adjacent vectors is in $[-\pi/2, \pi/2]$.



(a) 1 (b) 2 (c) 3 (d) 4

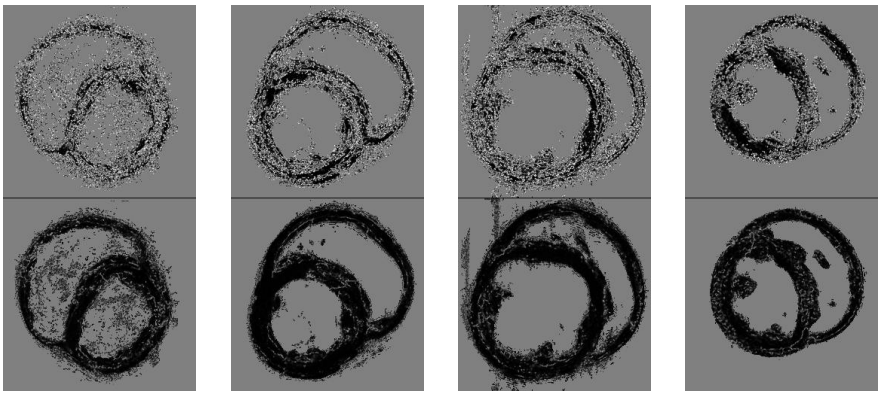


(e) 5 (f) 6 (g) 7 (h) 8

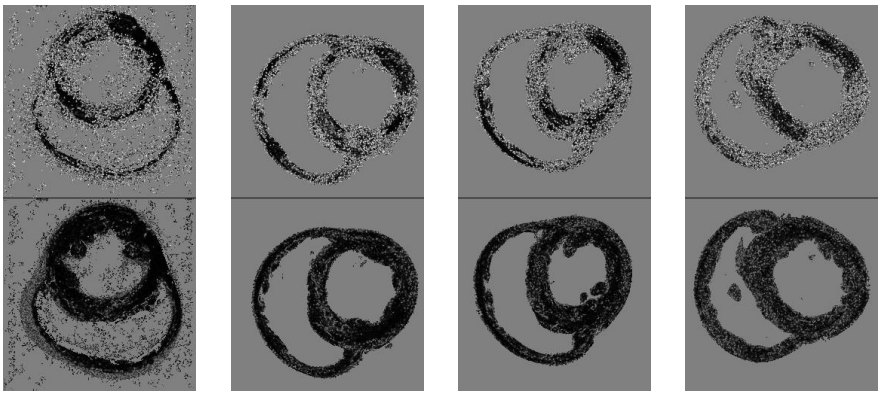


(i) 9 (j) 10 (k) 11

Figure 2. Re-aligning the principle eigenvector field to enforce local smoothness in direction for 11 heart diffusion tensor datasets. The plot on the top for each panel shows the interaction energy at each voxel calculated from Equation 1 and normalized over the size of the neighborhood i. e. the interaction energy value at each voxel location is normalized to be in the range $[-1, 1]$, -1 indicating all vectors in the neighborhood point in the same direction, $+1$ indicating that all vectors in the neighborhood point in the opposite direction. This range $[-1, 1]$ is mapped to the grayscale values $[0, 255]$ with -1 mapping to black(grayscale 0) and 1 mapping to white (grayscale 255). Locations where all neighboring vectors are pointing in opposing directions show as a white pixel in these images and where they are all pointing in the same direction show as black pixels.



(a) 1 (b) 2 (c) 3 (d) 4



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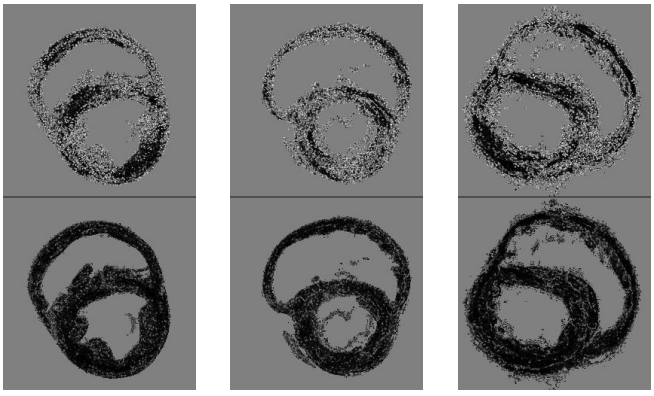
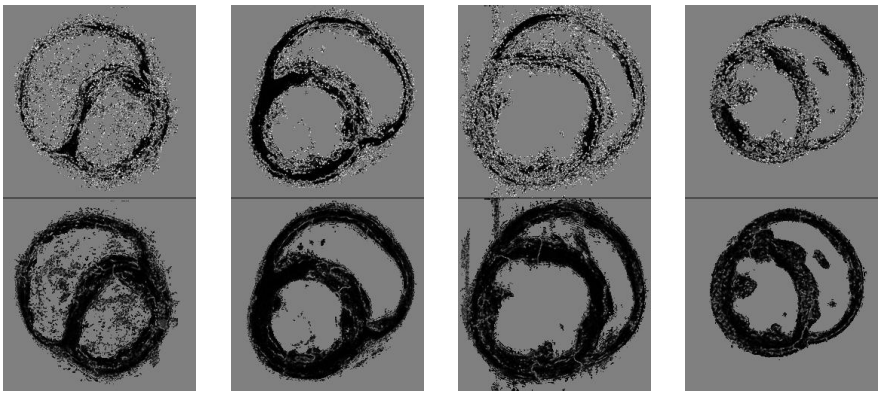
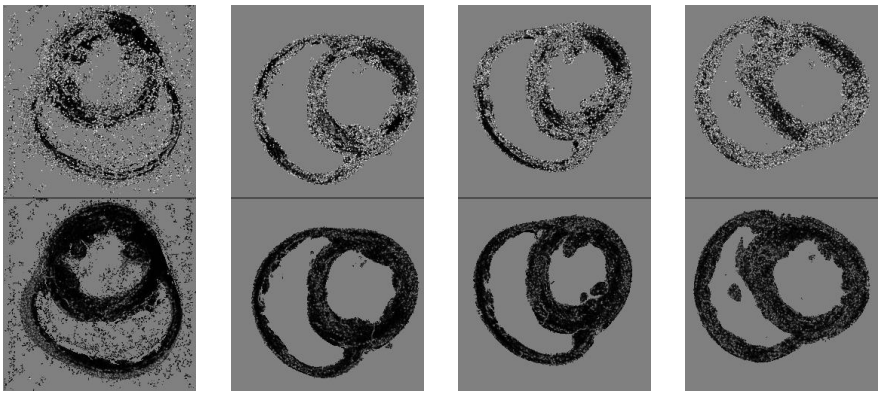


(i) 9 (j) 10 (k) 11

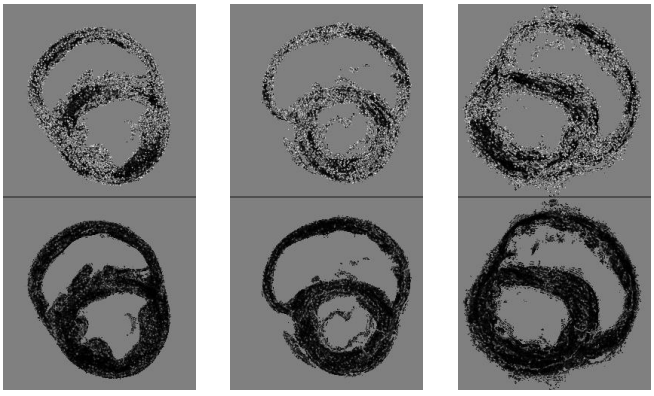
Figure 3. Re-aligning the secondary eigenvector field to enforce local smoothness in direction for 11 heart diffusion tensor datasets. The plot on the top for each panel shows the interaction energy at each voxel calculated from Equation 1 and normalized over the size of the neighborhood i. e. the interaction energy value at each voxel location is normalized to be in the range $[-1, 1]$, -1 indicating all vectors in the neighborhood point in the same direction, $+1$ indicating that all vectors in the neighborhood point in the opposite direction. This range $[-1, 1]$ is mapped to the grayscale values $[0, 255]$ with -1 mapping to black(grayscale 0) and 1 mapping to white (grayscale 255). Locations where all neighboring vectors are pointing in opposing directions show as a white pixel in these images and where they are all pointing in the same direction show as black pixels.



(a) 1 (b) 2 (c) 3 (d) 4



(e) 5 (f) 6 (g) 7 (h) 8



(i) 9 (j) 10 (k) 11

Figure 4. Re-aligning the tertiary eigenvector field to enforce local smoothness in direction for 11 heart diffusion tensor datasets. The plot on the top for each panel shows the interaction energy at each voxel calculated from Equation 1 and normalized over the size of the neighborhood i. e. the interaction energy value at each voxel location is normalized to be in the range $[-1, 1]$, -1 indicating all vectors in the neighborhood point in the same direction, $+1$ indicating that all vectors in the neighborhood point in the opposite direction. This range $[-1, 1]$ is mapped to the grayscale values $[0, 255]$ with -1 mapping to black(grayscale 0) and 1 mapping to white (grayscale 255). Locations where all neighboring vectors are pointing in opposing directions show as a white pixel in these images and where they are all pointing in the same direction show as black pixels.

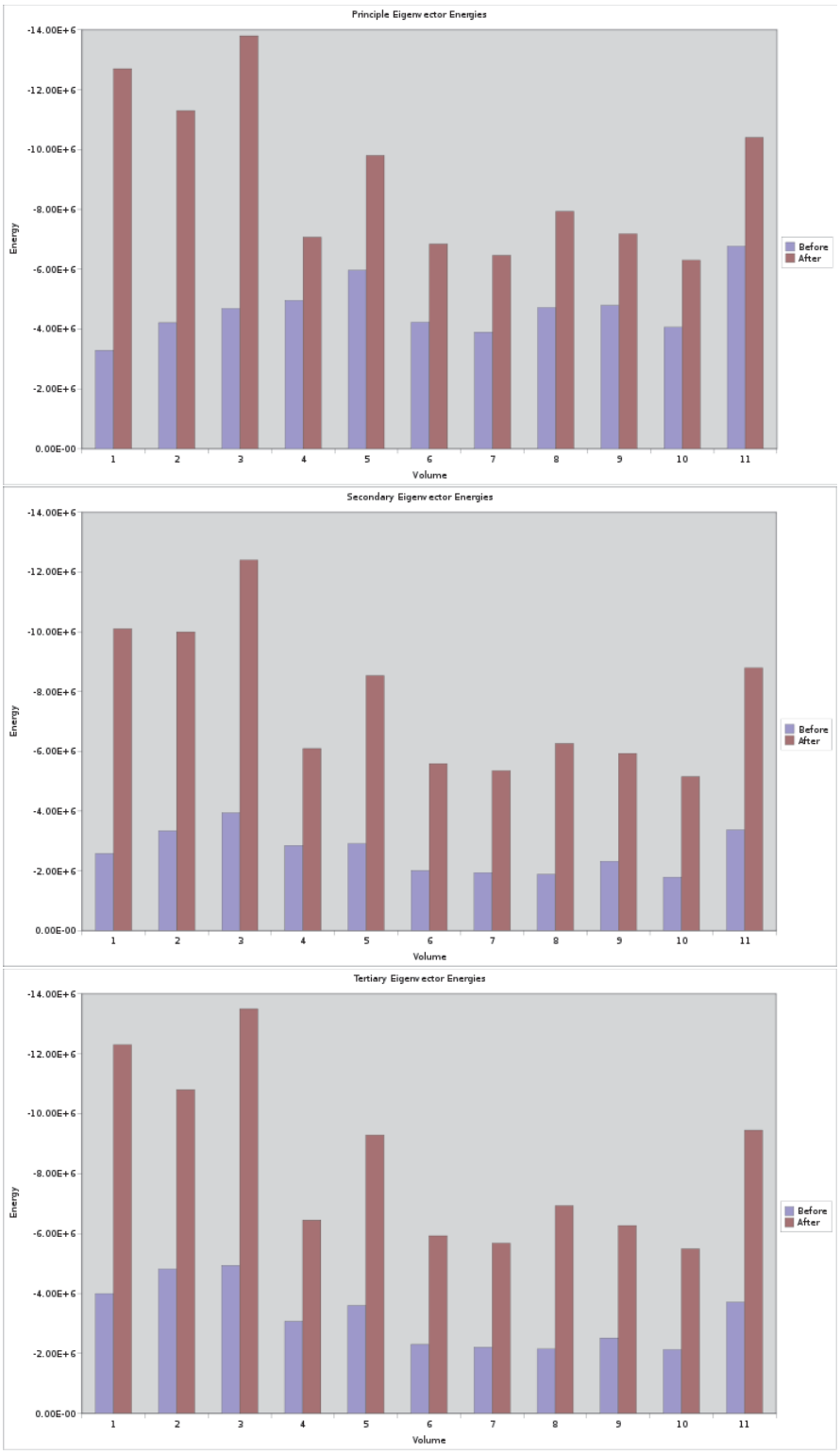


Figure 5. Whole 3D volume energy $E = \sum_{x \in \Omega} E(e_i, x)$ before and after re-alignment for 11 available heart datasets. The bar plots above show that there is a decrease in energy after the realignment procedure indicating equilibration to a more spatially smooth eigenvector field. Note that this realignment only reverses the direction of the eigenvector where needed with a sign change $e_i \mapsto -e_i$, and does not change the original orientation of the vectors in space.

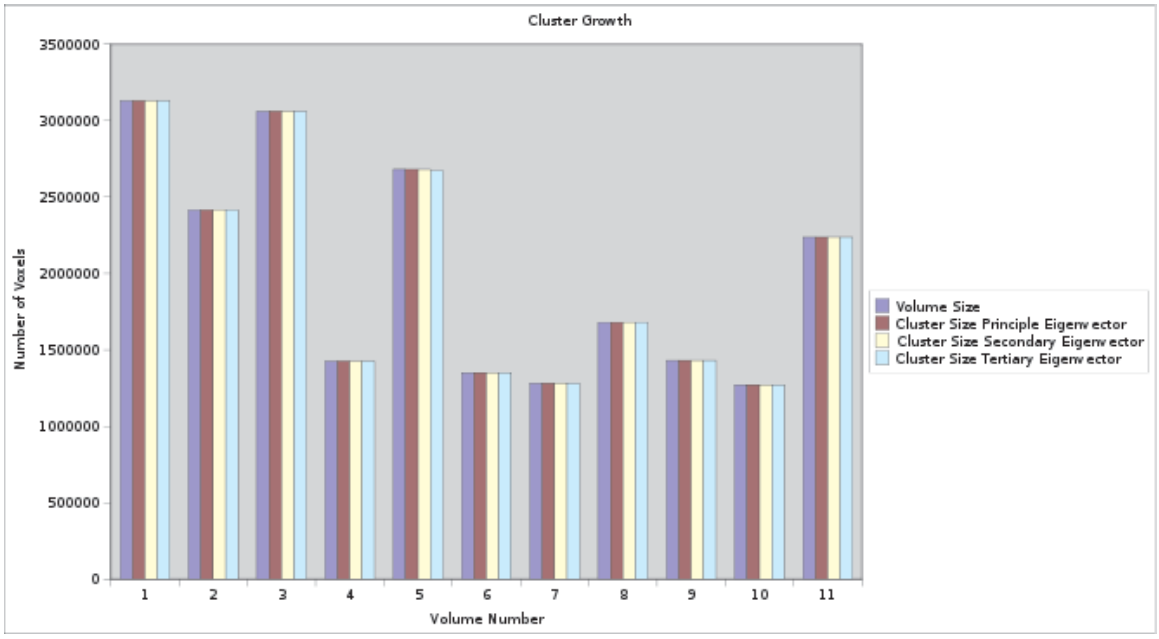


Figure 6. Size of the largest cluster of "like" spins is compared to the total number of voxels on which the re-alignment algorithm proposed here was run. This shows that the algorithm was able to find a cluster of "like" oriented vectors of size comparable to the total numbers of voxels in the dataset.